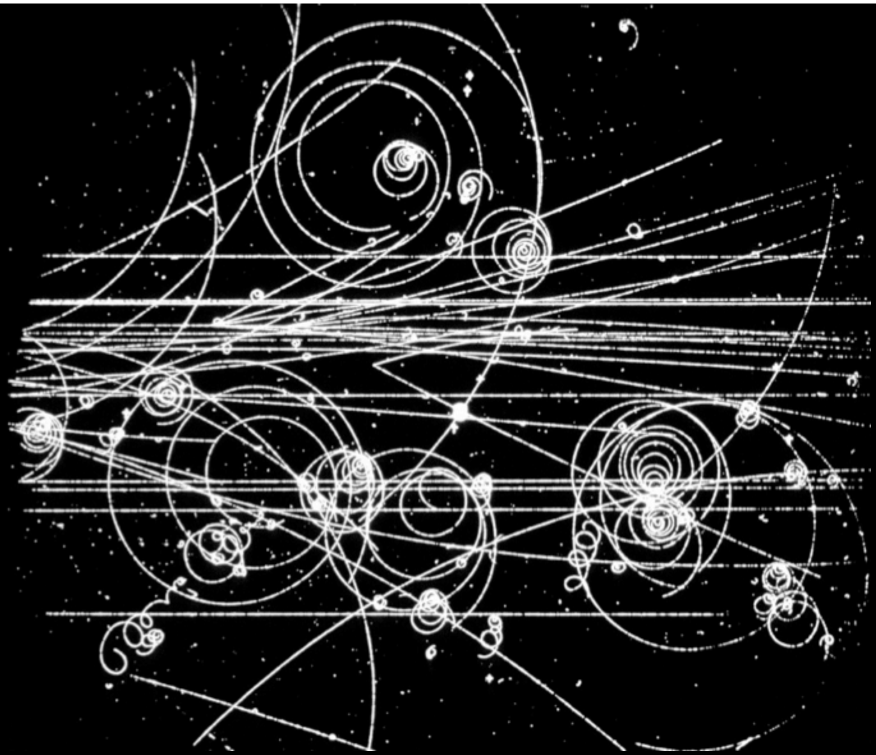
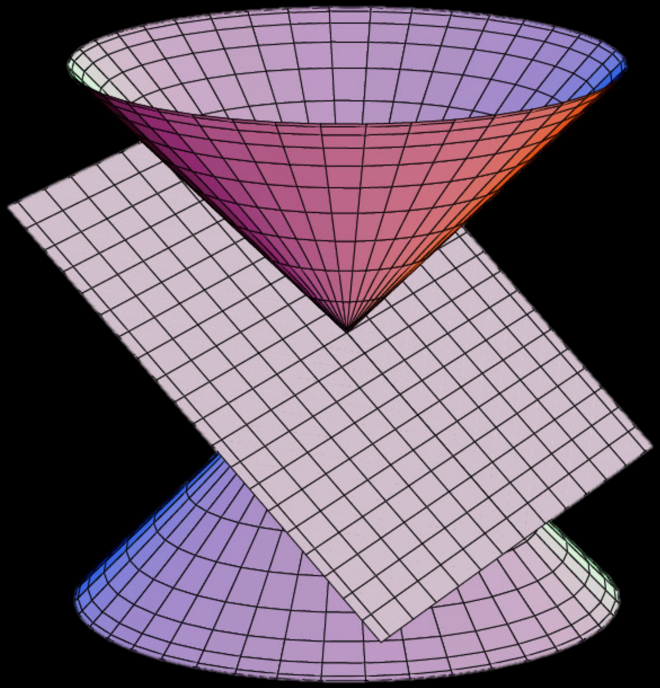


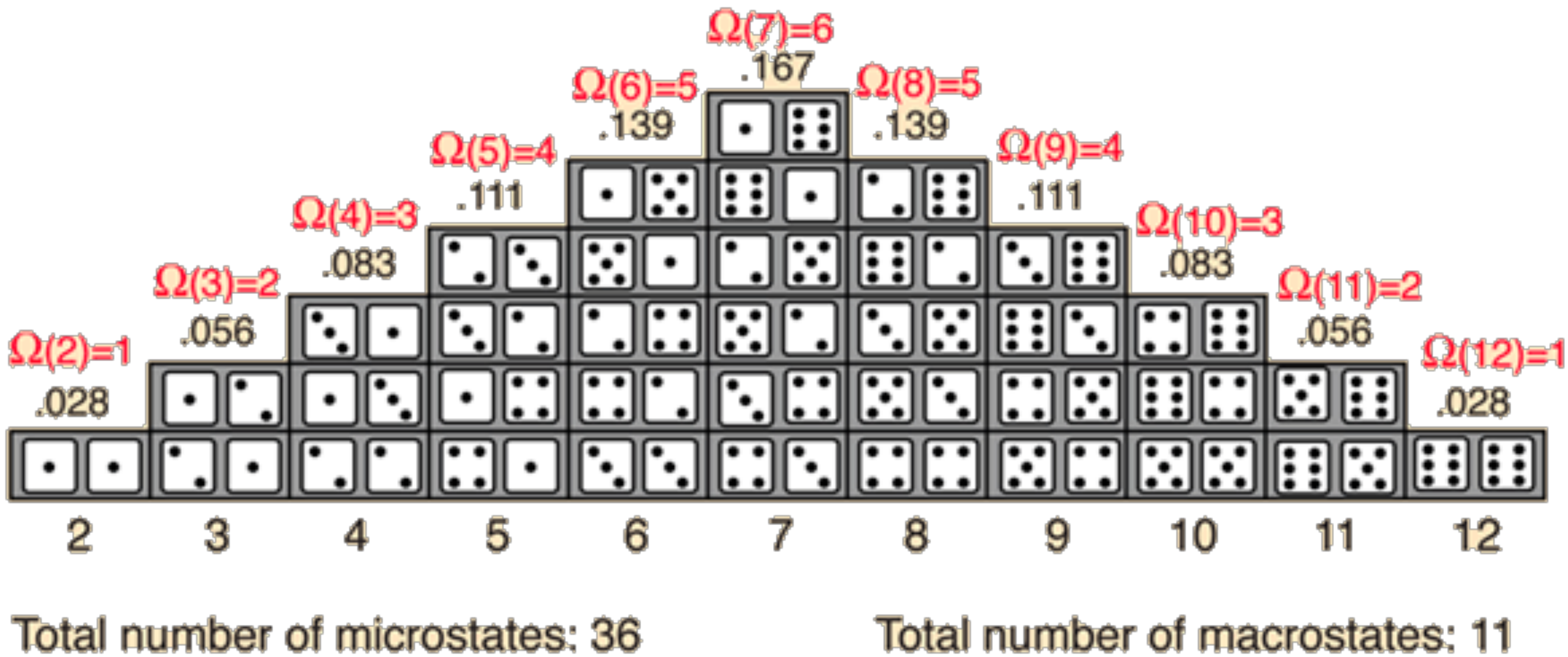


Modern Physics (Phys. IV): 2704

Professor Jasper Halekas
Van Allen 70

MWF 12:30-1:20 Lecture

Counting Macrostates/Microstates



Binomial Coefficients

Number of ways to choose 2 of 5 people

A B C D E

First pick one - 5 possibilities
Then pick second - 4 possibilities

Possible picks $= 5 \times 4$

But AB = BA (double counting)

Unique possible picks $= 5 \times 4 / 2$

$$\frac{5 \times 4}{2} = \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 2 \times 1} = \frac{5!}{3!\,2!}$$

Binomial Coefficient

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

Binomial Coefficients

- The binomial coefficient is the number of ways of picking k unordered outcomes from n possibilities ("n choose k")

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Multinomial Coefficients

Number of ways to put 5 people in 3 groups, w/ 2 in first bin, 2 in second, and 1 in 3rd.

- First pick 2 of 5

$$\Rightarrow \binom{5}{2} = \frac{5!}{3!\,2!}$$

- Then pick 2 of 3

$$\binom{3}{2} = \frac{3!}{2!\,1!}$$

- Then pick 1 of 1

$$\binom{1}{1} = \frac{1!}{1!}$$

- multiply $\frac{5!}{3!\,2!} \frac{3!}{2!\,1!} \frac{1!}{1!}$

$$= \frac{5!}{2!\,2!\,1!}$$

Generally $\binom{n}{k_1, \dots k_m} = \frac{n!}{k_1! \times \cdots k_m!}$

Multinomial Coefficients

- The multinomial coefficient is the number of ways of depositing n distinct objects into m distinct bins, with k_1 objects in the first bin, k_2 objects in the second bin, …

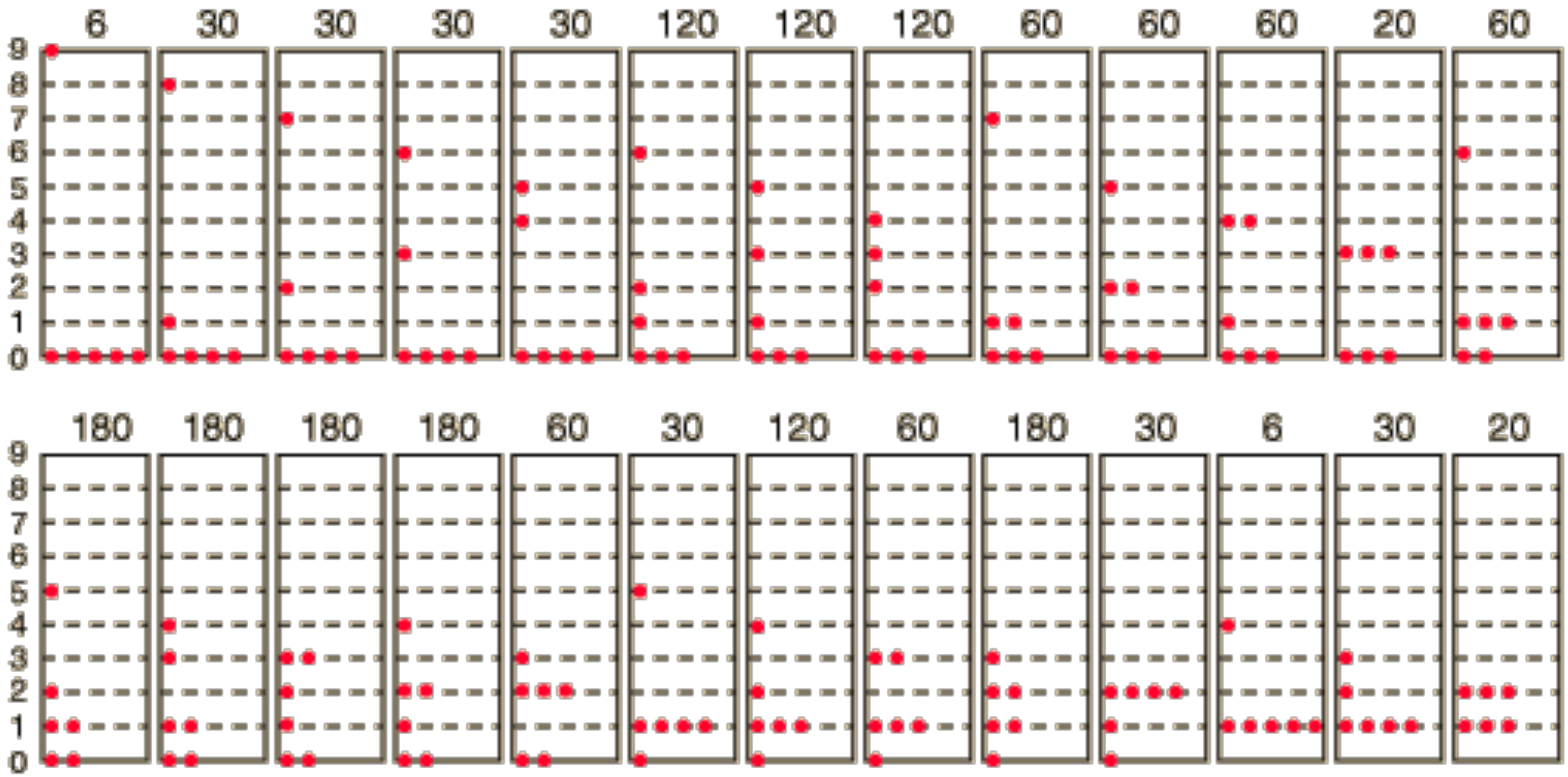
$$\binom{n}{k_1, k_2, \ldots, k_m} = \frac{n!}{k_1!\, k_2! \cdots k_m!}$$

Caveat

- If you find 10.1-10.2 hard to follow, I recommend the HyperPhysics web site for an alternate treatment (the following slides are partially derived from this site):
  - http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/disbol.html

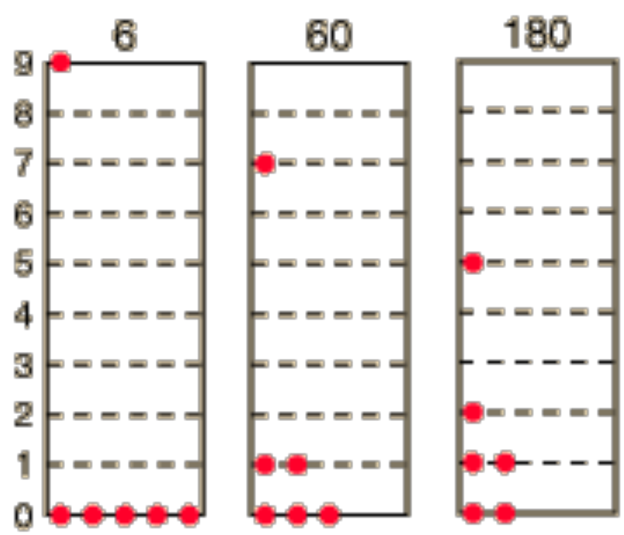
Energy Distribution (n=9, k=6)

9 units of energy distributed among six particles (i.e. <E> = 1.5)



Counting Microstates

$$\text{Number of microstates} = \frac{N!}{n_1!\, n_2!\, n_3! \ldots}$$ where N = total number of particles, n_i = number of particles in level i

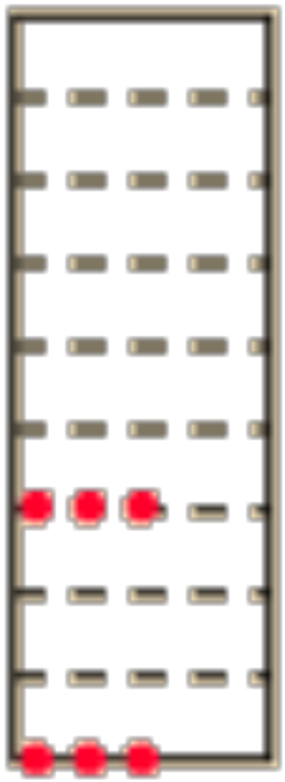


$$\frac{6!}{5!\,1!} = 6 \qquad \frac{6!}{3!\,2!\,1!} = 60 \qquad \frac{6!}{2!\,2!\,1!\,1!} = \frac{6\cdot5\cdot4\cdot3\cdot2}{4} = 180$$

Assumes particles are “distinguishable”

Concept Check

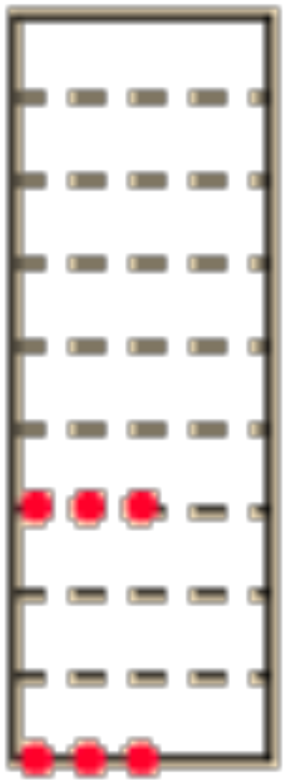
- How many different microstates are in this macrostate?



A. 6
B. 20
C. 30
D. 60
E. 180

Concept Check

- How many different microstates are in this macrostate?



A. 6
B. 20
C. 30
D. 60
E. 180

Total Multiplicity

Macrostates	Multiplicity
3 + 0 + 0	$3!/1!2! = 3$
2 + 1 + 0	$3!/1!1!1! = 6$
1 + 1 + 1	$3!/3! = 1$
	10

Same as $\binom{5}{2} = \frac{5!}{3!2!} = 10$

3 + 0 + 0	2 + 1 + 0	1 + 1 + 1
11000	100101	01010
10001	101001	
00011	110010	
	110100	
	010011	
	001011	

Total Multiplicity of Microstates

$$\Omega(N,q) = \binom{q+N-1}{q} = \frac{(q+N-1)!}{q!(N-1)!}$$

This is the formula for “distinguishable” particles

E.g. N = 4 particles, q = 8 units of energy

Concept Check

- How many total ways can I distribute three units of energy among three distinguishable particles?

- 4
- 6
- 10
- 20

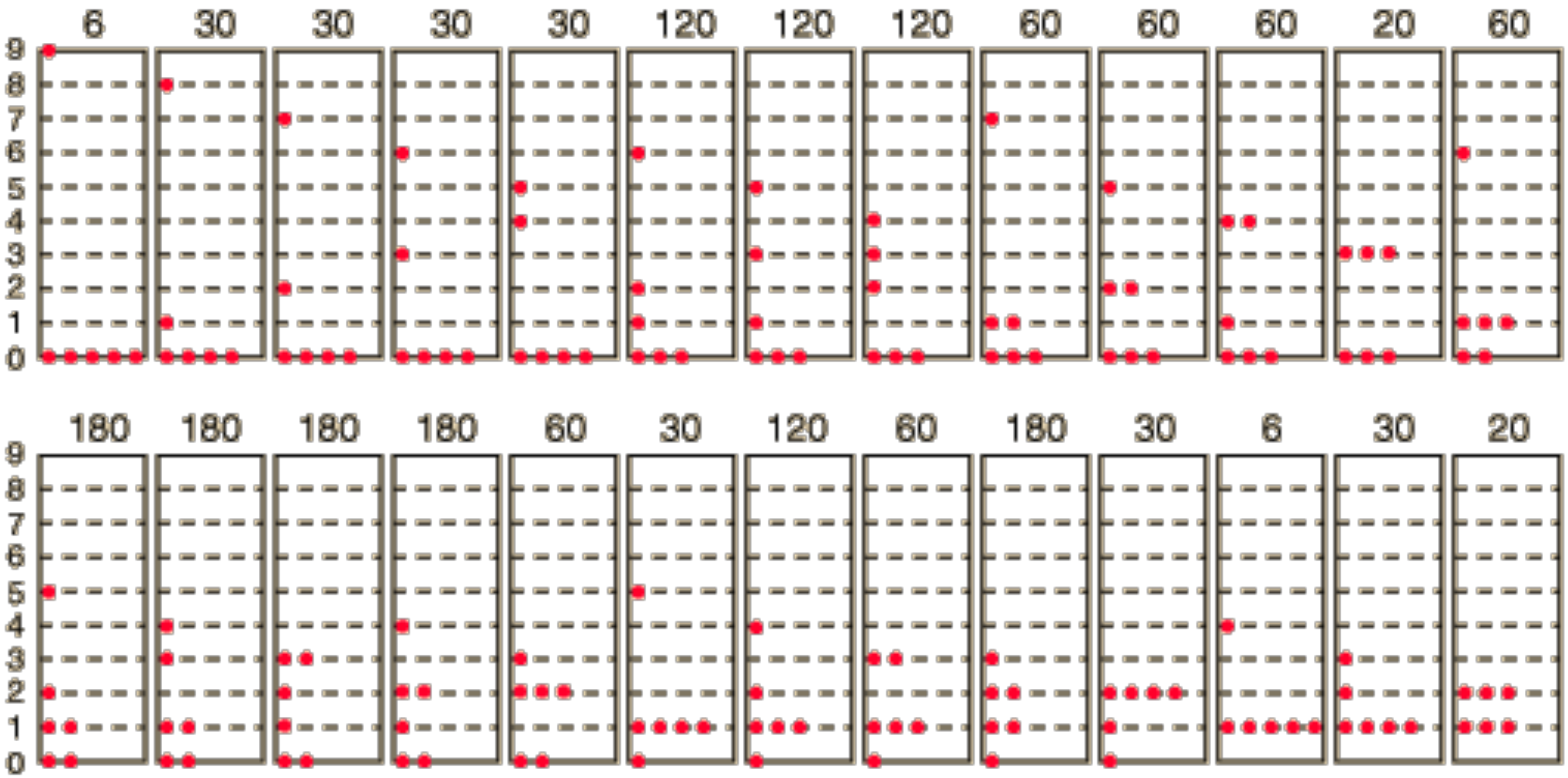
Concept Check

- How many total ways can I distribute three units of energy among three distinguishable particles?

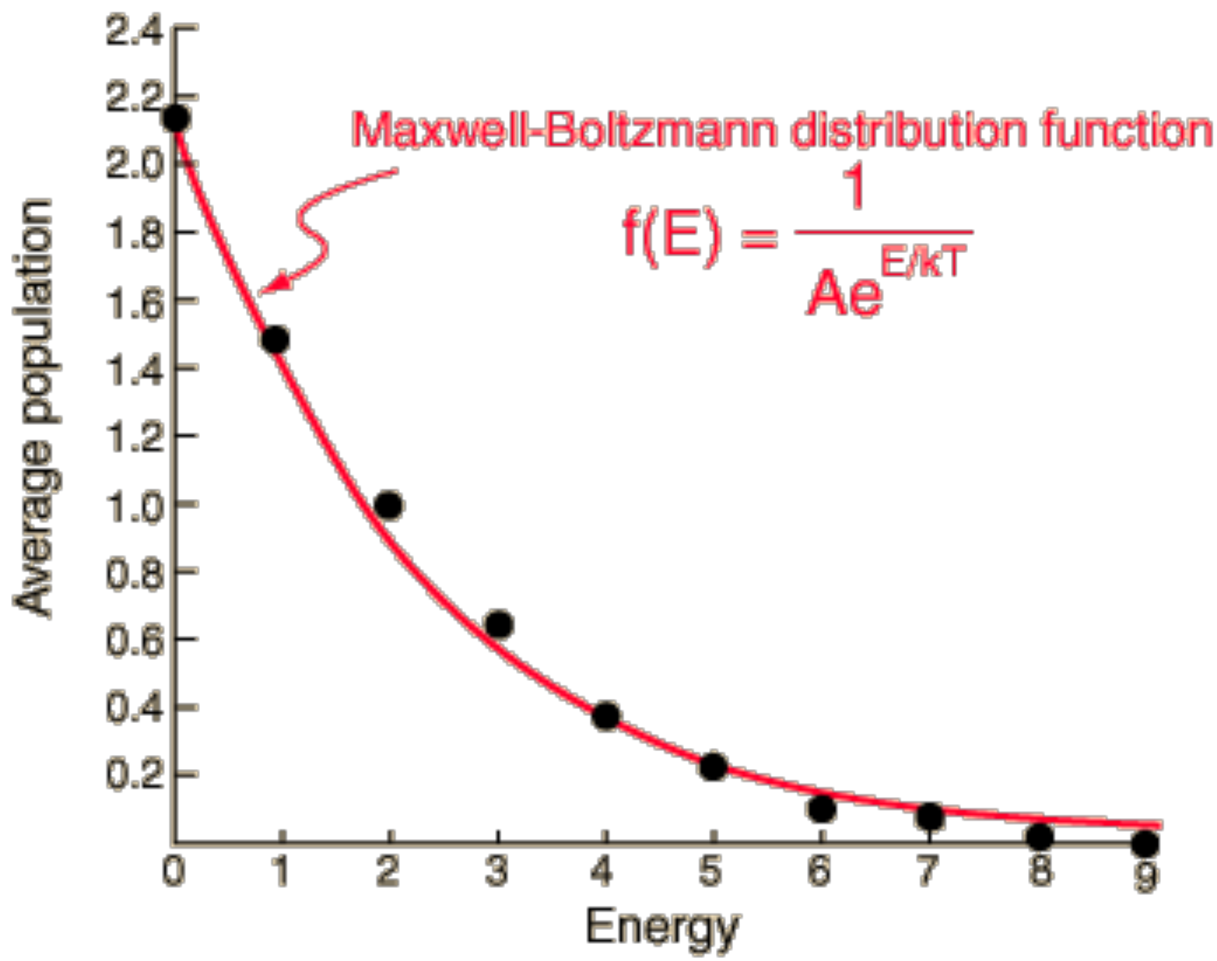
- 4
- 6
- 10
- 20

Energy Distribution (n=9, k=6)

2002 total states to count!



Maxwell-Boltzmann Distribution

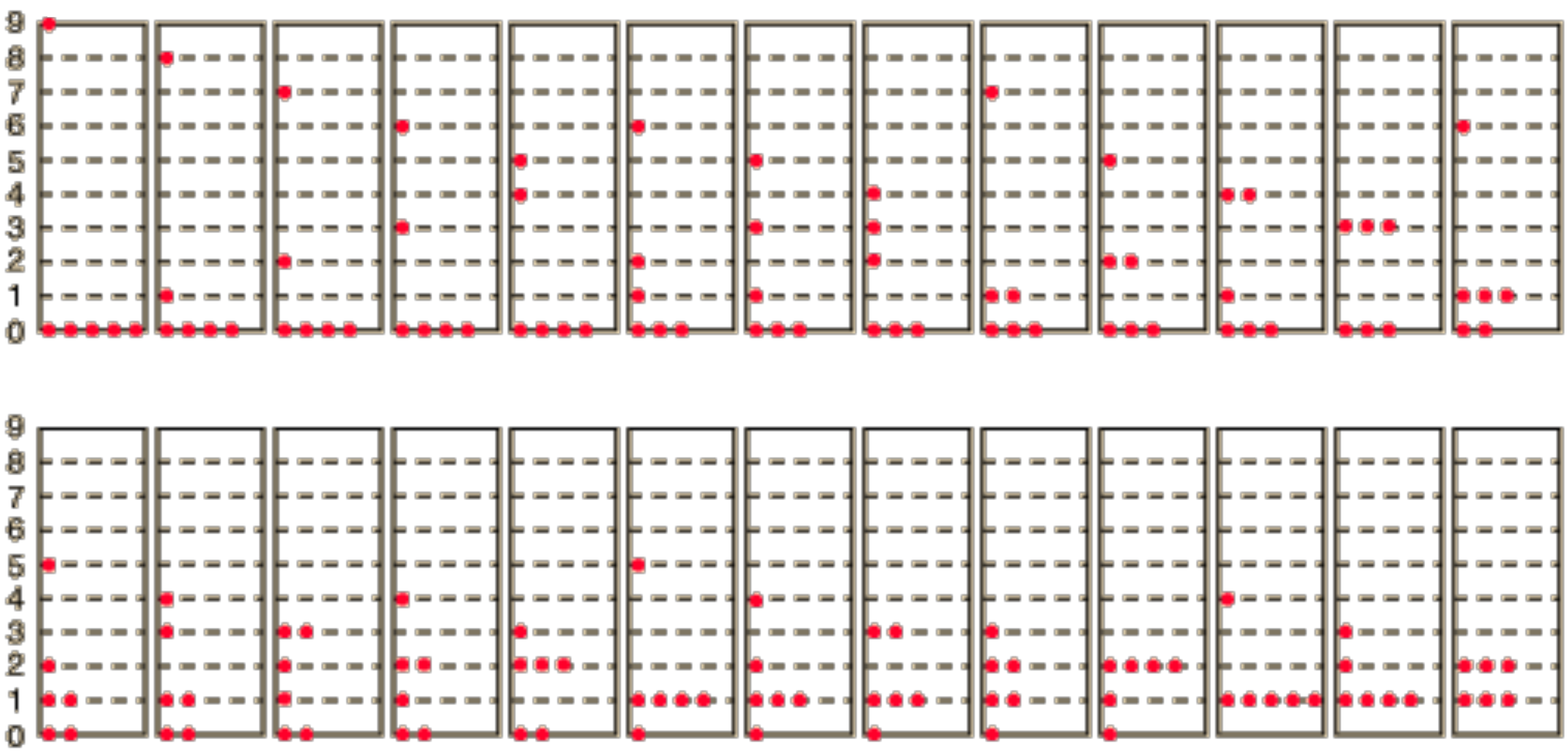


Quantum Particles are Indistinguishable

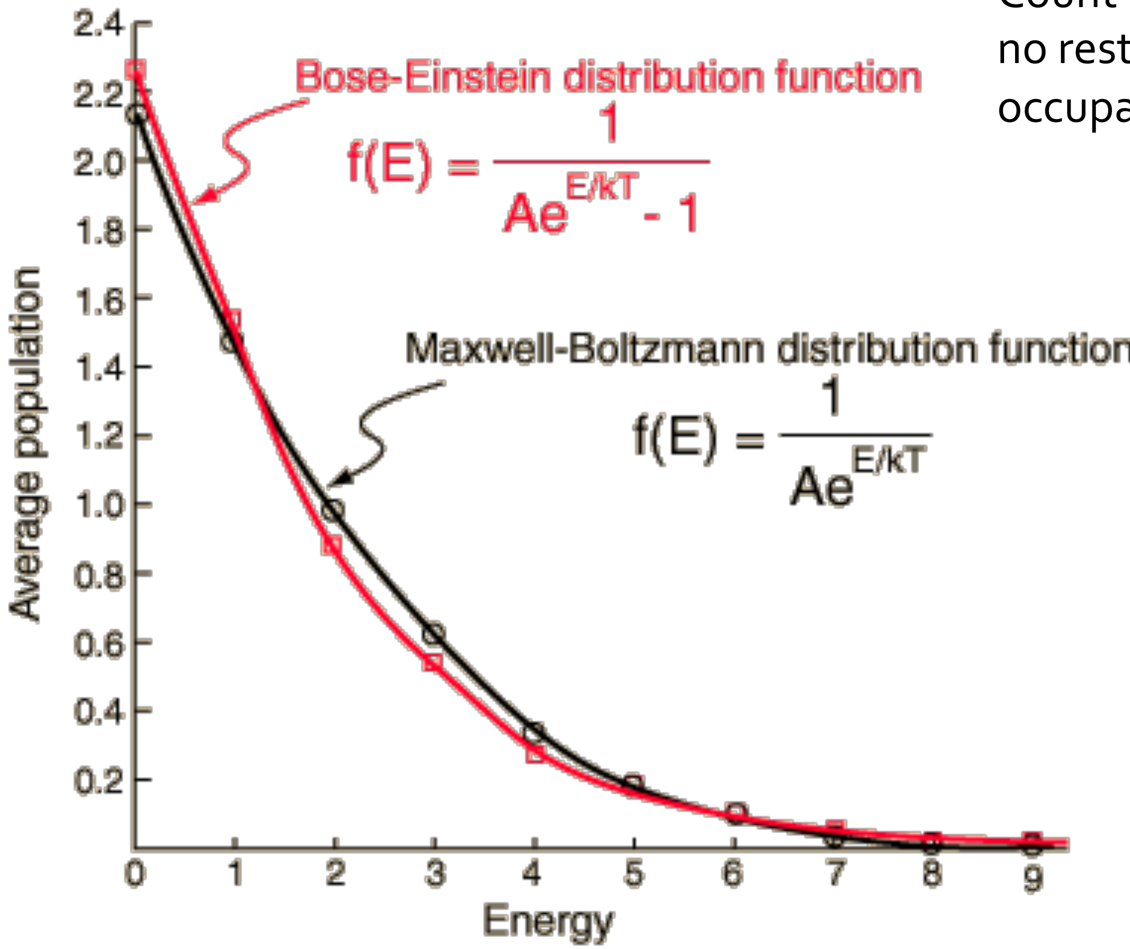
- Bosons (spin 1 particles)
  - E.g. photons
  - No restrictions on how many can be in the same quantum state
- Fermions (spin 1/2 particles)
  - E.g. electrons
  - At most two fermions (spin up and spin down) can be in any quantum state

Boson Energy Distribution (n=9, k = 6)

Only 26 total states to count

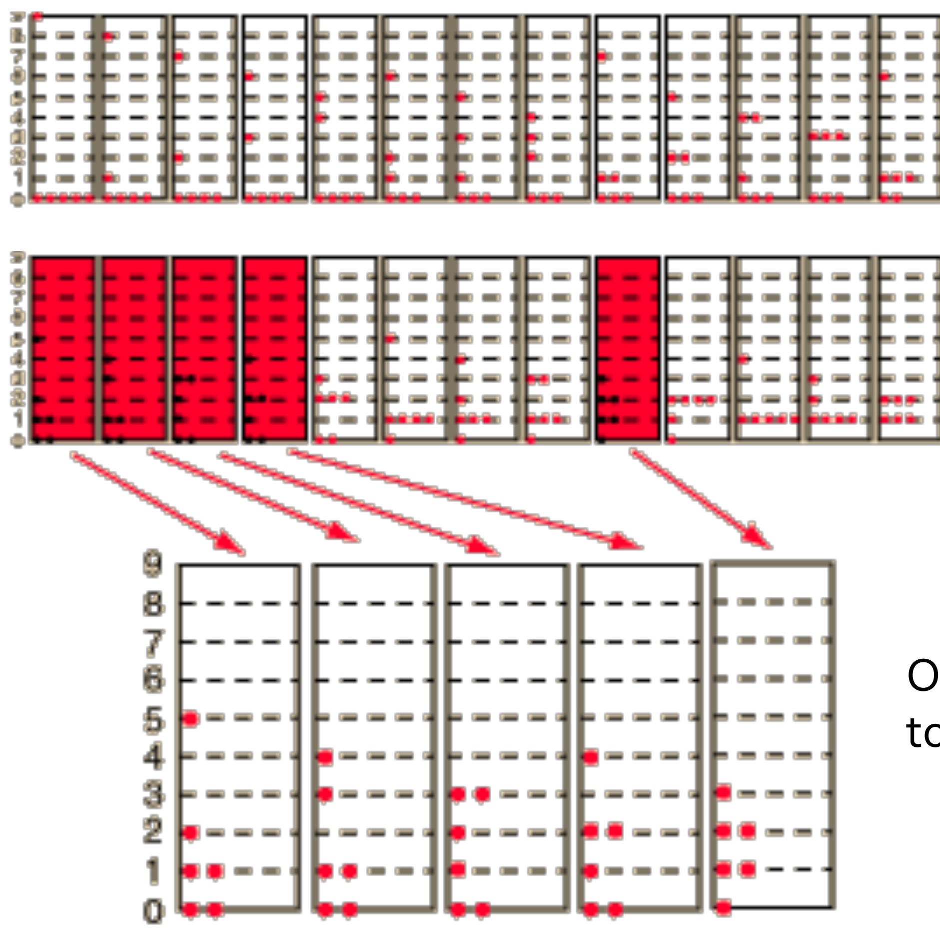


Bose-Einstein Statistics



Count only macrostates, no restrictions on occupancy of levels

Fermion Energy Distribution (n=9, k = 6)



Only 5 total states
to count!

Fermi-Dirac Statistics