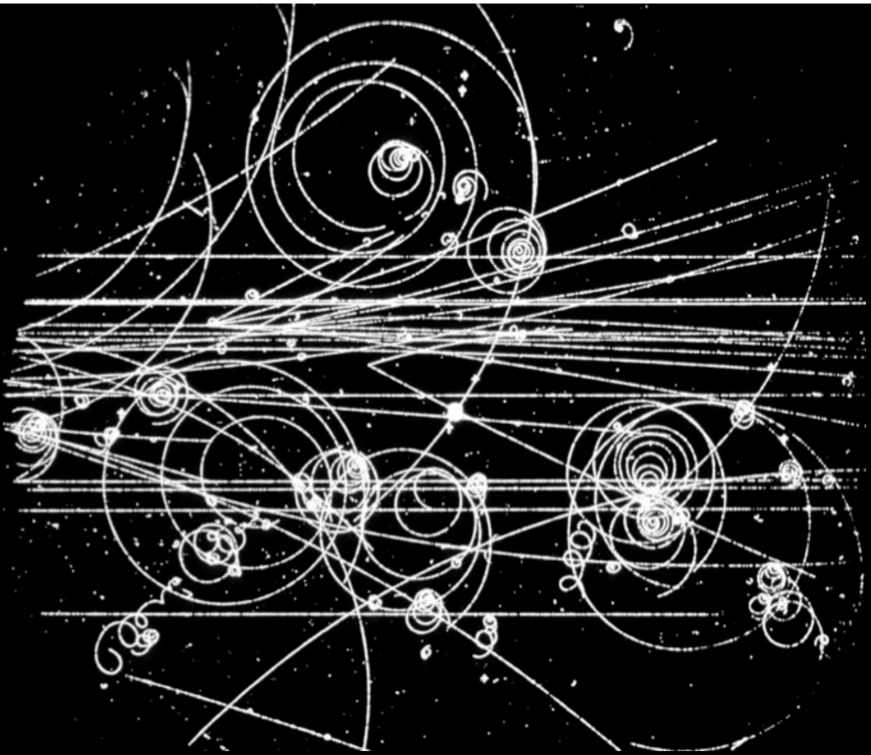
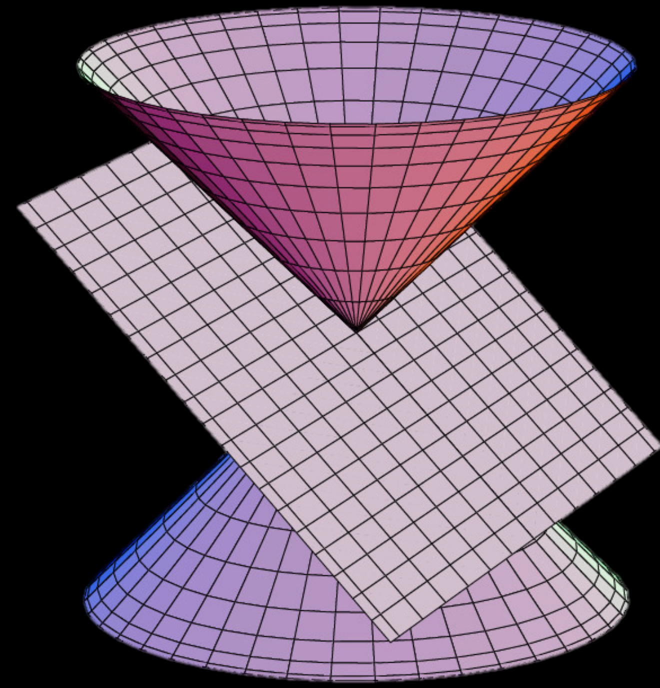




# Modern Physics (Phys. IV): 2704

Professor Jasper Halekas
Van Allen 70
MWF 12:30-1:20 Lecture

# Class Evaluations

- Class evaluations are open
  - Please please take a few minutes to fill them out!
  - They are completely anonymous and I don't see them until after grades are submitted
  - I rely heavily on your feedback to improve my teaching and make my classes better for all students

  - For the lab sections
    - If you have specific comments for/about Erik please put them in the evaluation for the individual lab sections (0A33/0A43)
    - If your comments are about the overall organization of the course, including labs, you can put them in the evaluation of the main section (0AAA/0BBB)

# Accessing Class Evaluations

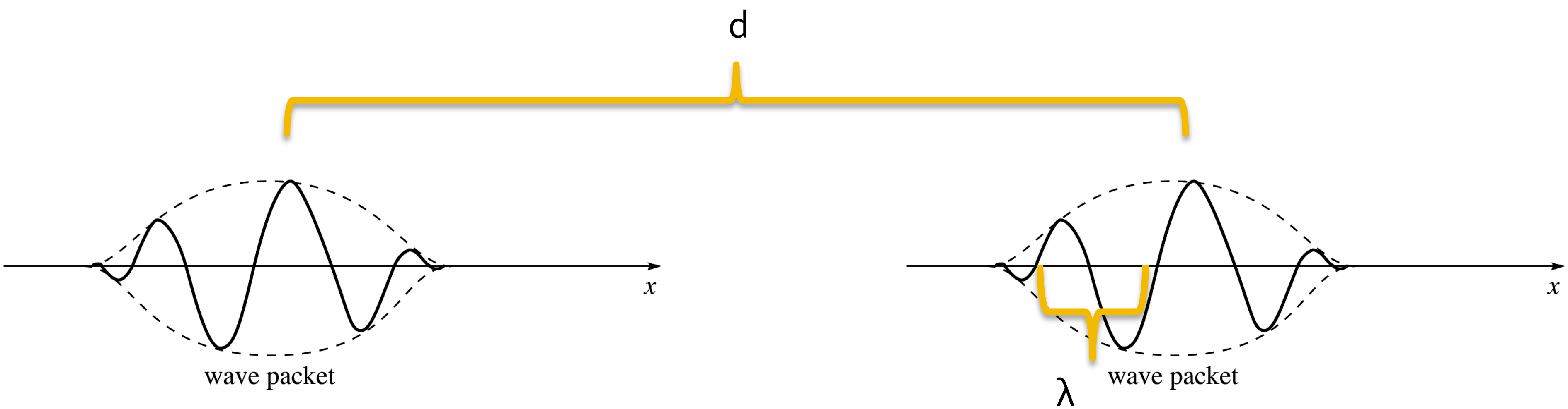
- Access through ICON:
  - Go to the ICON homepage (icon.uiowa.edu).
  - Enter your HawkID and password.
  - From the ICON Dashboard click on "Student Tools".
  - Click on "Course Evaluations (ACE)".
  - The online course evaluation system will open in a new tab.
  - You will need to log again using your HawkID and password.
- Access through myUI:
  - Go to the myUI homepage (myui.uiowa.edu).
  - Click on "Course Evaluations".
  - You will need to log again using your HawkID and password.

# Poll on Next Week's Schedule

- Next week will be two days of review and a special lecture on the "Quantum weirdness/ philosophy of quantum mechanics". Which order would people prefer?

A. Monday & Wednesday review, Friday special lecture
B. Monday special lecture, Wednesday & Friday review

# Classical Vs. Quantum Statistics



## Limit of Classical Statistics

Classical ; $\lambda << d$

$\lambda = h/p$

$KE = p^2/2m \sim kT$

$\Rightarrow p \sim \sqrt{2mkT}$

$\Rightarrow \lambda \sim h/\sqrt{2mkT}$

$d \sim (N/V)^{-1/3} = n^{-1/3}$

so if $\frac{h n^{1/3}}{\sqrt{2mkT}} << 1$

classical statistics okay

- quantum effects show up for large $n$ and low $T$

# Concept Check

- For which type of distribution does an assemblage of particles have the lowest average energy at a given temperature?

A. Bose-Einstein
B. Maxwell-Boltzmann
C. Fermi-Dirac
D. All the same

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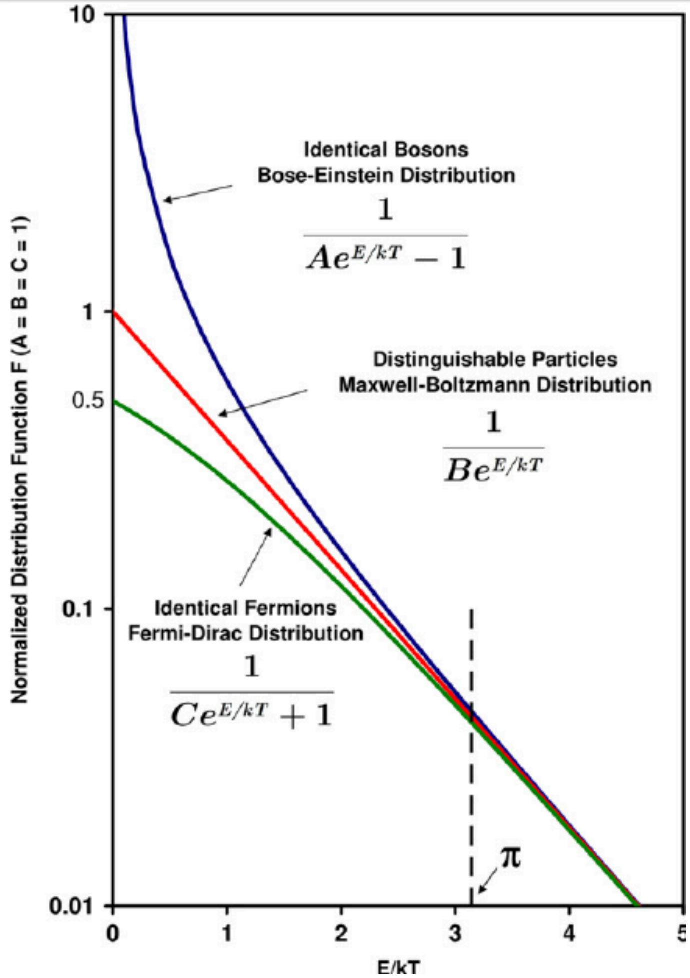
# Classical/Quantum Distribution Functions



# Low-T Limit

$$f_{BE} = \frac{1}{Ae^{E/kT} - 1}$$

$$E \gg kT \Rightarrow e^{E/kT} \to \infty$$
$$f_{BE} \to 0$$

$$E \ll kT \Rightarrow e^{E/kT} \to 1 + E/kT$$
$$f_{BE} \to \text{const.}$$

As $kT \to 0$
$f_{BE} \to \delta(0)$

---

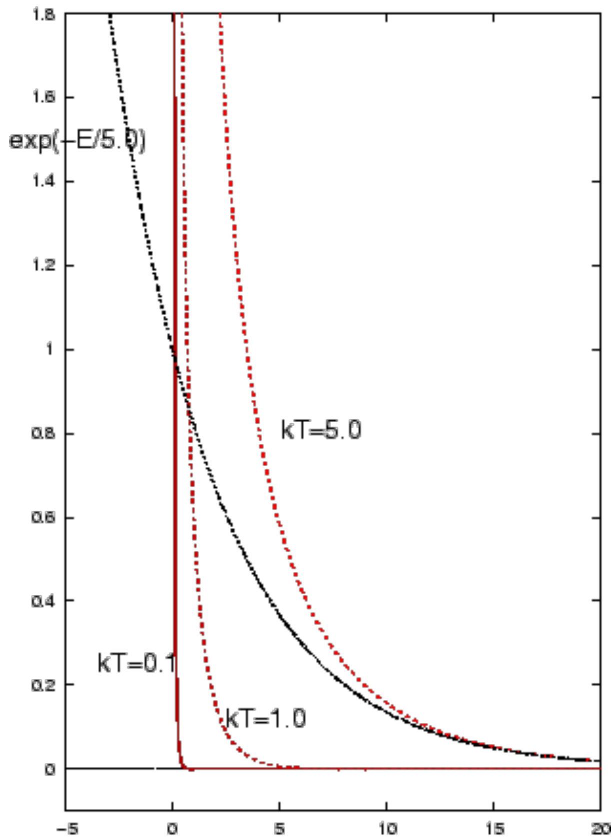
$$f_{FD} = \frac{1}{Ae^{E/kT} + 1} = \frac{1}{e^{(E-E_F)/kT} + 1}$$

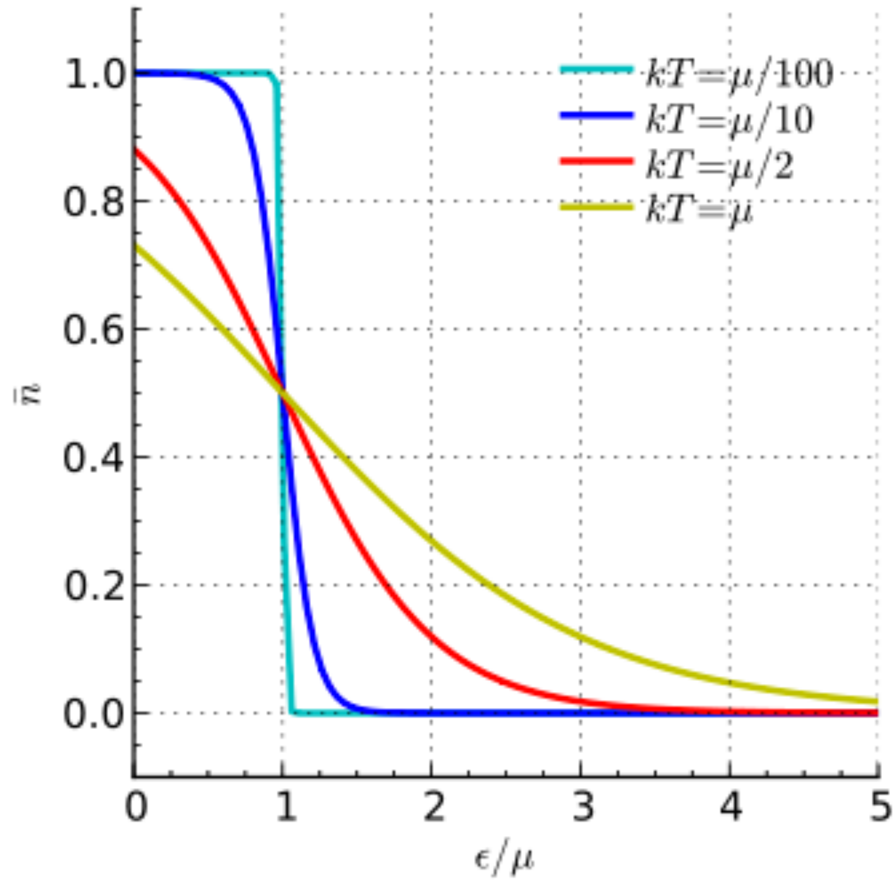
$$(E - E_F) \gg kT \qquad e^{(E-E_F)/kT} \to \infty$$
$$f_{FD} \to 0$$

$$|E - E_F| \gg kT \qquad e^{(E-E_F)/kT} \to 0$$
$$E < E_F \qquad f_{FD} \to 1$$

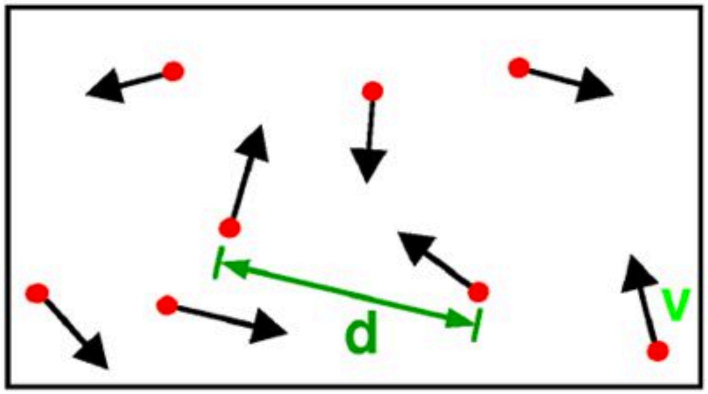
As $kT \to 0$
$f_{FD} \to$ step function
w/ step @ $E = E_F$

# Low Temperature Limit of Statistical Distributions

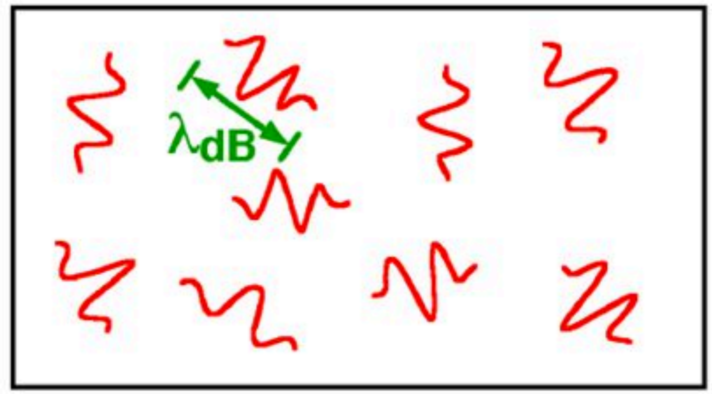


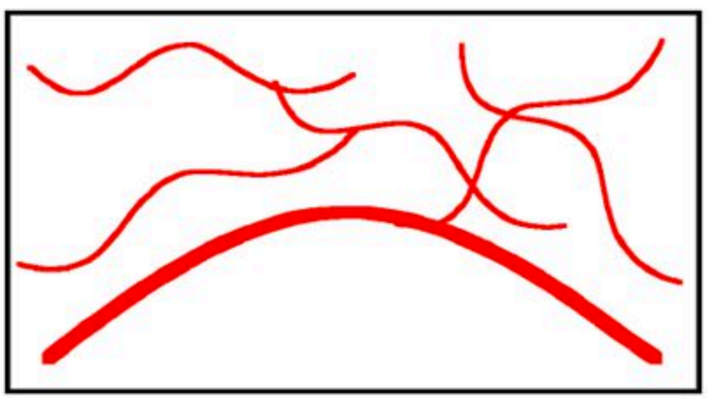
# Bose-Einstein Condensate



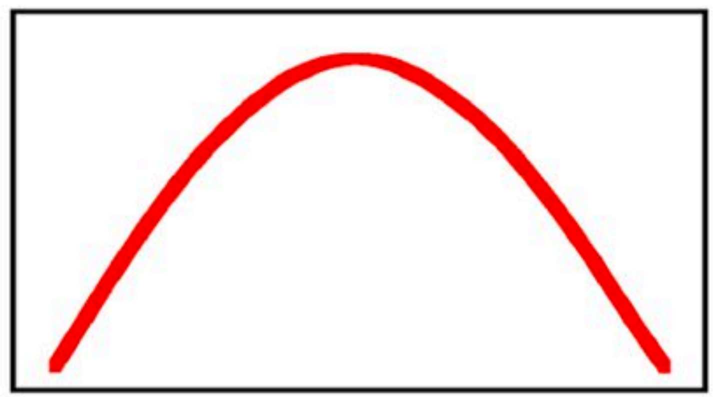

1. High temperature particle behaviour dominated




2. Low temperature $\lambda_{dB} \propto T^{-0.5}$



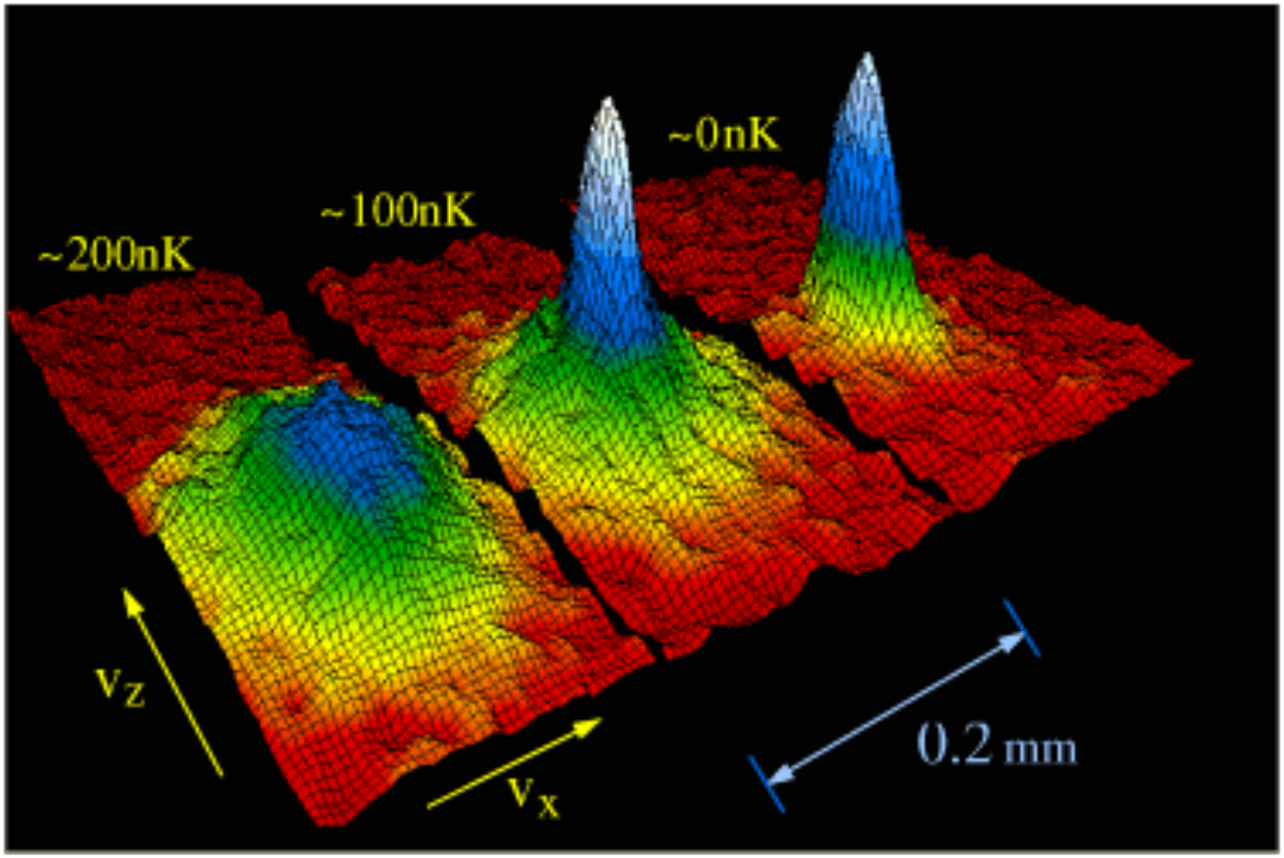
3. T=Tcrit Bose Einstein Condensate



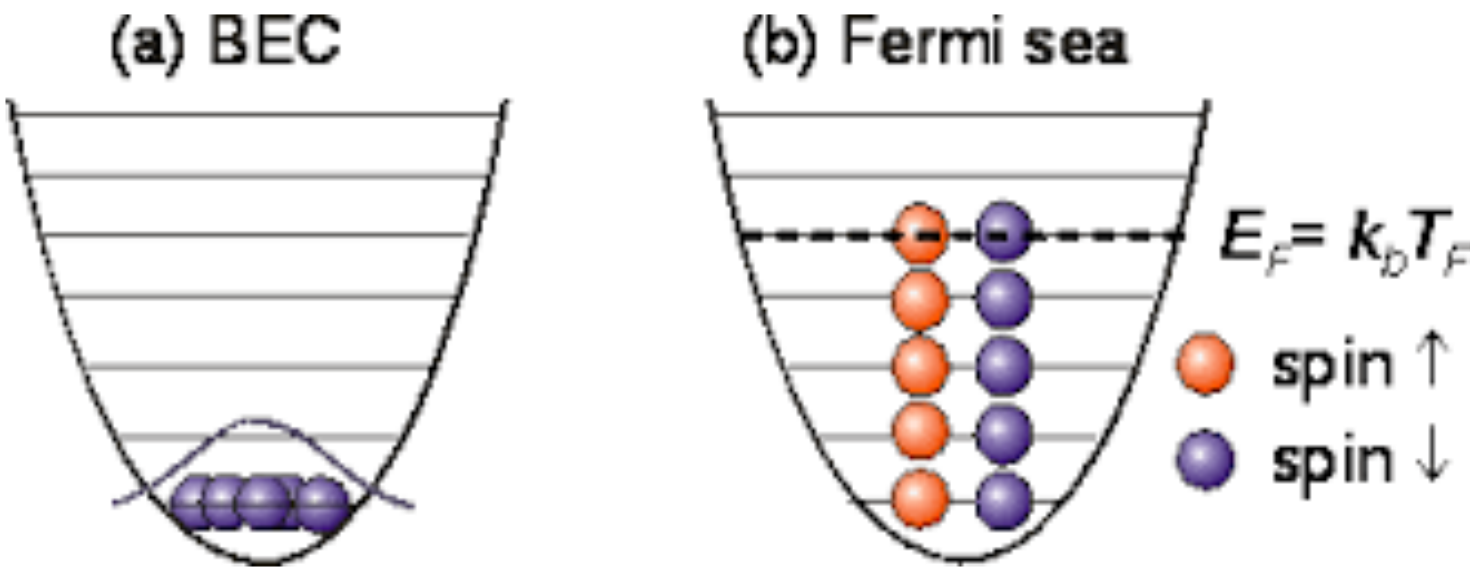
4. T=0 Giant Matter Wave

# BEC Velocity Distributions

2 D velocity distributions



# BEC vs. Low-Temperature Fermions



# Free Electrons in Metals

$$g(E) \text{ for gas} = \frac{2s+1}{\sqrt{2}} \frac{m^{3/2}}{\pi^2 \hbar^3} \sqrt{E}$$

$$s = 1/2 \Rightarrow 2s+1 = 2$$

(spin up, spin down)

$$g(E) = \sqrt{2} \frac{m^{3/2}}{\pi^2 \hbar^3} \sqrt{E} = \sqrt{2} \frac{8\pi m^{3/2}}{h^3} \sqrt{E}$$

$$N(E) = V g(E) f(E) = V\sqrt{2} \frac{8\pi m^{3/2}}{h^3} \frac{\sqrt{E}}{e^{(E-E_F)/kT}+1}$$

$$N = V\sqrt{2} \frac{8\pi m^{3/2}}{h^3} \int_0^\infty \frac{\sqrt{E}\, dE}{e^{(E-E_F)/kT}+1}$$

$$@\ T = 0 \quad \frac{1}{e^{(E-E_F)/kT}+1} \rightarrow \Theta(E_F)$$

step function

$$\Rightarrow N = V\sqrt{2} \frac{8\pi m^{3/2}}{h^3} \int_0^{E_F} \sqrt{E}\, dE$$

$$= V\sqrt{2} \frac{8\pi m^{3/2}}{h^3} \frac{2}{3} E_F^{3/2}$$

$$\Rightarrow \boxed{E_F = \frac{h^2}{2m} \left(\frac{3N}{8\pi V}\right)^{2/3} = \frac{h^2}{2m} \left(\frac{3n}{8\pi}\right)^{2/3}}$$

$$\langle E \rangle = \frac{1}{N} \int E N(E)\, dE$$

$$= \int_0^{E_F} E^{3/2}\, dE \Big/ \int_0^{E_F} E^{1/2}\, dE$$

$$= \frac{2}{5} E_F^{5/2} \Big/ \left(\frac{2}{3} E_F^{3/2}\right)$$

$$= \boxed{\frac{3}{5} E_F}$$

$$\langle E \rangle = \frac{3}{5} \frac{h^2}{2m} \left( \frac{3n}{8\pi} \right)^{2/3}$$

even @ $T = 0$ !

Compare to ideal gas

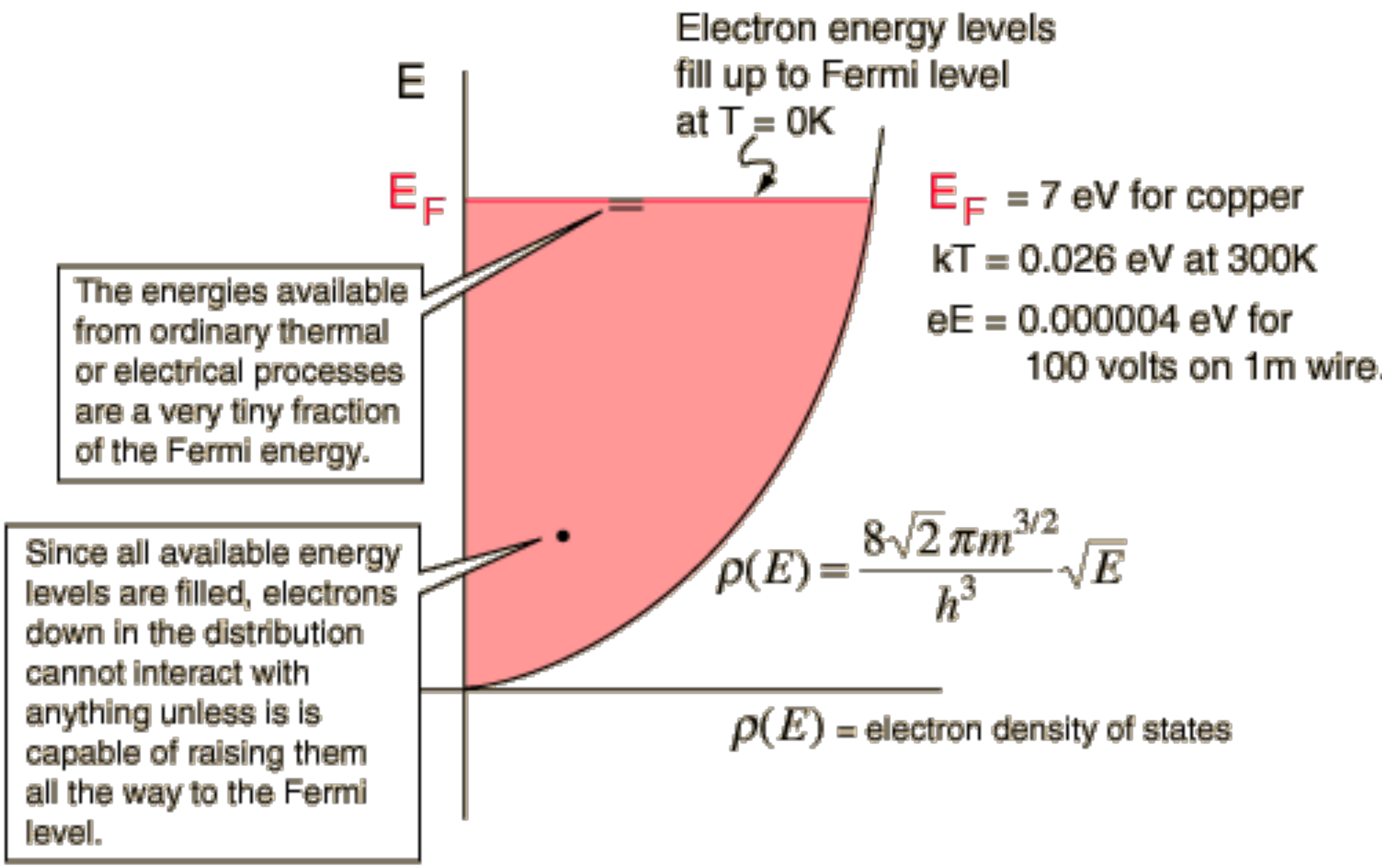
$$\langle E \rangle = \frac{3}{2} kT$$

$\rightarrow 0$ as $T \rightarrow 0$

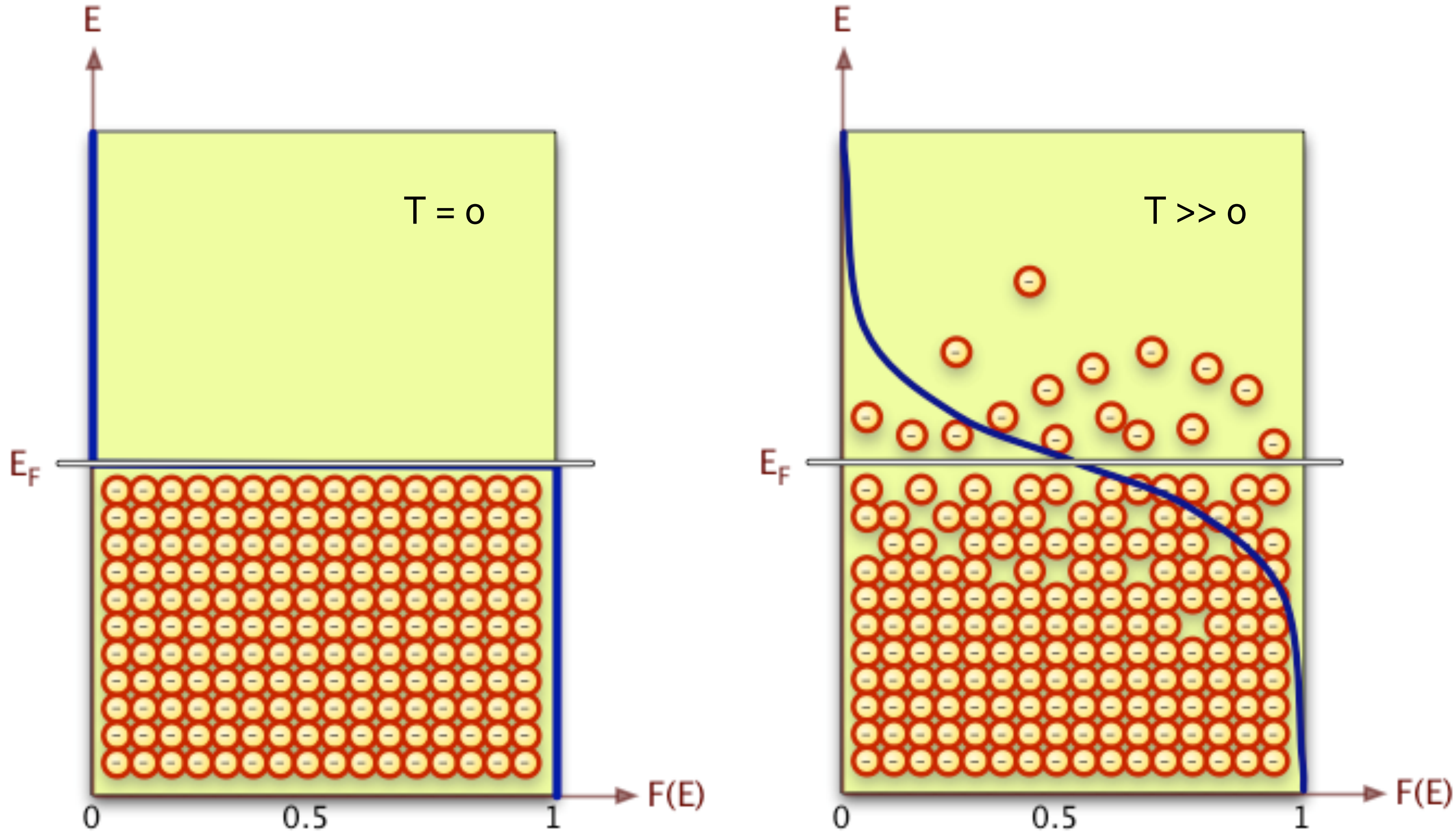
Bose - Einstein

$\langle E \rangle \rightarrow 0$ @ finite $T$ !

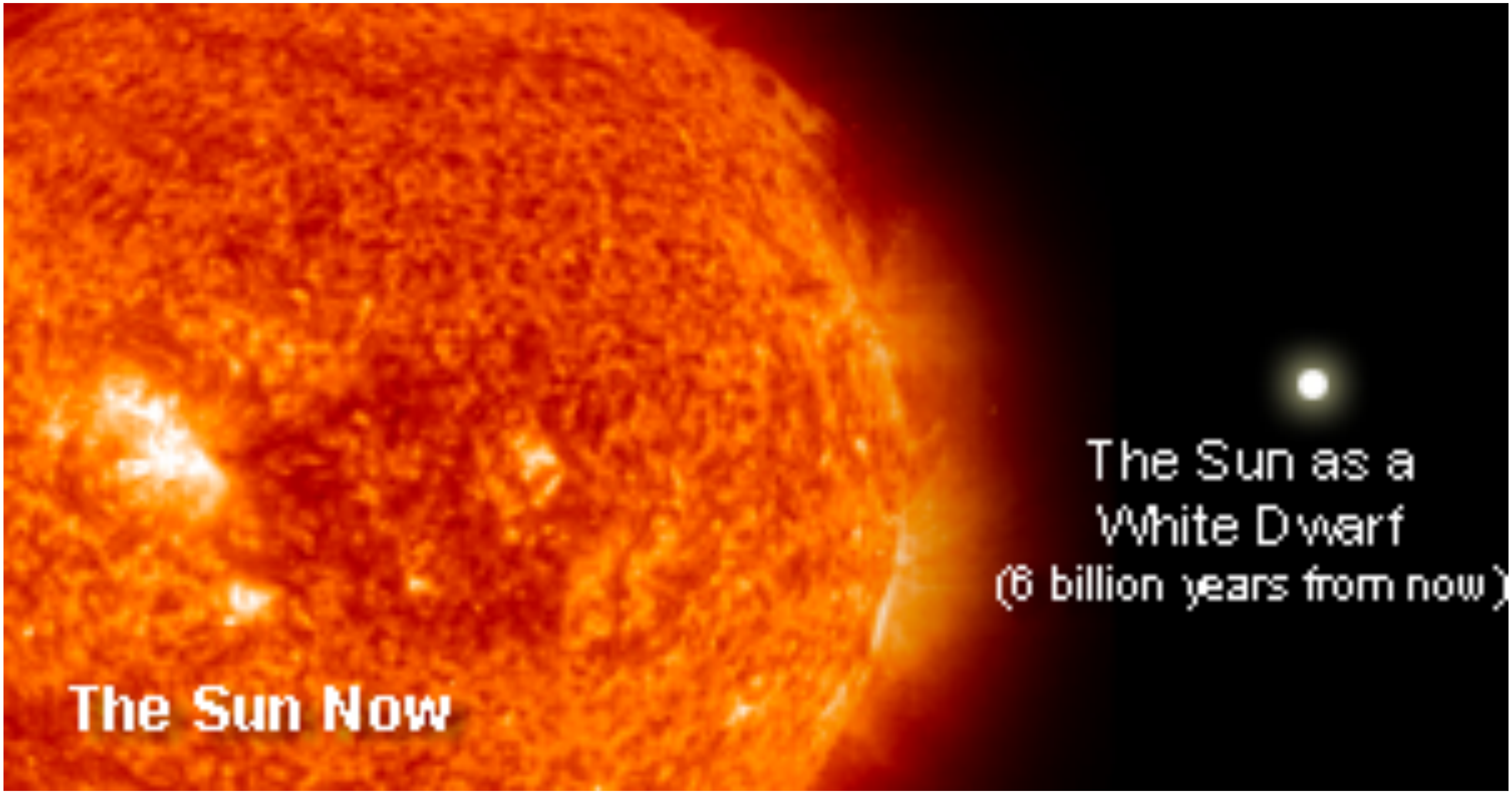
# Fermi Energy of Metals



# Electron Energy Distribution in Metals



# The Fate of our Sun



# White Dwarf

$$E_e = N\langle E\rangle \quad \text{(total electron E)}$$
$$= 3/5\, N E_F$$
$$= 3/5\, N \frac{h^2}{2m_e}\left(\frac{3N}{8\pi V}\right)^{2/3}$$

$$V_{sphere} = 4/3\, \pi R^3$$
$$\Rightarrow E_e = \frac{3Nh^2}{10m_e R^2}\left(\frac{9N}{32\pi^2}\right)^{2/3}$$

$$E_{gravity} = -3/5\, \frac{GM^2}{R} = -3/5\, \frac{GN^2 m_{He}^2}{4R}$$

since $M = N/2\, m_{He}$
($2\, e^-$ per He)

$$E_{total} = E_e + E_{grav}$$

Equilibrium $\quad dE_{total}/dR = 0$

$$\Rightarrow \frac{-6Nh^2}{10m_e R^3}\left(\frac{9N}{32\pi^2}\right)^{2/3} + 3/5\, \frac{GN^2 m_{He}^2}{4R^2} = 0$$

$$\Rightarrow R_{eq} = \frac{6Nh^2}{10m_e}\left(\frac{9N}{32\pi^2}\right) \Big/ \left(\frac{3GN^2 m_{He}^2}{20}\right)$$

$$= \frac{4h^2}{G m_e m_{He}^2 N^{1/3}}\left(9/32\pi^2\right)^{2/3}$$

$$= \boxed{\frac{h^2}{G m_e m_{He}^2 N^{1/3}}\left(\frac{9}{4\pi^2}\right)^{2/3}}$$

For ~ 1 solar mass

$$R = R_E$$
$$\rho \sim 10^9 \text{ kg/m}^3 = 10^6 \text{ g/cm}^3$$
$$= 10^6 \times \text{ normal matter}$$

$$E_F \sim 200 \text{ keV}$$
$$= 0.4\, m_e c^2$$

---

## Degeneracy Pressure

Ideal Gas $\quad PV = NkT$

$$\Rightarrow P = NkT/V$$
$$= N \cdot \tfrac{2}{3} \langle E \rangle / V$$
$$= \tfrac{2}{3} E_{total}/V$$

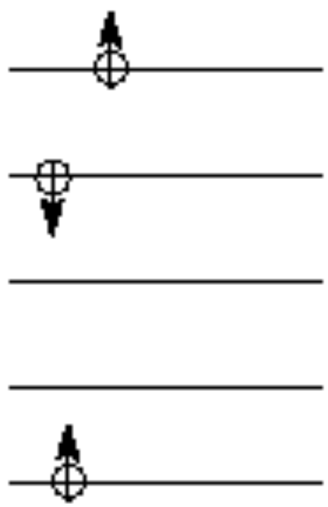
$$P_e = \tfrac{2}{3} E_e / V$$
$$= \tfrac{2}{3} \cdot \tfrac{3}{5} N E_F / V = \tfrac{2}{5} n_e E_F$$

$$= \tfrac{2}{5} n_e \cdot \frac{h^2}{2m_e} \left(\frac{3n_e}{8\pi}\right)^{2/3}$$

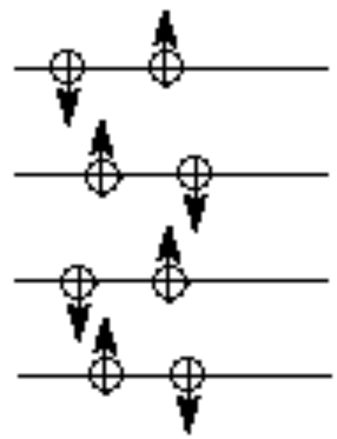
$$= \boxed{\frac{h^2}{5m_e} \left(\frac{3}{8\pi}\right)^{2/3} \cdot n_e^{5/3}}$$

purely due to fermion's
anti social nature!!

# Degenerate Matter



Regular gas: many unfilled energy levels. Particles free to move about and change energy levels.



**Degenerate** gas: all lower energy levels filled with two particles each (opposite spins). Particles **locked** in place.

# Stellar Endgames