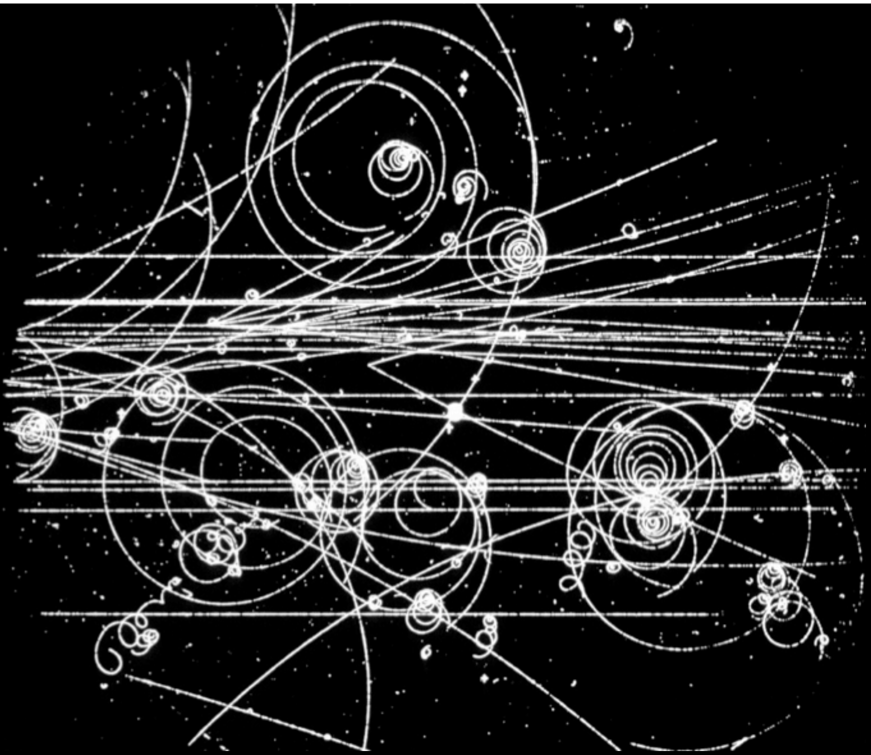
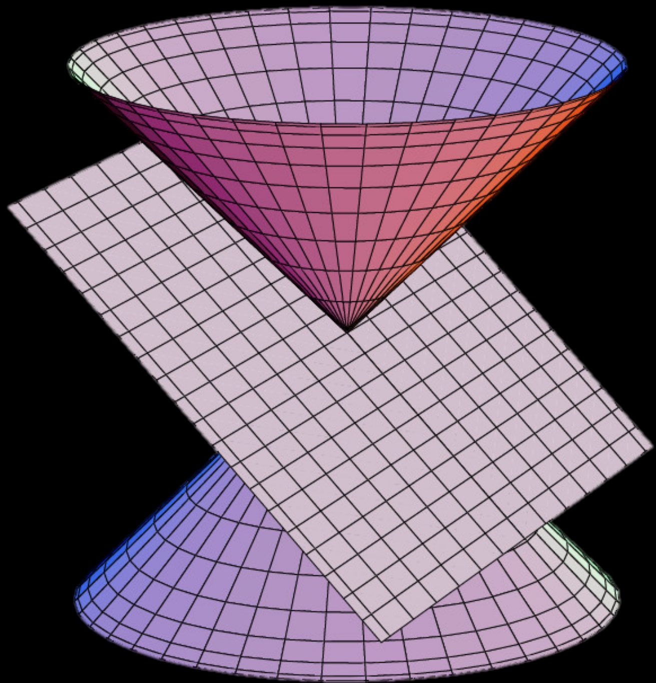




Modern Physics (Phys. IV): 2704

Professor Jasper Halekas
Van Allen 70
MWF 12:30-1:20 Lecture

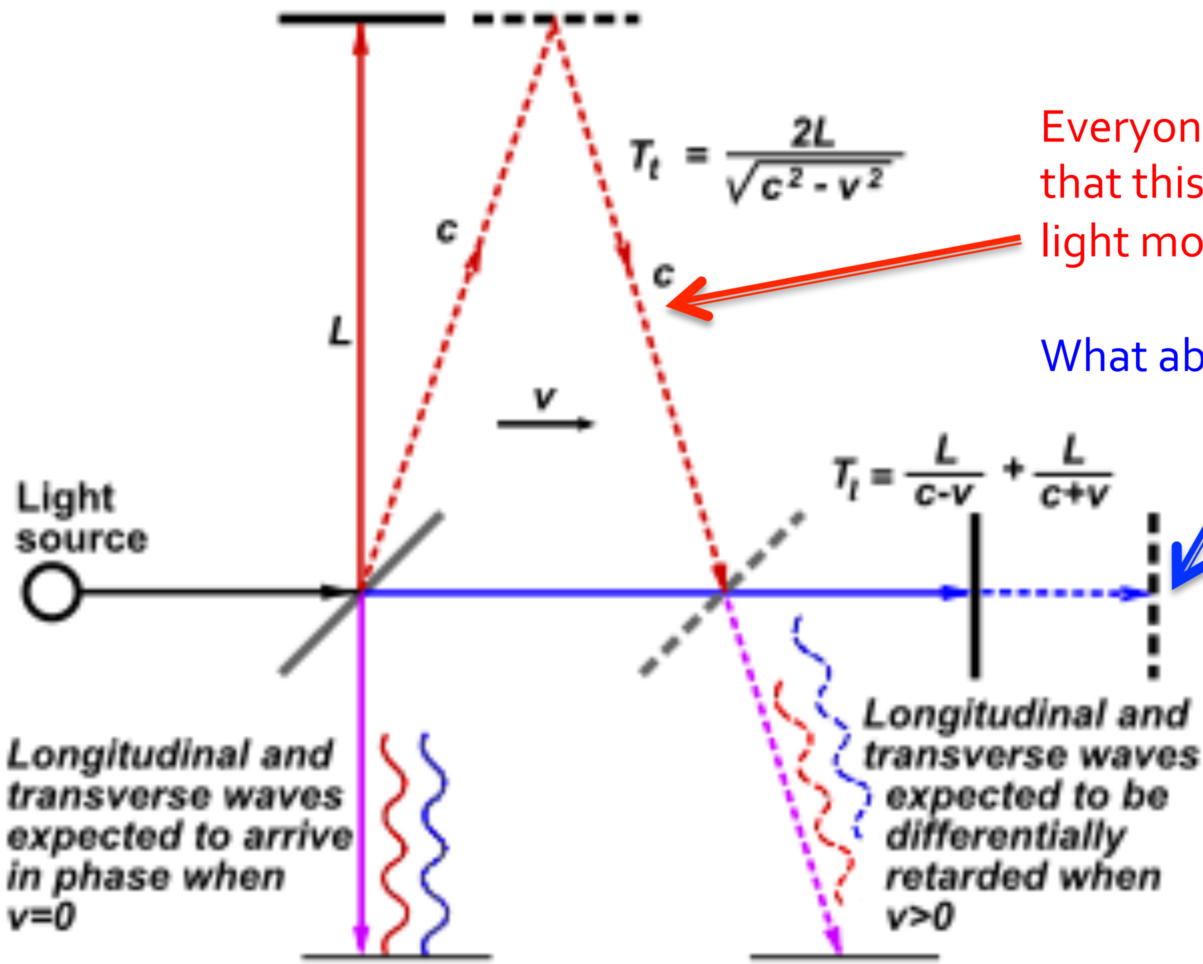
Announcements

- First labs start today!
- First homework assignment due in class (or before) on Friday

Last Time Recap

- Events that are simultaneous in one inertial frame are not necessarily simultaneous in another inertial frame
- The time duration measured between two events is different in different frames
  - $\Delta t = \gamma \Delta \tau$
    - $\Delta \tau = \Delta t_o$ is the “proper time” measured in a frame where the two events take place at the same location

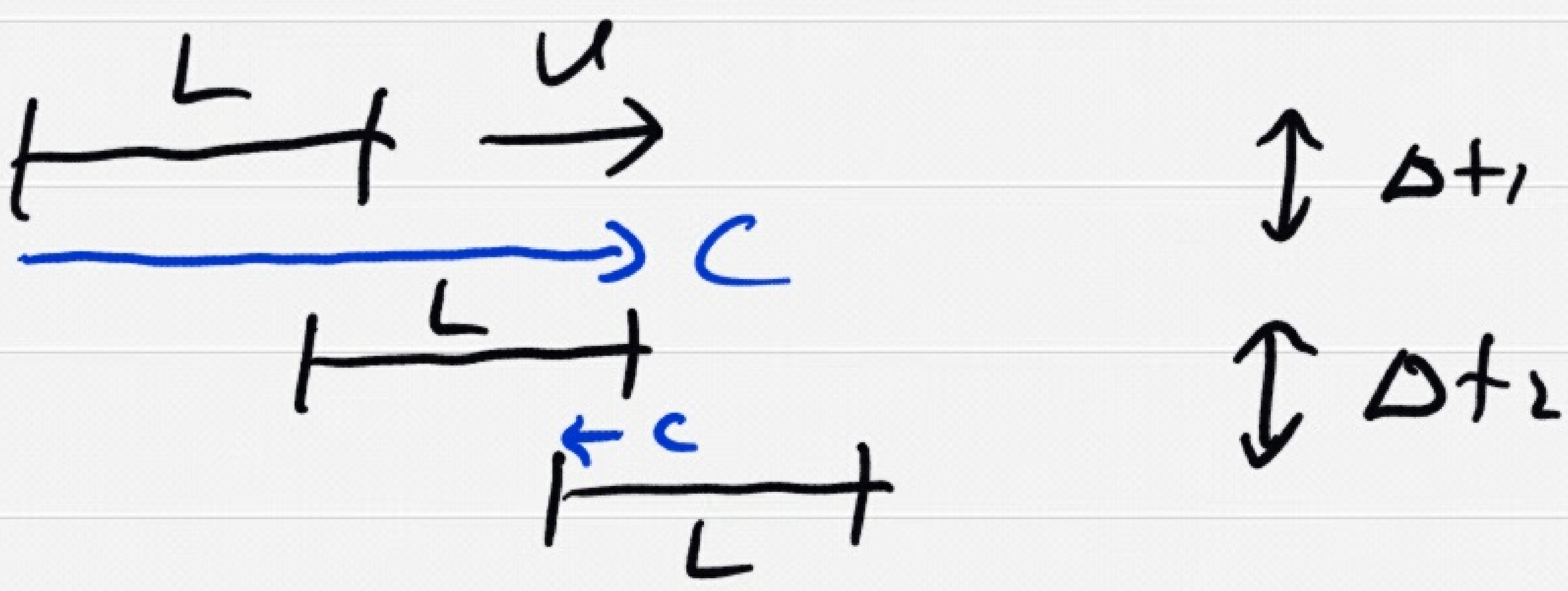
Back to Michelson-Morley



Everyone now agrees that this half of the light moves at c

What about this half?

Length Contraction



Ether Frame

$$c\,\Delta t_1 = L + u\,\Delta t_1$$
$$c\,\Delta t_2 = L - u\,\Delta t_2$$

$$\Delta t = \Delta t_1 + \Delta t_2$$
$$= \frac{L}{c-u} + \frac{L}{c+u}$$
$$= 2L/c \cdot \frac{1}{1-u^2/c^2}$$
$$= 2L/c \cdot \gamma^2$$

But $\Delta t = \gamma\,\Delta t' = \gamma\,\Delta t_0$

w/ $\Delta t_0 = 2L_0/c$

= time interval in Earth frame

$\Rightarrow 2L/c \cdot \gamma^2 = 2L_0/c \cdot \gamma$

$\Rightarrow \gamma L = L_0$

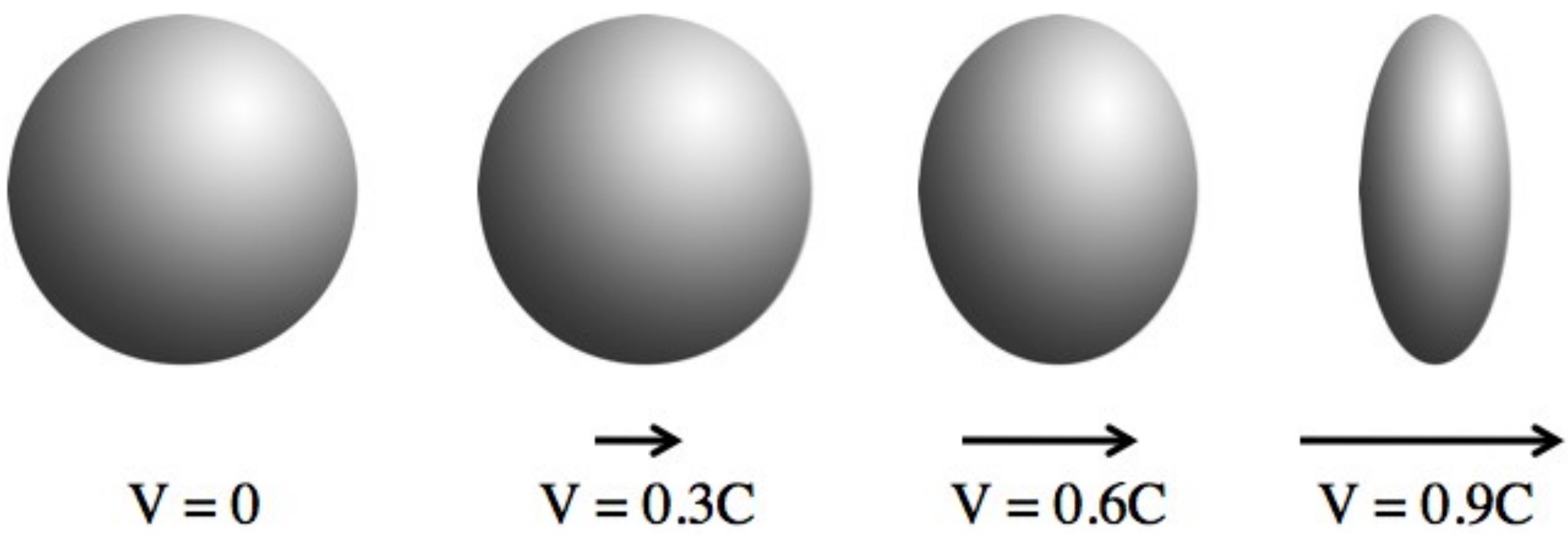
$\Rightarrow L = L_0/\gamma$

w/ L_0 = "proper length"

Proper Length

- "Proper Length" = L_o
  - The length of an object measured in a frame where it is at rest
- The length L measured in **any** other frame moving with a velocity with respect to this frame that has a component along the length will be shorter
  - $L = L_o/\gamma$

Length Contraction



Concept Check: Proper Time/Length

- A star (assumed to be at rest relative to the Earth) is 100 light-years from Earth. (A light-year is the distance light travels in one year.) An astronaut sets out from Earth on a journey to the star at a constant speed of 0.98*c*. (Note: At $u = 0.98c$, $\gamma=5$)

- How long does a light signal take to reach the star, and how long does it take the astronaut to reach the star, both according to Earth?

A. 100 years, 98 years
B. 98 years, 100 years
C. 100 years, 102 years
D. 100 years, 100 years

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- A star (assumed to be at rest relative to the Earth) is 100 light-years from Earth. An astronaut sets out from Earth on a journey to the star at a constant speed of $0.98c$. (Note: At $u = 0.98c$, $\gamma=5$)
- According to the astronaut, what is the distance to the star, and how long does it take to get there?

A. 100 light years, 100 years
B. 20 light years, 20 years
C. 20 light years, 20.4 years
D. 100 light years, 20.4 years
E. 20 light years, 102 years

Concept Check: Proper Time/Length

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E. 20 light years, 102 years

Concept Check: Proper Time/Length

- Light takes 100 years to travel to the star, but according to the astronaut he makes it there in 20.4 years. Does that mean he travels faster than light?

A. Yes
B. No
C. Maybe?
D. WTF?

Concept Check: Proper Time/Length

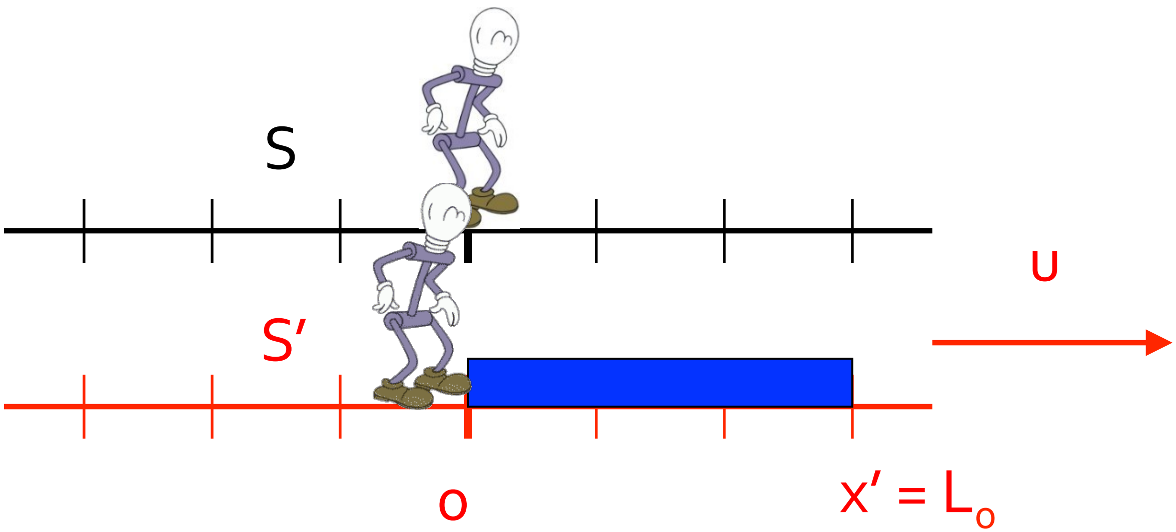
- Light takes 100 years to travel to the star, but according to the astronaut he makes it there in 20.4 years. Does that mean he travels faster than light?

A. Yes
B. No
C. Maybe?
D. WTF?

Moral of the story

- The proper time and the proper length do not have to be in the same frame!
  - Astronaut says he travels distance $L = L_o/\gamma$ in proper time t_o
  - From Earth, we see astronaut travel proper length L_o in time $t = \gamma t_o$
  - A muon's trip through the atmosphere is a similar story

The Lorentz Transformation: Preamble

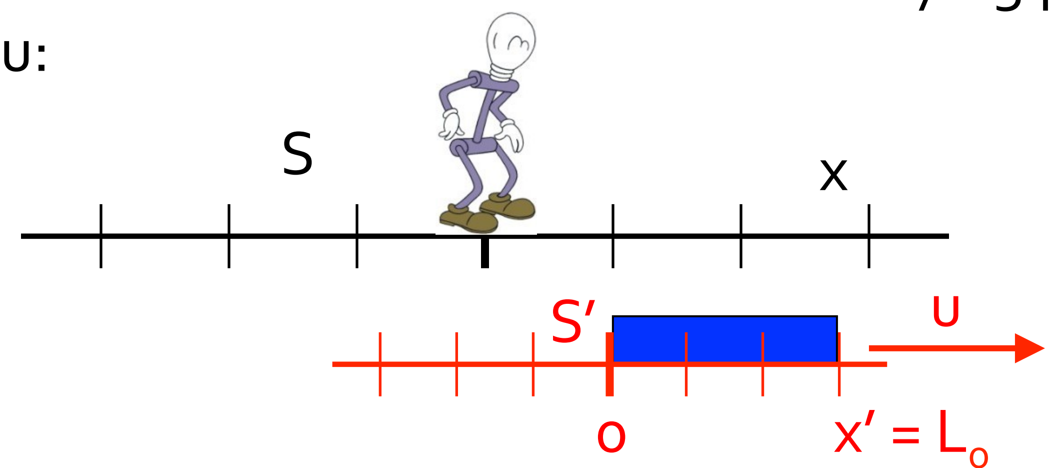


A stick with length L_o is at rest in S′. Its endpoints are the events (x,t) = (0,0) and (x′,0) in S′. S′ is moving to the right with respect to frame S at a velocity u.

Event 1 – left of stick passes origin of S. Its coordinates are (0,0) in S and (0,0) in S′.

Concept Check: Lorentz Transformation

An observer at rest in frame S sees a stick flying past him with velocity u:



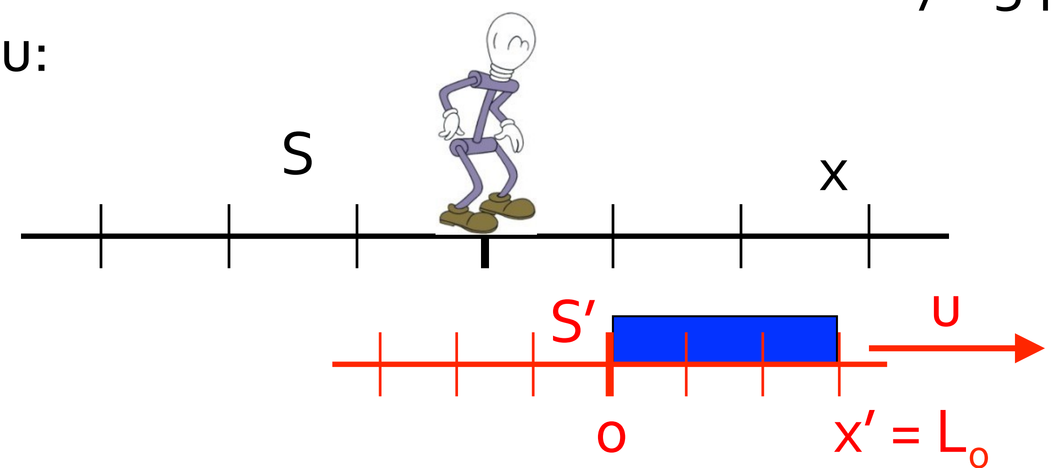
As viewed from S, the stick's length is L_o/γ. Time t passes. According to S, where is the *right* end of the stick? (Assume the *left* end of the stick was at the origin of S at time t=o.)

A) $x = \gamma ut$ B) $x = ut + x'/\gamma$ C) $x = -ut + x'/\gamma$

D) $x = ut - x'/\gamma$ E) Something else…

Concept Check: Lorentz Transformation

An observer at rest in frame S sees a stick flying past him with velocity u:



As viewed from S, the stick's length is L_o/γ. Time t passes. According to S, where is the *right* end of the stick? (Assume the *left* end of the stick was at the origin of S at time t=0.)

A) $x = \gamma ut$ B) $x = ut + x'/\gamma$ C) $x = -ut + x'/\gamma$

D) $x = ut - x'/\gamma$ E) Something else…

Lorentz Transformation

$$x = ut + x'/\gamma$$

$$\Rightarrow x' = \gamma(x - ut)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma(t - u/c^2 \, x)$$

- Align origins

$$[x, y, z, t]$$

$$= [x', y', z', t']$$

$$= [0, 0, 0, 0]$$

when left end of stick @ origin

- At time t

$$x_L = ut$$

$$x_L' = \gamma(ut - ut) = 0$$

$$x_R = ut + L_0/\gamma$$

$$x_R' = \gamma(ut + L_0/\gamma - ut) = L_0$$

$$\Delta x = x_R - x_L = L_0/\gamma$$

$$\Delta x' = L_0$$

//

- $t_L' = \gamma(t - u/c^2 \, x_L) = \gamma(t - u^2/c^2 \, t)$

$t_R' = \gamma(t - u/c^2 \, x_R) = \gamma(t - u^2/c^2 \, t - u/c^2 \, L_0/\gamma)$

$\Delta t' = -u/c^2 \, L_0$

- clocks out of sync according to moving observer

Transformations

If S′ is moving with speed v in the positive x direction relative to S, then the coordinates of the same event in the two frames are related by:

Galilean transformation
(classical)

$$x' = x - ut$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

Lorentz transformation
(relativistic)

$$x' = \gamma(x - ut)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - \frac{u}{c^2}x)$$

Note: This assumes (0,0,0,0) is the same event in both frames.

Interval Transformations

If S' is moving with speed v in the positive x direction relative to S, then the coordinates of the same event in the two frames are related by:

Galilean transformation
(classical)

$$\Delta x' = \Delta x - u\Delta t$$

$$\Delta y' = \Delta y$$

$$\Delta z' = \Delta z$$

$$\Delta t' = \Delta t$$

Lorentz transformation
(relativistic)

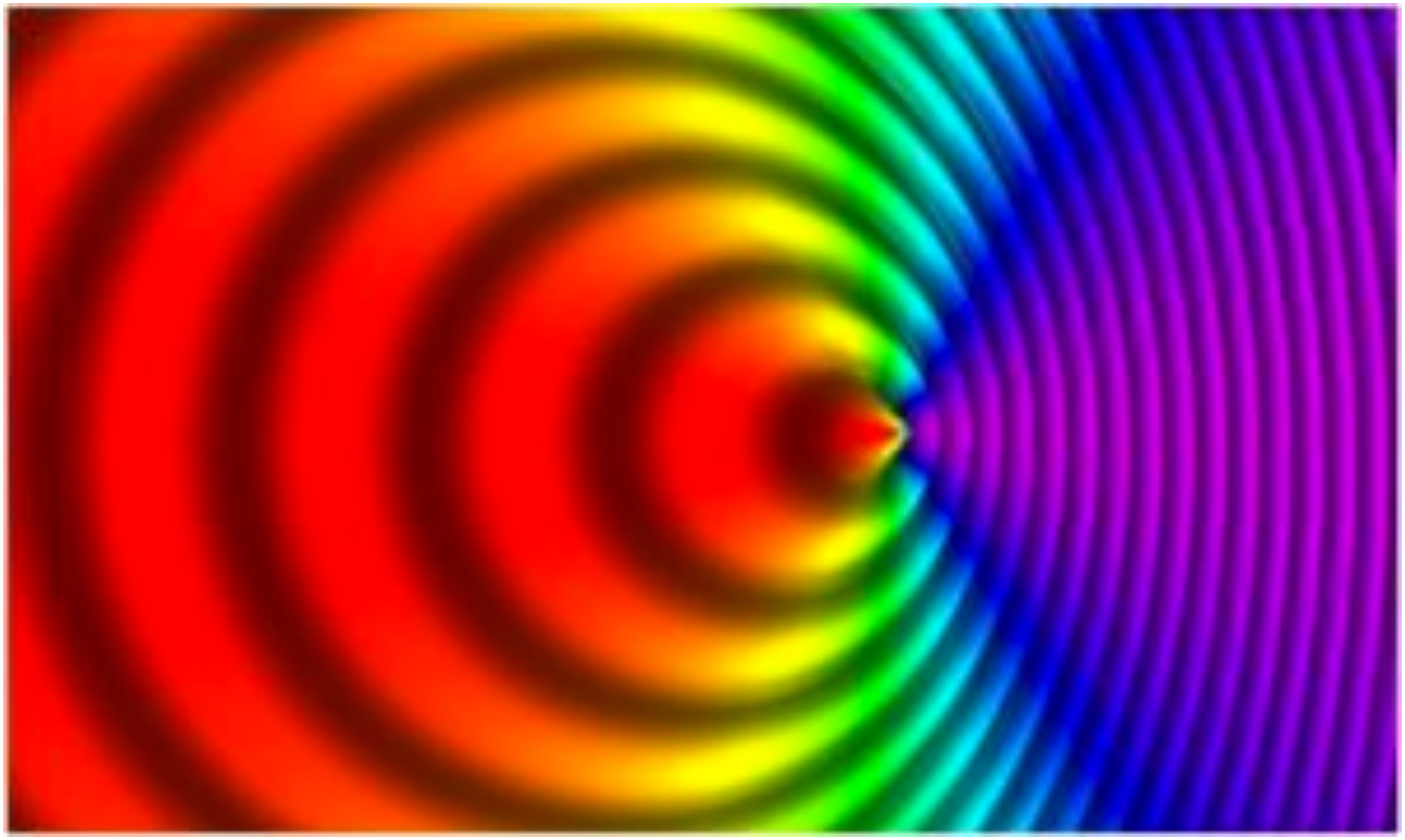
$$\Delta x' = \gamma(\Delta x - u\Delta t)$$

$$\Delta y' = \Delta y$$

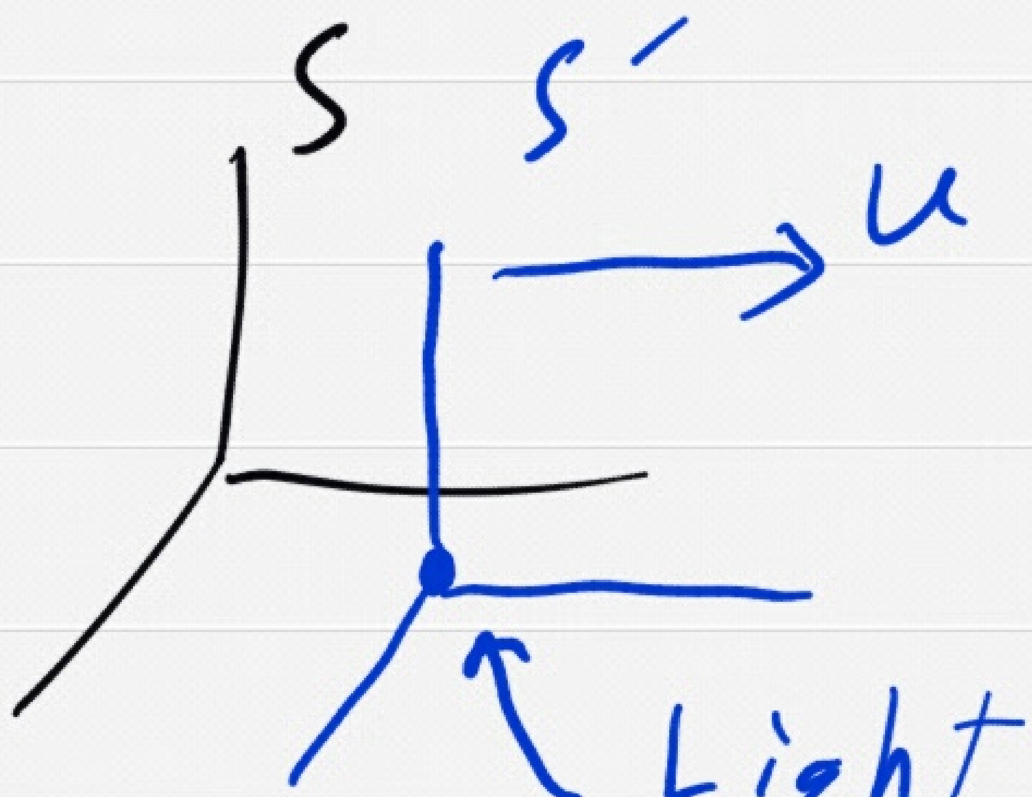
$$\Delta z' = \Delta z$$

$$\Delta t' = \gamma(\Delta t - \frac{u}{c^2}\Delta x)$$

Relativistic Doppler Shift



Relativistic Doppler Shift



Light source w/ frequency f' @ origin

	S'	S
Pulse 1	$x_1' = 0$	$x_1 = 0$
	$t_1' = 0$	$t_1 = 0$
Pulse 2	$x_2' = 0$	$x_2 = \gamma u t_2'$
	$t_2' = 1/f'$	$t_2 = \gamma t_2'$

- Pulse 2 takes $\Delta t = \dfrac{x_2 - x_1}{c}$ to reach origin in S

- Pulse 2 received at

$$\begin{aligned} 1/f &= t_2 - t_1 + \Delta t \\ &= \gamma t_2' + \gamma u t_2'/c \\ &= \gamma (1 + u/c)\, t_2' \\ &= \gamma (1 + u/c) \cdot 1/f' \end{aligned}$$

or $$f = \frac{1}{\gamma} \frac{1}{1 + u/c} f'$$

$$f = \frac{\sqrt{1 - u^2/c^2}}{1 + u/c} f'$$

$$= \frac{\sqrt{(1 + u/c)(1 - u/c)}}{1 + u/c} f'$$

$$\boxed{f = \sqrt{\frac{1 - u/c}{1 + u/c}} \, f'}$$

- Can write as

$$f = \sqrt{\frac{1-\beta}{1+\beta}} \, f'$$

w/ $\beta = u/c$

Relativistic Doppler Shift