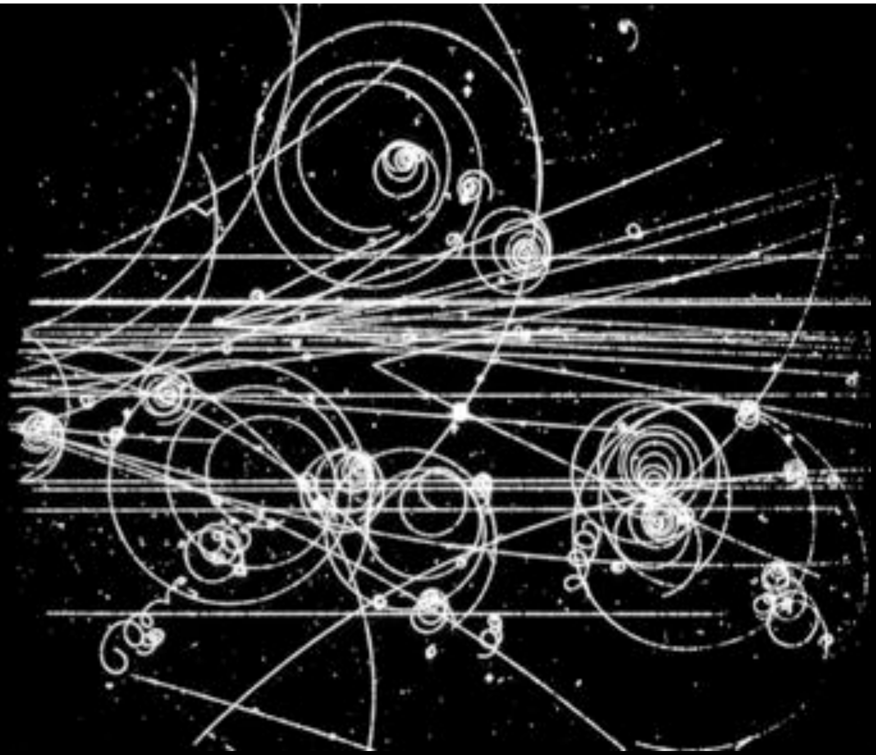
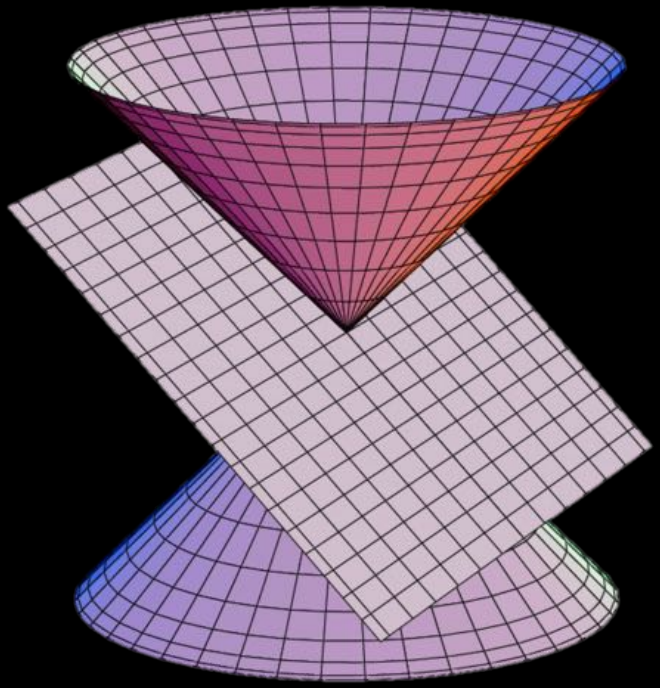




Modern Physics (Phys. IV): 2704

Professor Jasper Halekas
Van Allen 70
MWF 12:30-1:20 Lecture

Final Exam Details

- Final Exam is Ready
  - 10 questions
  - ~1/3 points on material on first midterm
  - ~1/3 points on material on second midterm
  - ~1/3 points on material since second midterm

  - Sample final and extra questions on last 1/3 posted

- You are allowed a calculator, and both sides of an 8.5x11 page for an equation sheet

- Final 7:30-9:30 am Friday 5/11 in Van 70 (this room)
  - Donuts on me!

Special Relativity

- The laws of physics are the same in all inertial reference frames
- The speed of light is the same in all inertial frames

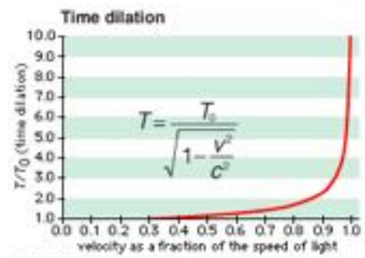
$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

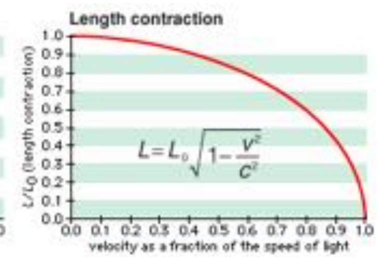
$$y' = y$$

$$z' = z$$

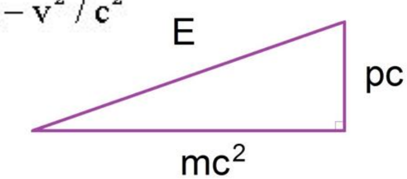
$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Time dilation
Length contraction
Velocity addition
Doppler effect
Relativity of simultaneity





$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$



$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Rest energy

Sample Question

- A muon is produced in the upper atmosphere. At rest, the lifetime of a muon is 2.5 ns. However, we observe it traveling the entire 100 km to the Earth's surface (which at its velocity takes 250 ns according to our clock) without decaying. In the frame of the muon, how far does it travel?

A. 1 km
B. 10 km
C. 100 km
D. 1000 km
E. 10000 km

$$\Delta t = 2.5 \text{ ns}$$

$$\Delta t' = \gamma \Delta t = 250 \text{ ns}$$

$$\Rightarrow \gamma = 100$$

$$\Delta L = \Delta L' / \gamma = \boxed{1 \text{ km}}$$

Sample Question

- Two particles of mass m are moving at $v = 0.866c$ from opposite directions towards a fixed point. They collide head-on, resulting in the formation of a new particle of mass M at rest. What is M? Note that at $v = 0.866c$, $\gamma = 2$.

A. $M = m$
B. $M = 2m$
C. $M = 4m$
D. $M = 1.5m$
E. None of the above

$$E = E_1 + E_2 = \gamma_1 mc^2 + \gamma_2 mc^2$$
$$= 4mc^2$$
$$\vec{p} = \vec{p}_1 + \vec{p}_2 = 0$$

$$Mc^2 = \sqrt{E^2 - (pc)^2}$$
$$= 4mc^2$$
$$\Rightarrow \boxed{M = 4m}$$

Particle Nature of EM Radiation

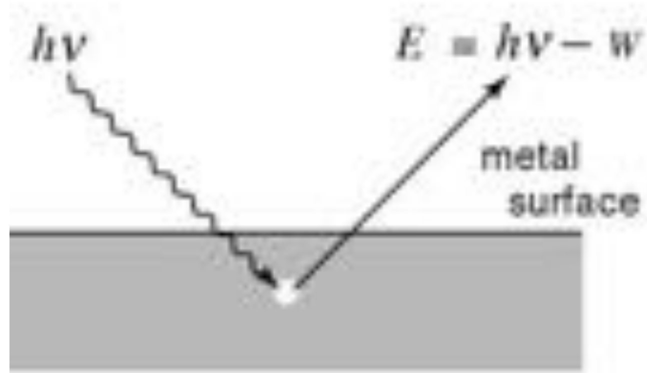
$$E = hf = \frac{hc}{\lambda} = \hbar\omega$$

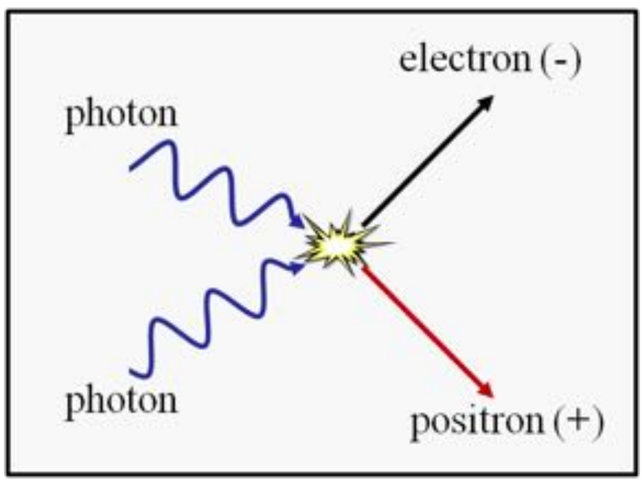
h = 6.63 x 10^{-34} Js → Planck constant

f = frequency of photon/electromagnetic radiation

c = 3 x 10^{8} m/s → speed of light in a vacuum

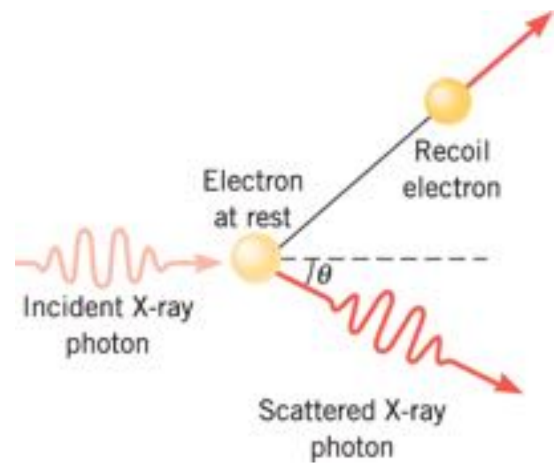
λ = wavelength of photon/electromagnetic radiation





Collisions must satisfy conservation of (relativistic) energy and linear momentum

E.g. pair production, which can occur if the photon energy is more than 2*511 keV ($m_e c^2$ = 511 keV)



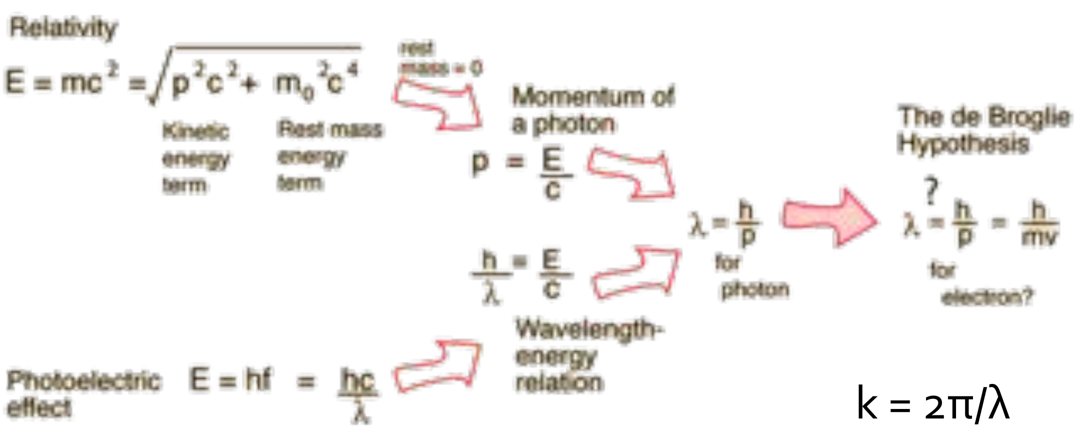
Sample Question

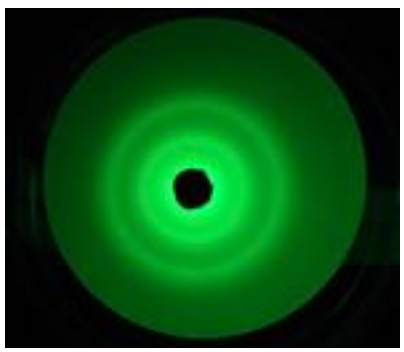
- An electron and a positron with total energies E_+ and E_- collide and produce two photons with wavelengths λ_1 and λ_2. What is the correct conservation of energy equation?

A. $E_+ + E_- = hc/\lambda_1 + hc/\lambda_2$
B. $KE_+ + KE_- = hc/\lambda_1 + hc/\lambda_2$
C. $E_+ + E_- + m_e c^2 + m_e c^2 = hc/\lambda_1 + hc/\lambda_2$
D. $E_+ + E_- = h\lambda_1 + h\lambda_2$
E. $KE_+ + KE_- = h/\lambda_1 + h/\lambda_2$

$$E_+ + E_- = h\nu_1 + h\nu_2$$
$$= hc/\lambda_1 + hc/\lambda_2$$

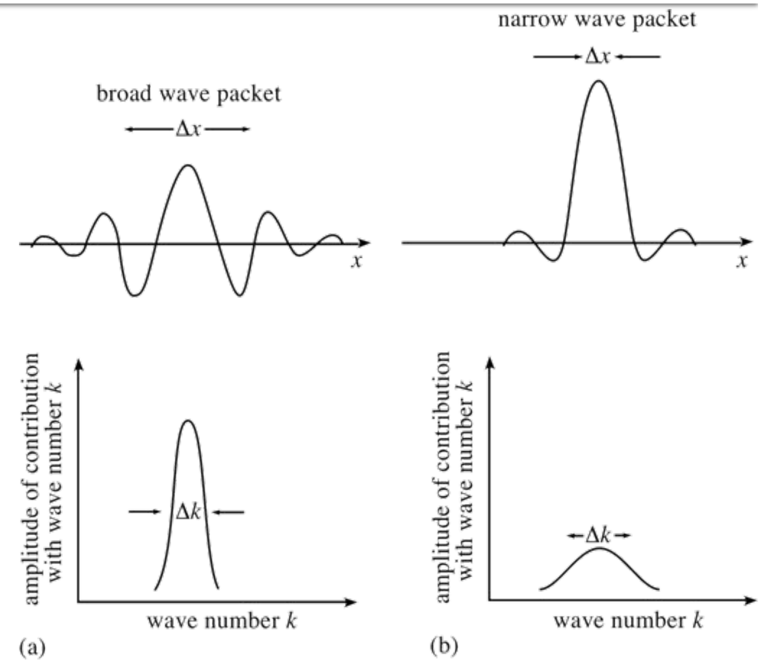
Wavelike Properties of Particles





$k = 2\pi/\lambda$

$k = p/\hbar$



$$\Delta p \, \Delta x \geq \frac{1}{2}\hbar \qquad \Delta E \, \Delta t \geq \frac{1}{2}\hbar$$

Sample Question

- A beam of electrons is incident on a slit, producing an interference pattern of maxima and minima. The slit width is halved. What happens to the spacing of the maxima/minima?

A. They get closer
B. They get farther apart
C. They stay the same

$\Delta p_y \Delta y \sim \hbar$

$\Rightarrow \Delta p_y \sim \hbar / \Delta y$

$\Delta y \downarrow \Rightarrow \Delta p_y \uparrow$

$\Rightarrow$ **spacing $\uparrow$**

Sample Question

- How do the de Broglie wavelengths of a 100 eV electron and a 1 eV proton compare? The mass of a proton is ~1800 times greater than that of an electron. (both are non-relativistic)

A. The electron wavelength is greater
B. The proton wavelength is greater
C. The wavelengths are approximately the same
D. Protons don't have a de Broglie wavelength

$$\lambda_D = h/p = h/\sqrt{2mK}$$

$$\lambda_{De}/\lambda_{Dp} = \frac{h/\sqrt{2m_e \cdot 100\,eV}}{h/\sqrt{2m_p \cdot 1\,eV}}$$

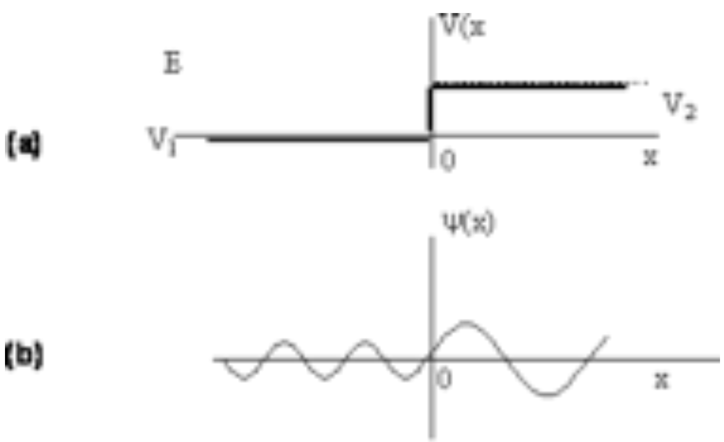
$$= \sqrt{\frac{m_p}{100\,m_e}} = \sqrt{\frac{1840}{100}} > 1$$

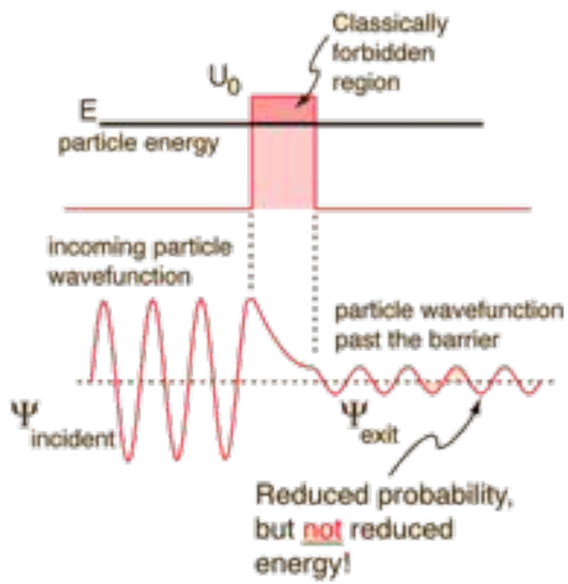
Schrödinger Equation

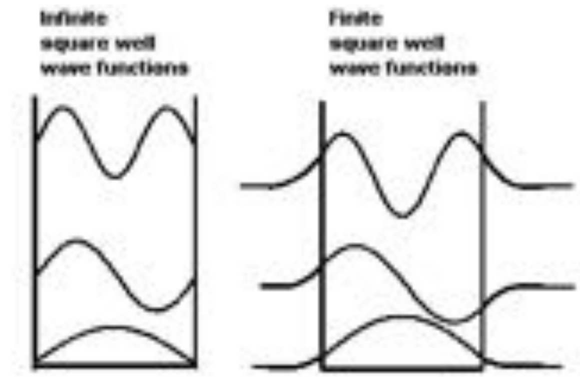
$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi(x)}{\partial x^2}+U(x)\Psi(x)=E\Psi(x)$$

Time independent equation

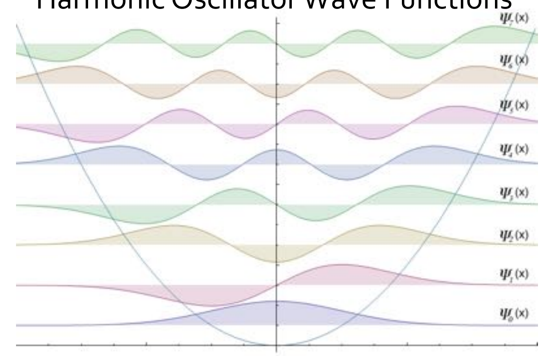
Solutions must be continuous
Continuous in slope unless U = ∞





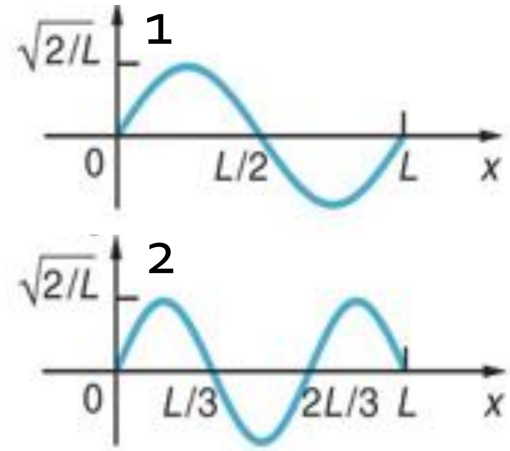


Harmonic Oscillator Wave Functions



Sample Question

- Two particles are trapped in separate square well potentials (with the same size, depth, etc). The first particle's wave function has one full wavelength across the box. The second particle's wave function has 1.5 wavelengths across the box. Which is in a higher energy level?



A. First particle
B. Second particle
C. Both are the same
D. No way to tell from information given

$$E = p^2/2m + U$$

$$= h^2/2m\lambda^2 + U$$

smaller λ $\Rightarrow$ higher kinetic energy

$\Rightarrow$ higher E for const. U

Sample Question

- A particle incident on a potential step at $x = 0$ has the wave function $\psi_1 = Ae^{ikx}+Be^{-ikx}$ for $x < 0$, and the wave function $\psi_2 = Ce^{-k'x}$ for $x > 0$. Which equations are correct?

A. $A + B = C$
B. $ik(A-B) = -k'C$
C. $A^2 + B^2 = C^2$
D. $A = -B, A = C$
E. $A = C, B = 0$

ψ continuous @ $x = 0$

$$Ae^0 + Be^0 = Ce^0$$

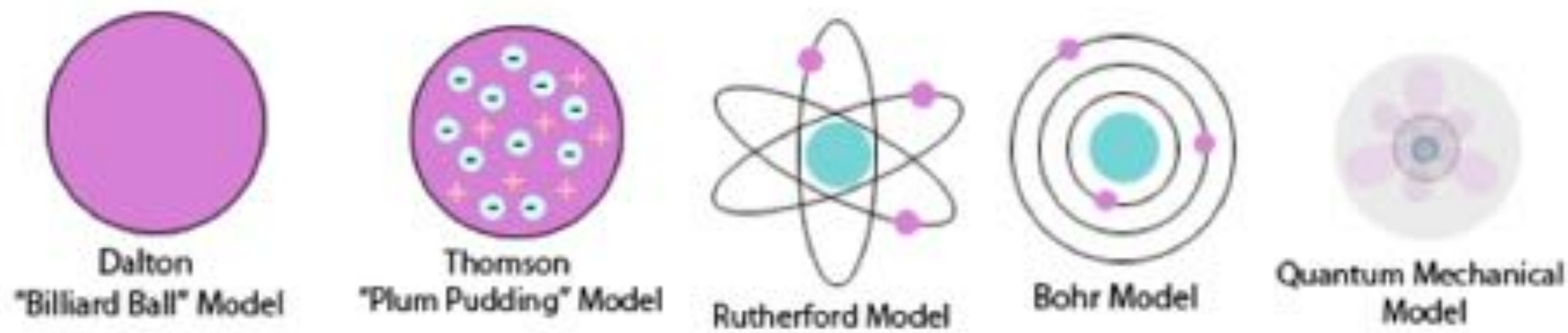
$$\Rightarrow \boxed{A + B = C}$$

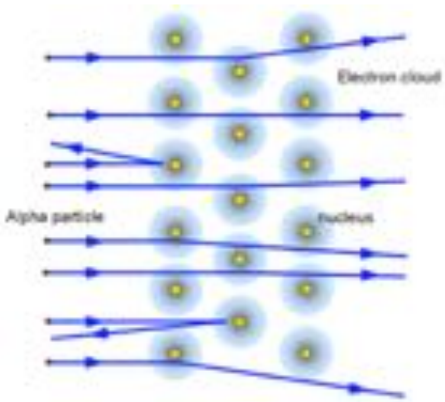
ψ' continuous @ $x = 0$

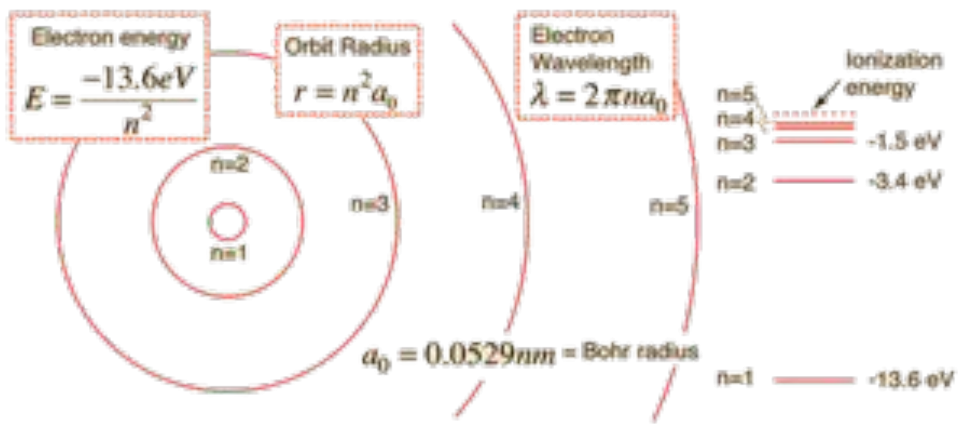
$$ikAe^0 - ikBe^0 = -\kappa' C e^0$$

$$\Rightarrow \boxed{ik(A - B) = -\kappa' C}$$

Bohr Atom


Dalton
"Billiard Ball" Model
Thomson
"Plum Pudding" Model
Rutherford Model
Bohr Model
Quantum Mechanical
Model


Electron cloud
Alpha particle
nucleus


Electron energy
$E = \frac{-13.6eV}{n^2}$
Orbit Radius
$r = n^2 a_0$
Electron
Wavelength
$\lambda = 2\pi n a_0$
Ionization
energy
n=5
n=4
n=3
n=2
n=1
n=2
n=1
n=3
n=4
n=5
-1.5 eV
-3.4 eV
$a_0 = 0.0529nm$ = Bohr radius
-13.6 eV

Sample Question

- Imagine an atom of Boron (Z = 5) which has been ionized four times, leaving it with a single electron. How much energy would it take to remove that final electron?

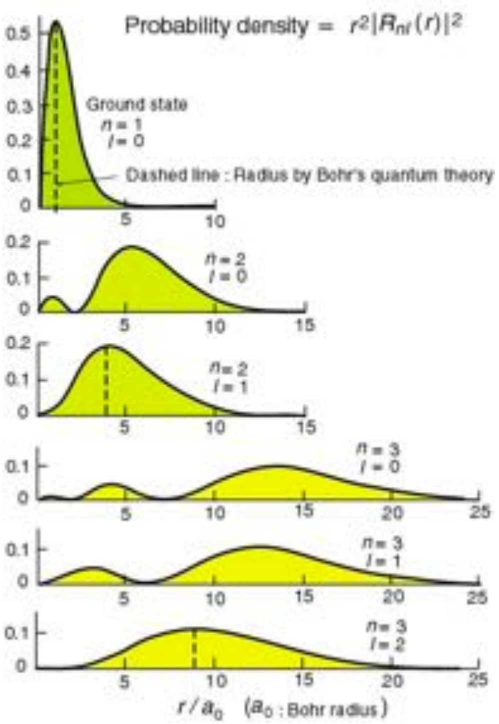
A. 13.6 eV
B. 13.6/25 = 0.544 eV
C. 13.6/5 = 2.72 eV
D. 13.6*5 = 68 eV
E. 13.6*25 = 340 eV

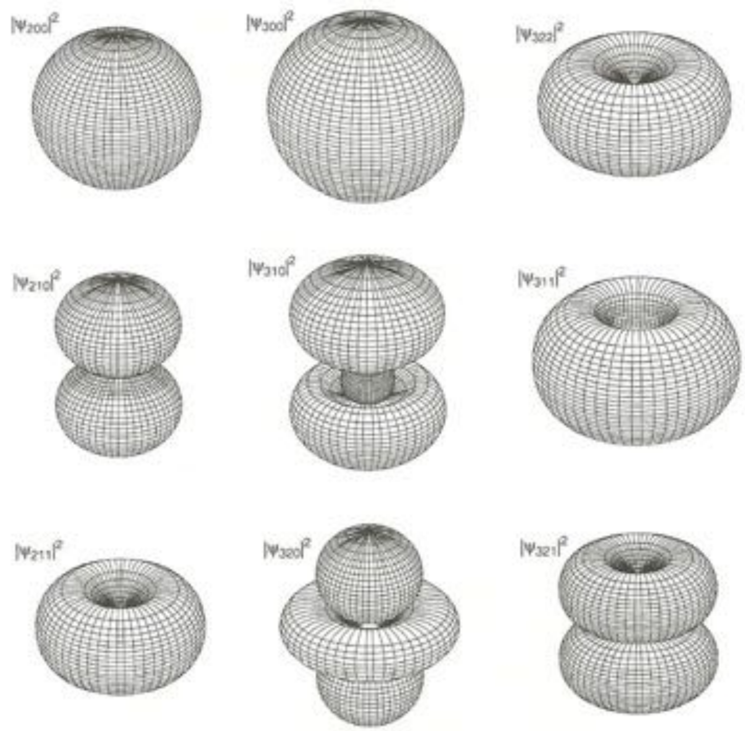
$$\Delta E = 13.6\ Z^2/4^2$$

$$= 13.6 \cdot Z^2$$

$$= \boxed{340\ eV}$$

Hydrogen Atom





	n	*l*	*m*	*s*
1s	1	0	0	½, - ½
2s	2	0	0	½, - ½
2p	2	1	1, 0, -1	½, - ½
3s	3	0	0	½, - ½
3p	3	1	1, 0, -1	½, - ½
3d	3	2	2, 1, 0, -1, -2	½, - ½
4s	4	0	0	½, - ½
4p	4	1	1, 0, -1	½, - ½
4d	4	2	2, 1, 0, -1, -2	½, - ½
4f	4	3	3, 2, 1, 0, -1, -2, -3	½, - ½

n -> Energy
l -> Angular momentum
m_l -> Orientation of l
m_s -> Electron spin