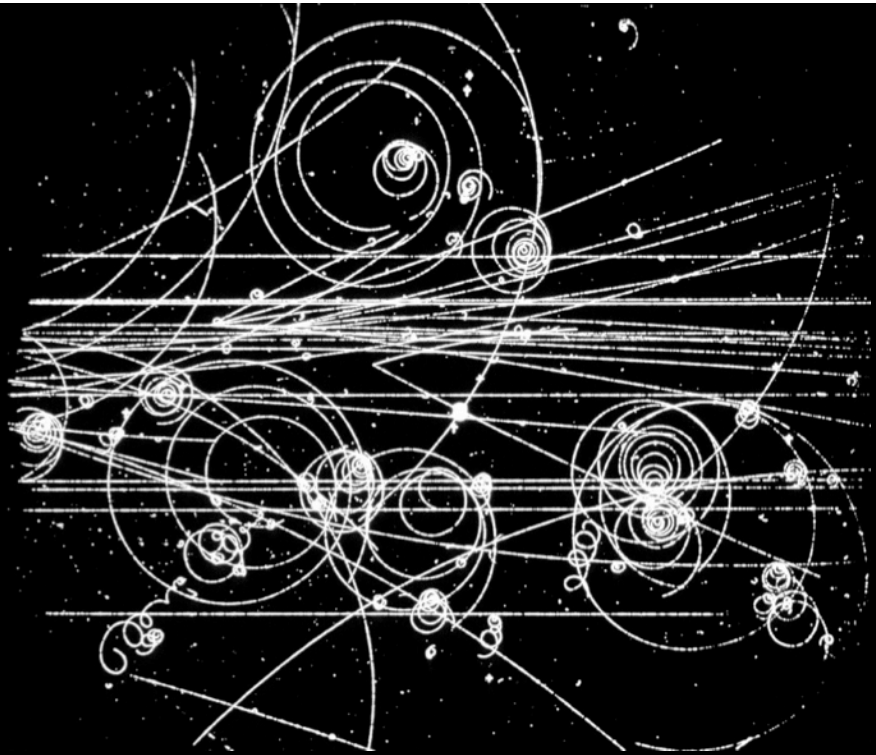
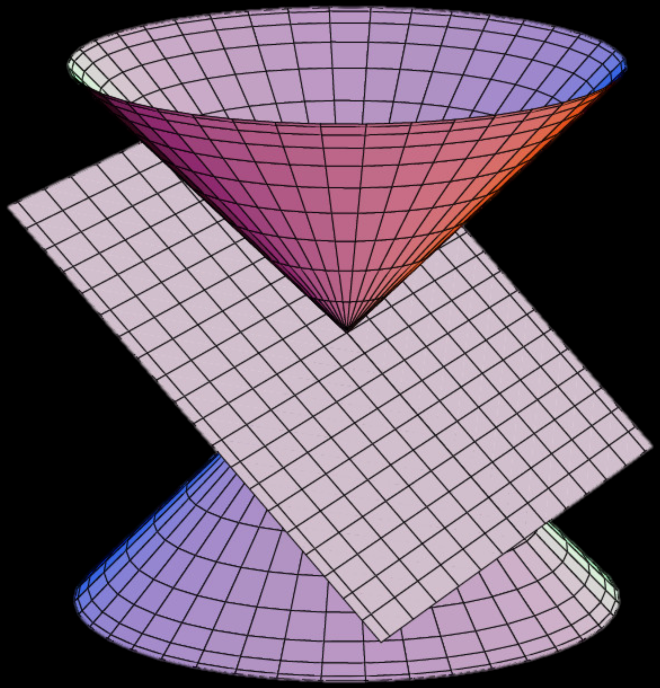




Modern Physics (Phys. IV): 2704

Professor Jasper Halekas
Van Allen 70
MWF 12:30-1:20 Lecture

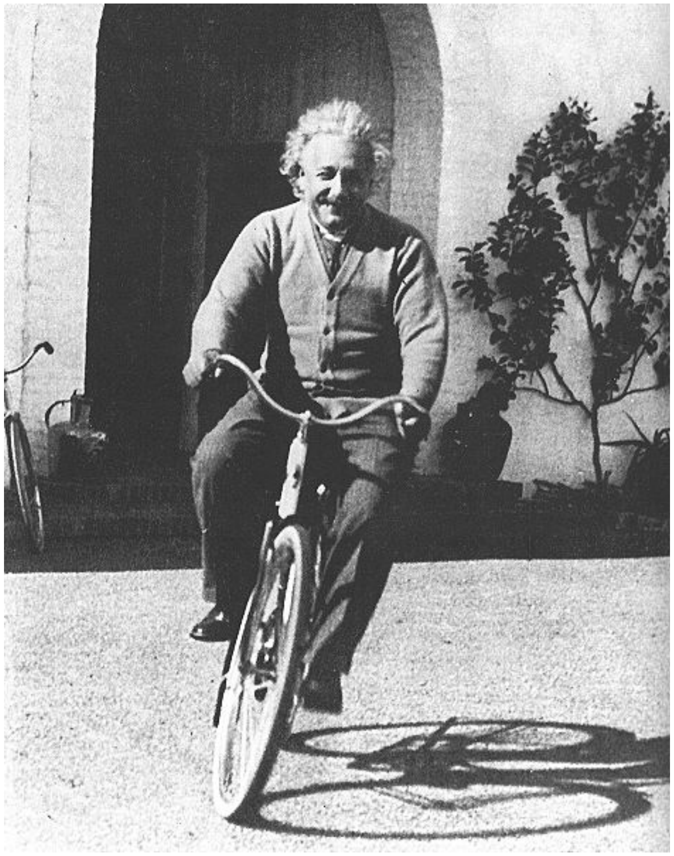
Homework #1 Statistics

- For the thirty submitted assignments, the average score was 8.82/10
  - Good job!
- However, eight students did not submit an assignment…
  - What happened?

Einstein's Postulates

The laws of physics are the same in all inertial frames of reference.

The speed of light is the same in all inertial frames of reference.



Velocity transformation (3D)

Classical:

$$v'_x = v_x - u$$

$$v'_y = v_y$$

$$v'_z = v_z$$

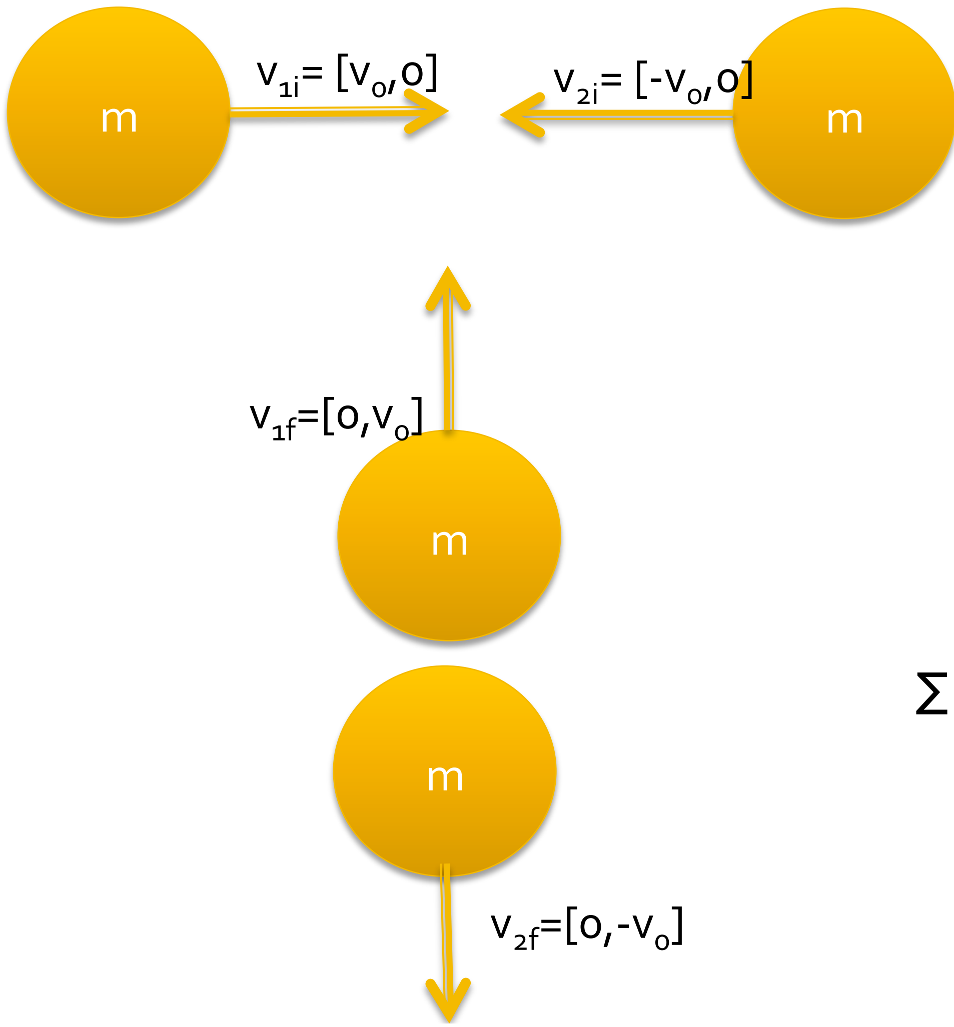
Relativistic:

$$v'_x = \frac{v_x - u}{1 - v_x u / c^2}$$

$$v'_y = \frac{v_y}{\gamma\left(1 - v_x u / c^2\right)}$$

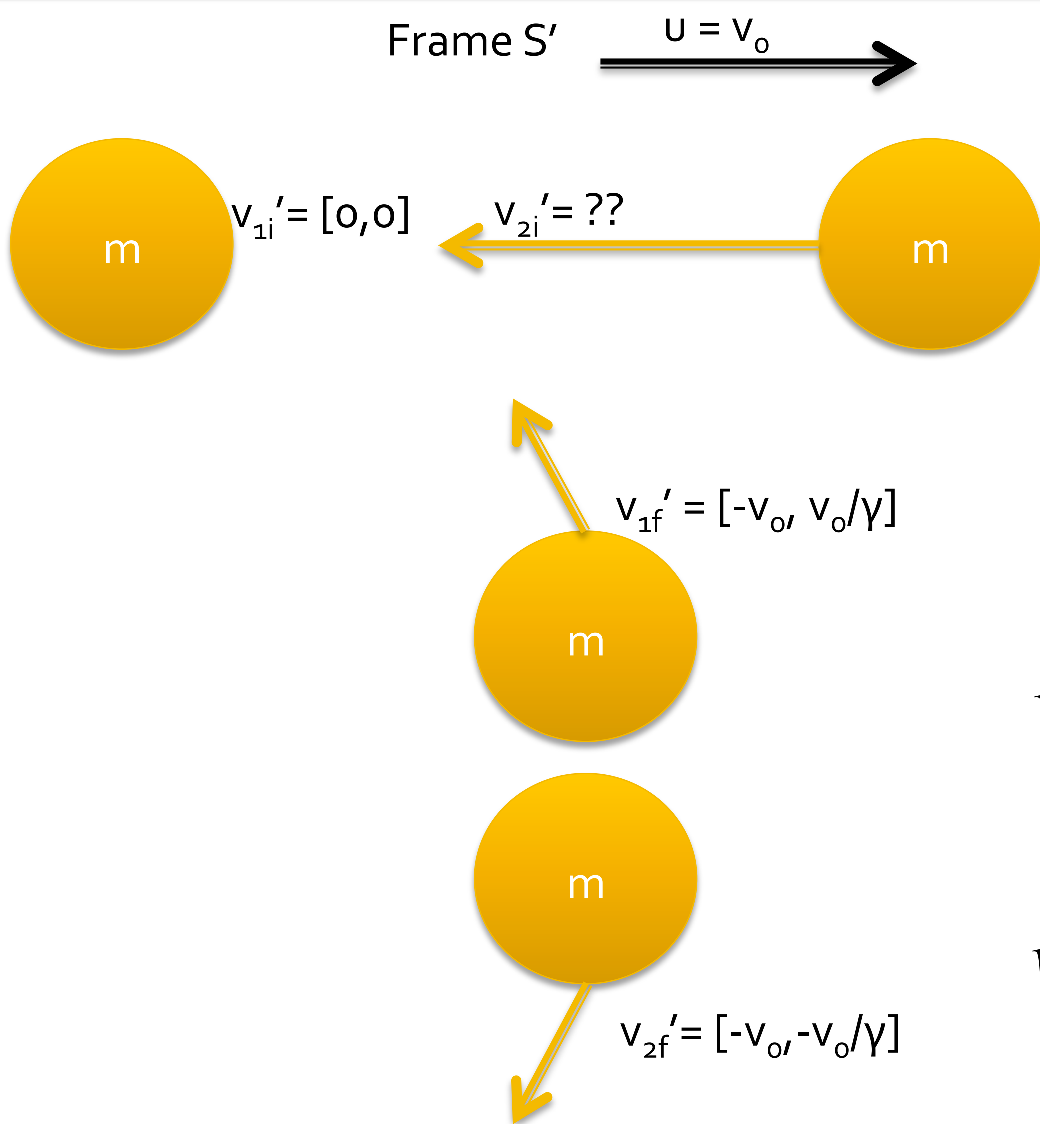
$$v'_z = \frac{v_z}{\gamma\left(1 - v_x u / c^2\right)}$$

Conservation of Classical Momentum in an Elastic Collision



$$\Sigma m v_i = \Sigma m v_f = 0$$

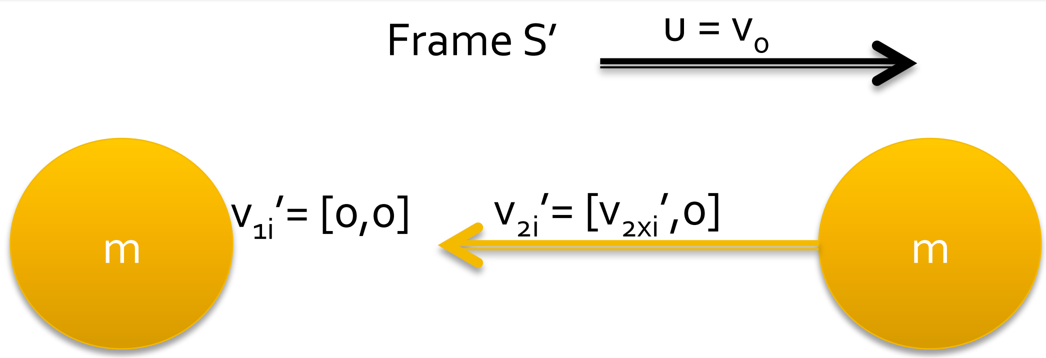
Conservation of Momentum in a Moving Frame

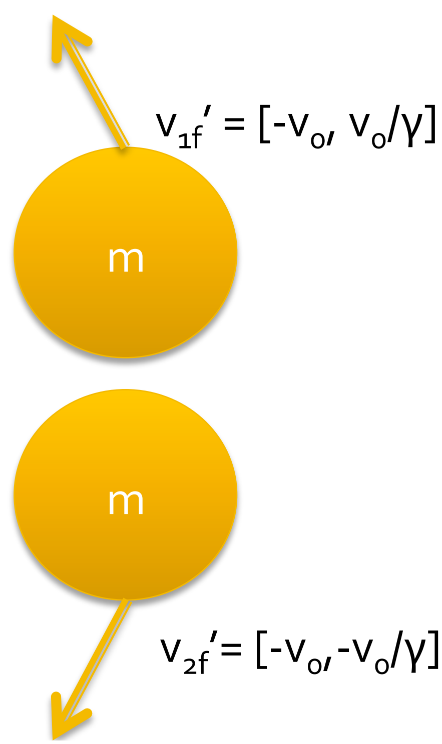


$$v'_x = \frac{v_x - u}{1 - v_x u / c^2}$$

$$v'_y = \frac{v_y}{\gamma\left(1 - v_x u / c^2\right)}$$

Concept Check





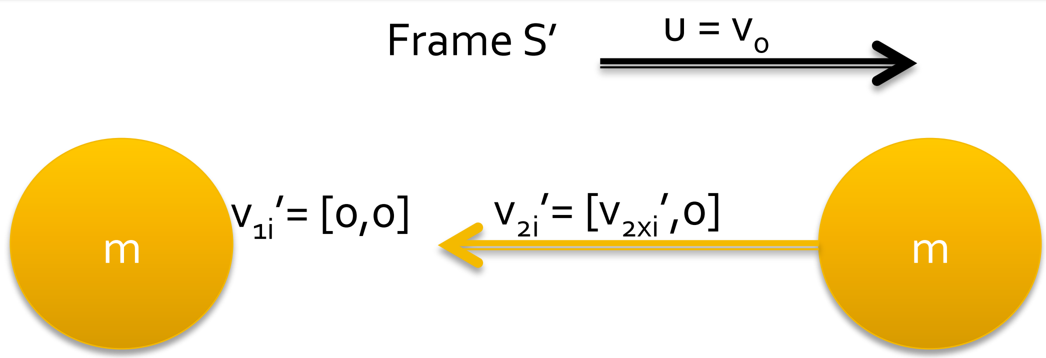
If we transform to frame S′, moving at u = v_o w/ respect to S, What is the value of v_{2xi}′ as observed in frame x′??

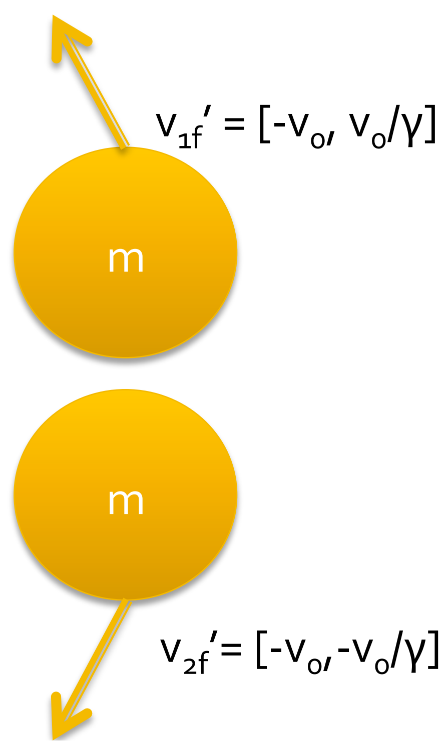
A. $-2v_o$
B. $2v_o$
C. $-2v_o/(1+v_o^2/c^2)$
D. $-2v_o/(\mathrm{sqrt}(1-v_o^2/c^2))$

$$v'_x = \frac{v_x - u}{1 - v_x u / c^2}$$

$$v'_y = \frac{v_y}{\gamma\left(1 - v_x u / c^2\right)}$$

Concept Check

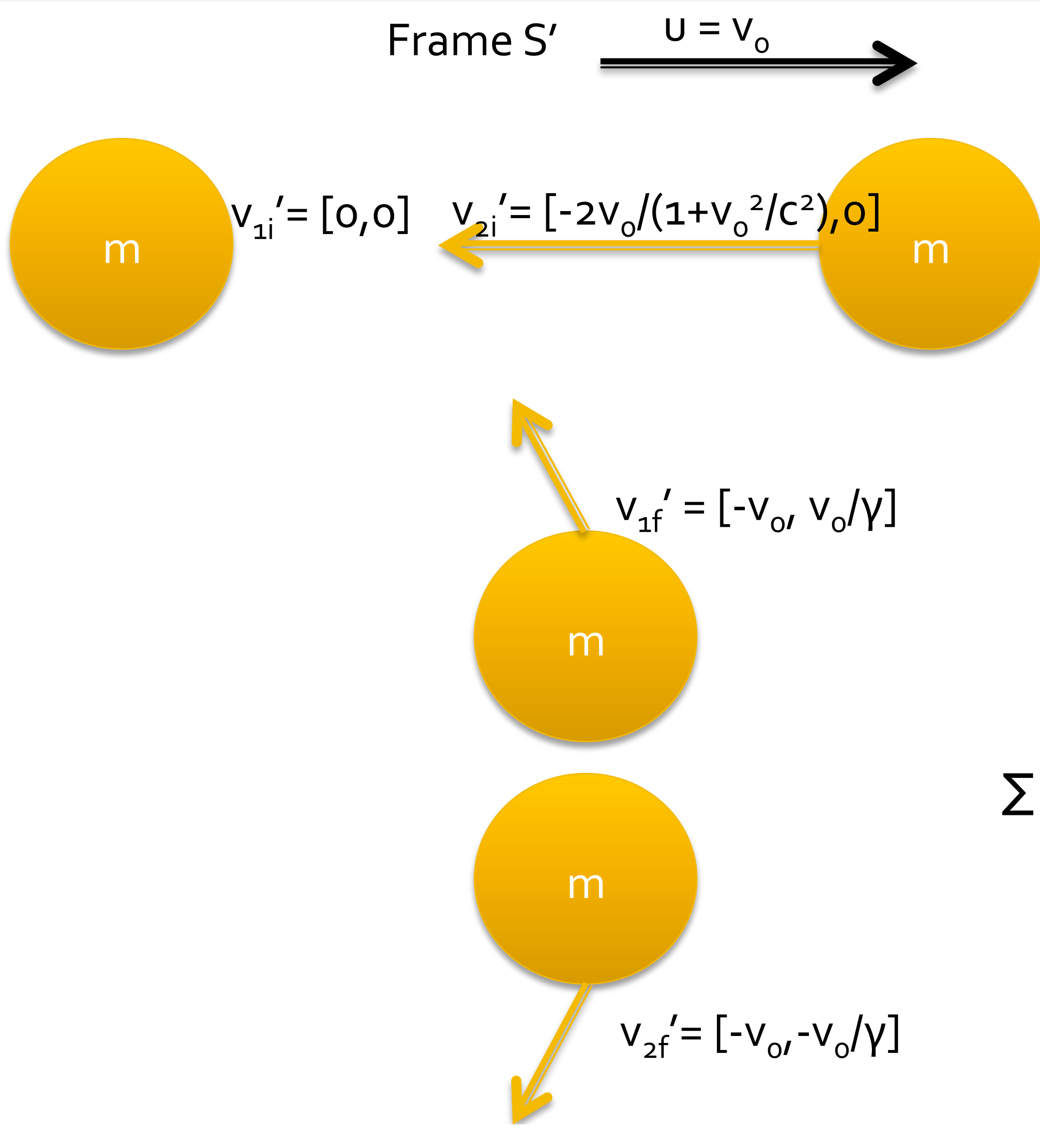




If we transform to frame S′, moving at u = v_0 w/ respect to S, What is the value of v_{2xi}' as observed in frame x′??

A. $-2v_0$
B. $2v_0$
C. $-2v_0/(1+v_0^2/c^2)$
D. $-2v_0/(\mathrm{sqrt}(1-v_0^2/c^2))$

Conservation of Classical Momentum in a Moving Frame



$\Sigma m v_i' \neq \Sigma m v_f'$

Relativistic Momentum

The relativistic momentum p of a particle with mass *m* and velocity **v** is:

$$\boldsymbol{p} = \gamma m \mathbf{v}$$

- Note the gamma factor is that corresponding to **v**, not to a transformation velocity u!
- This new definition of the relativistic momentum fulfills the condition of conservation of momentum (given either no or balanced external forces) in all frames.

Relativistic Momentum

$$\vec{p} = m\vec{v}/\sqrt{1-v^2/c^2}$$

$$v_i' = -2v_0/(1+v_0^2/c^2)$$

$$p_{xi}' = m v_{xi}'/\sqrt{1-v_i'^2/c^2}$$

$$v_i'^2/c^2 = \frac{4v_0^2}{c^2}\cdot\frac{1}{(1+v_0^2/c^2)^2}$$

$$1-v_i'^2/c^2 = \frac{c^2(1+v_0^2/c^2)^2 - 4v_0^2}{c^2(1+v_0^2/c^2)^2}$$

$$= (1-v_0^2/c^2)^2/(1+v_0^2/c^2)^2$$

$$\Rightarrow p_{xi}' = -\frac{2mv_0}{1+v_0^2/c^2}\cdot\frac{1+v_0^2/c^2}{1-v_0^2/c^2}$$

$$= -2mv_0/(1-v_0^2/c^2)$$

$$p_{xf}' = -2mv_0/\sqrt{1-v_f'^2/c^2}$$

$$v_f' = \sqrt{v_0^2+(v_0/\gamma)^2}$$

$$= v_0\sqrt{1+1/\gamma^2}$$

$$= v_0\sqrt{1+1-v_0^2/c^2}$$

$$v_f'^2 = v_0^2(2-v_0^2/c^2)$$

$$1 - v_f'^2/c^2 = 1 - \frac{2v_0^2}{c^2} + v_0^4/c^4$$

$$= (1 - v_0^2/c^2)^2$$

so $1/\sqrt{1 - v_f'^2/c^2} = 1/(1 - v_0^2/c^2)$

$$\Rightarrow p'_{xf} = -2mv_0/(1 - v_0^2/c^2)$$

$$p'_{xf} = p'_{xi}$$ //

Classical vs. Relativistic Momentum

An electron has a mass m = 9.1 x 10^{-31} kg. The table below shows the classical and relativistic momentum of the electron at various speeds (units of 10^{-22} kg·m/s):

v	$p=m\cdot v$ classical	$p=\gamma m\cdot v$ relativistic	difference [%]
0.05c	0.1365	0.1367	0.125
0.1c	0.273	0.2743	0.50
0.5c	1.37	1.58	15.5
0.9c	2.46	5.64	129
0.99c	2.7	19.2	609

Lab 1

Relativistic Force

$$\mathbf{F} = m\mathbf{a} \qquad \mathbf{F} = \frac{d\mathbf{p}}{dt}$$

Using the definition of the relativistic momentum we obtain a suitable definition for a relativistic force:

$$\mathbf{F} \equiv \frac{d\mathbf{p}}{dt} = \frac{d}{dt}(\gamma m v)$$

Work - Energy

$$F = dp/dt = d/dt\,(\gamma m v)$$
$$= dx/dt \cdot d/dx\,(\gamma m v)$$
$$= v\, d/dx\,(\gamma m v)$$

$$W = \int F\,dx = \Delta KE$$

$$\int F\,dx = \int v\, d/dx\,(\gamma m v)\,dx$$

Integrate by parts

$$\int v\,dp = vp - \int p\,dv$$

$$= v\gamma m v - \int_0^v \frac{m v'}{\sqrt{1 - v'^2/c^2}}\,dv'$$

$$= \gamma m v^2 + mc^2 \sqrt{1 - v'^2/c^2}\,\Big|_0^v$$
$$= \gamma m v^2 + \frac{1}{\gamma} mc^2 - mc^2$$
$$= \gamma\left(mv^2 + (1 - v^2/c^2)\, mc^2\right) - mc^2$$
$$= \gamma mc^2 - mc^2$$
$$= (\gamma - 1)\, mc^2$$
$$= \Delta KE$$

$$(= KE \text{ since } KE(v=0) = 0)$$

Relativistic Kinetic Energy

The relativistic kinetic energy K of a particle with a rest mass *m* is:

$$K = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2$$

For slow velocities the relativistic energy equation gives the same value as the classical equation! Using the binomial approximation for γ: $\gamma \approx 1 + \frac{1}{2}v^2/c^2$

$\rightarrow K = \gamma mc^2 - mc^2 \approx mc^2 + \frac{1}{2} mc^2 v^2/c^2 - mc^2 = \frac{1}{2} mv^2$

Total Energy

We rewrite the equation for the relativistic kinetic energy and define the total energy of a particle as:

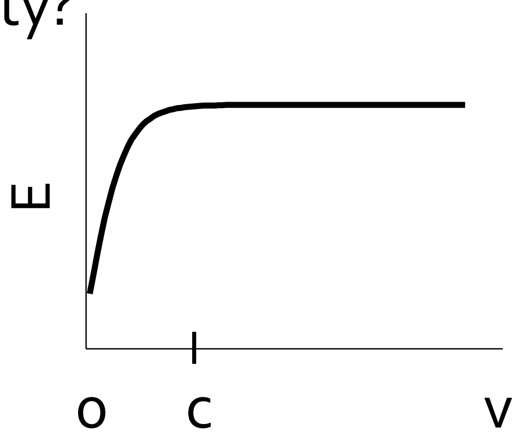
$$E = \gamma mc^2 = K + mc^2$$

This definition of the relativistic *mass-energy E* fulfills the condition of conservation of total energy.

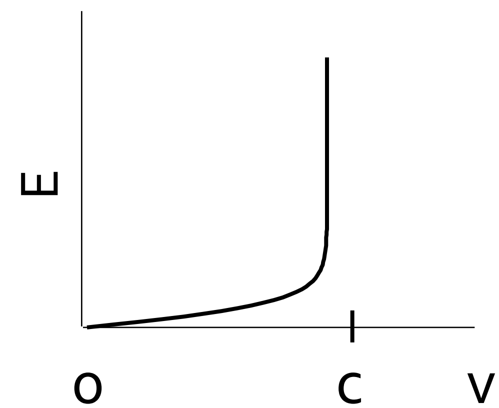
Concept Check

Which graph best represents the total energy of a particle (particle's mass $m>0$) as a function of its velocity v, in special relativity?

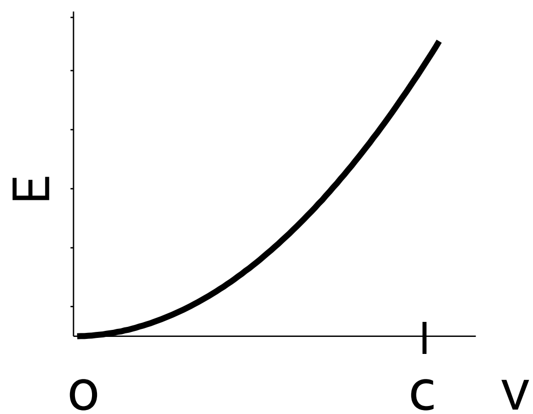
a)



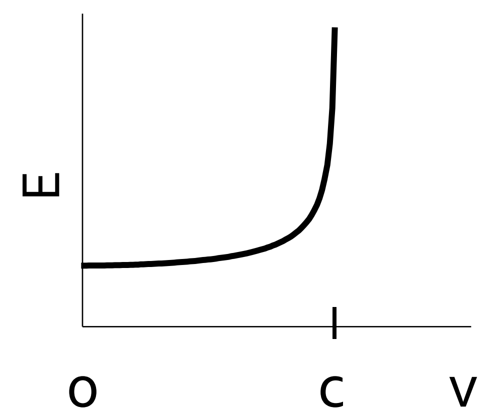
b)



c)



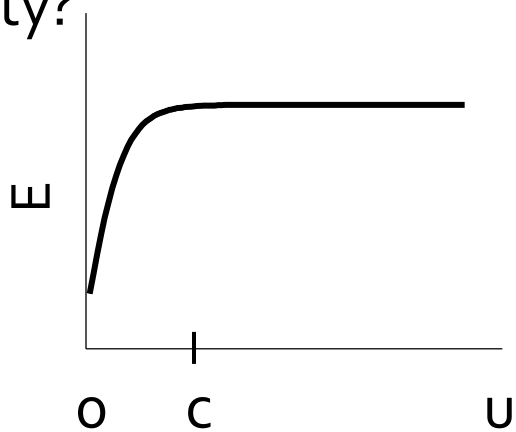
d)



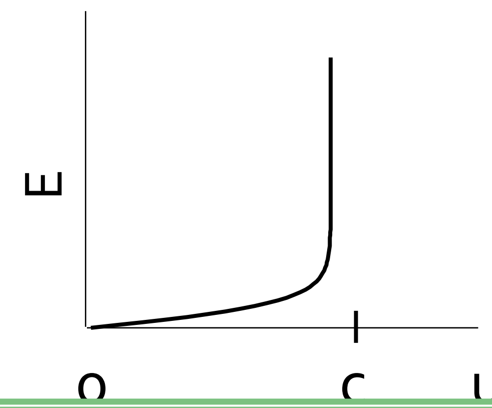
Concept Check

Which graph best represents the total energy of a particle (particle's mass m>0) as a function of its velocity u, in special relativity?

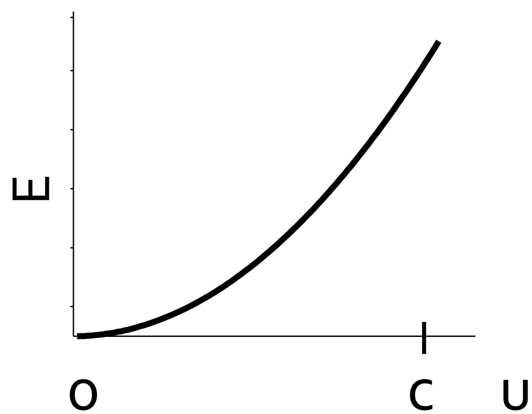
a)



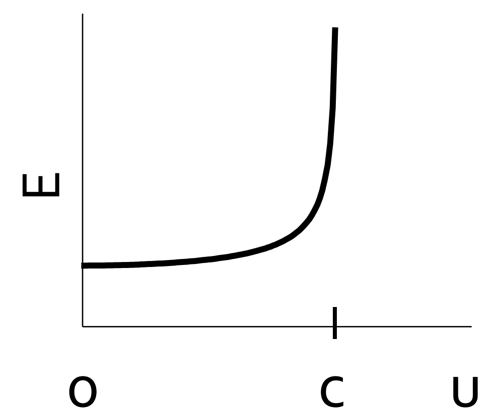
b)



c)



d)



Rest Energy

- A particle at rest still has non-zero energy
  - The rest energy $E = mc^2$
  - For a proton, the rest energy is 0.94 GeV
  - Even for an electron, the rest energy is 511 keV
    - Compare to the measly 500 eV of kinetic energy we gave our electron in the first lab…

Concept Check: Inelastic Collision

$$E_1 = \gamma mc^2 = K + mc^2 \qquad E_2 = \gamma mc^2 = K + mc^2$$

Two equal masses approach at equal and opposite relativistic speed and undergo a totally inelastic collision. What is the mass of the final combined object?

A. M = 2m
B. M < 2m
C. M > 2m

Concept Check: Inelastic Collision

$$E_1 = \gamma mc^2 = K + mc^2 \qquad E_2 = \gamma mc^2 = K + mc^2$$

Two equal masses approach at equal and opposite relativistic speed and undergo a totally inelastic collision. What is the mass of the final combined object?

A. M = 2m
B. M < 2m
C. M > 2m

Equivalence of Mass and Energy

$$E_{tot,final} = Mc^2 = 2K + 2mc^2 = E_{tot,initial}$$

We find that the total mass M of the final system is bigger than the sum of the masses of the two parts! M>2m.

Potential energy inside an object contributes to its mass!!!

Energy and Momentum

$p = \gamma m v$, $E = \gamma m c^2$

$$E^2 = \gamma^2 m^2 c^4$$
$$= \frac{m^2 c^4}{1 - v^2/c^2}$$

$$p^2 = \gamma^2 m^2 v^2$$
$$= \frac{m^2 v^2}{1 - v^2/c^2}$$

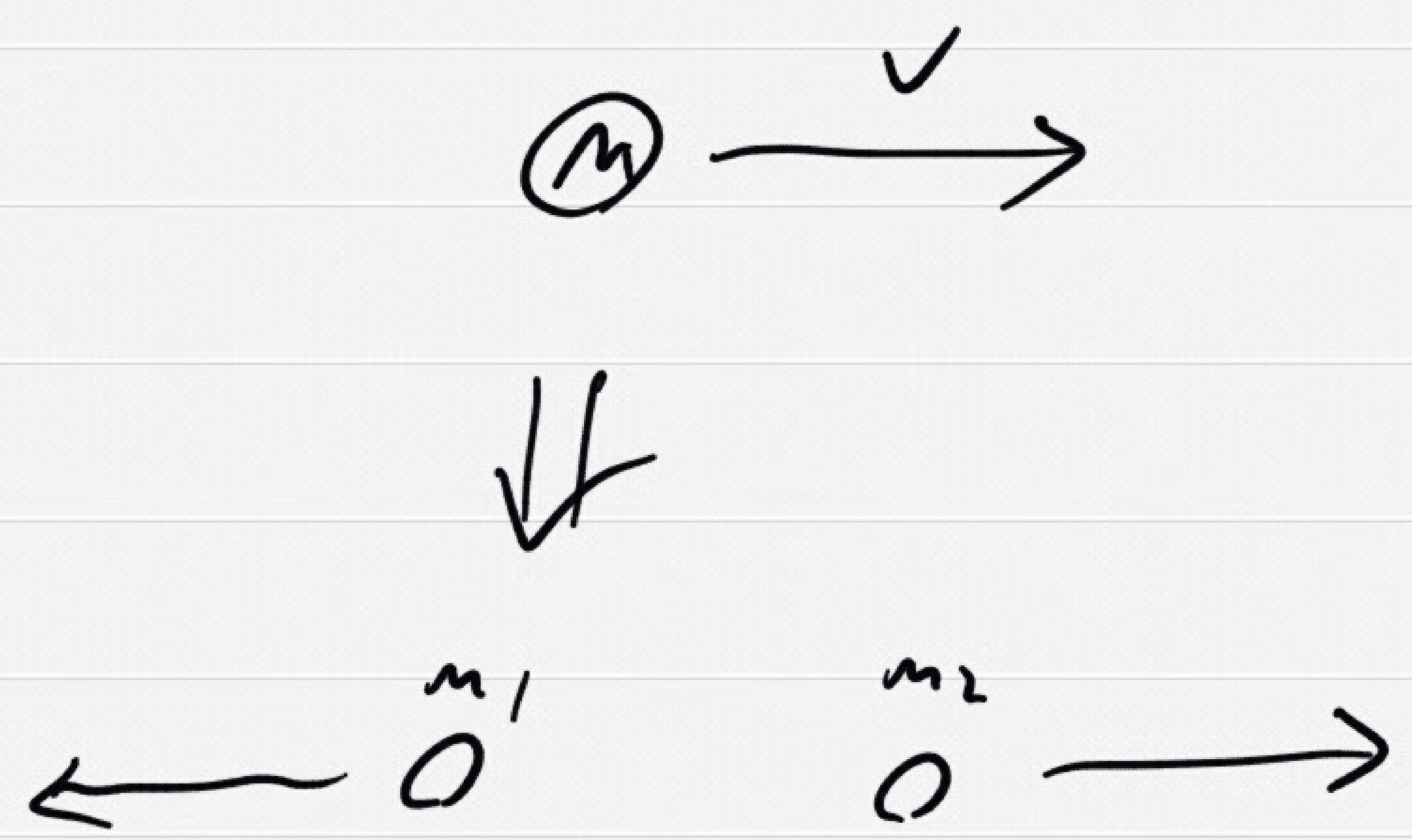
$$E^2 - p^2 c^2$$

$$= \frac{m^2 c^4 - m^2 c^2 v^2}{1 - v^2/c^2}$$

$$= m^2 c^4$$

or $$\boxed{E^2 = (pc)^2 + (mc^2)^2}$$

Sample Problem



$KE_1 = 25\ \text{MeV}$ $\qquad$ $KE_2 = 282\ \text{MeV}$

$m_1c^2 = m_2c^2 = 140\ \text{MeV}$

$M = ?\ ,\quad v = ?$

$E_1 = m_1c^2 + KE_1 = 165\ \text{MeV}$

$E_2 = m_2c^2 + KE_2 = 422\ \text{MeV}$

$E_0 = E_1 + E_2 = 587\ \text{MeV}$

$p_1c = \sqrt{E_1^2 - (m_1c^2)^2} = 87\ \text{MeV}$

$p_2c = \sqrt{E_2^2 - (m_2c^2)^2} = 398\ \text{MeV}$

$p_0c = p_2c - p_1c = 398 - 87 = 311\ \text{MeV}$

$Mc^2 = \sqrt{E_0^2 - (p_0c)^2} = 498\ \text{MeV}$

$E_0 = \gamma Mc^2$

$\Rightarrow \gamma = 587/498 = 1.18$

$\Rightarrow v/c = \sqrt{1 - (1/1.18)^2} = \boxed{0.53}$