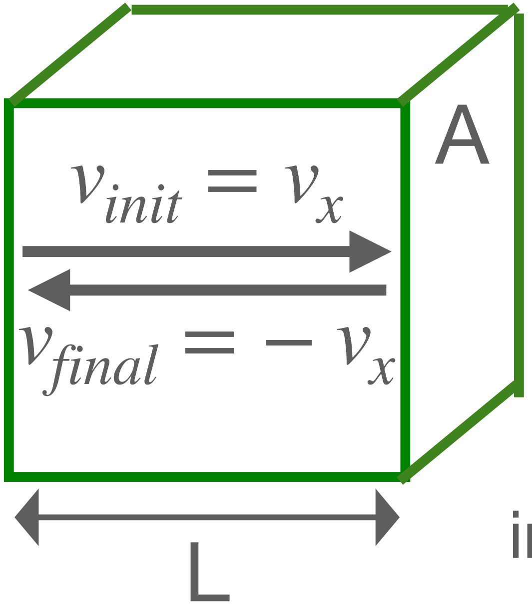


Assume a molecule makes many round trips before colliding with another molecule

time between collisions with one wall

$$\Delta t = \frac{2L}{v_x}$$

impulse per collision: $\Delta p = m(v_x - (-v_x)) = 2v_x$

one molecule:

$$P = \frac{F_x}{A} = \frac{\Delta p / \Delta t}{A} = \frac{mv_x^2}{AL}$$

N molecules:

$$PV = Nm\overline{v_x^2} = NkT$$

KE for x-motion:

$$KE = N\frac{1}{2}m\overline{v_x^2} = N\frac{1}{2}kT$$

all 3 directions

$$KE = N\left(\underbrace{\frac{1}{2}m\overline{v_x^2}}_{\frac{1}{2}kT} + \underbrace{\frac{1}{2}m\overline{v_y^2}}_{\frac{1}{2}kT} + \underbrace{\frac{1}{2}m\overline{v_z^2}}_{\frac{1}{2}kT}\right) = 3N\frac{1}{2}kT$$

special case of **Equipartion Theorem**

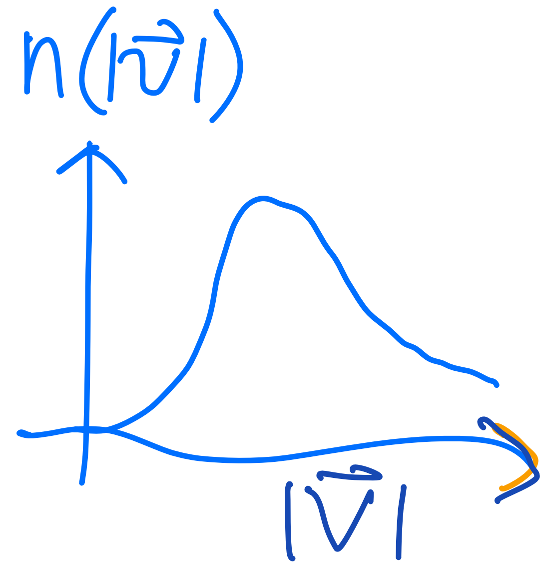
at thermal equilibrium, each quadratic contribution to energy (Degree of Freedom) will have an avg of $\frac{1}{2}kT$ energy

$$KE_{trans} = 3N\frac{1}{2}kT = N\frac{1}{2}m\overline{v^2}$$

$$v^2 = \vec{v}\cdot\vec{v}$$

$$\frac{3}{2}kT = \frac{1}{2}m\overline{v^2}$$

$$v_{RMS} = \sqrt{\overline{v^2}} = \sqrt{3kT/m}$$



$$v_{RMS} = \sqrt{\overline{v^2}} \neq \overline{|\vec{v}|}$$

Equipartition theorem

$$U_{thermal} = N * f * \frac{1}{2} kT$$

N: # of particles

f: # of degrees of freedom

degrees of freedom ↓ **quadratic** meaning the energy depends on square of a coord or momentum.

$$E = \sum_{n=1}^{N} \frac{p_{x,n}^2}{2m} + \frac{p_{y,n}^2}{2m} + \frac{p_{z,n}^2}{2m}$$

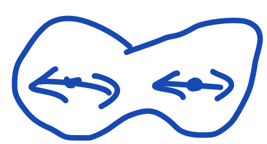
} only translation

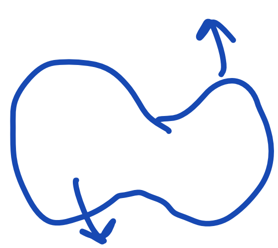
$$E = \sum_{n=1}^{N} \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}m\omega^2 y^2 + \frac{1}{2}m\omega^2 z^2$$

6 DoF per particle. Each particle moves in 3D in a harmonic oscillator potential.

system has 6N DoF

average: $U_{thermal} = 6N \times \frac{1}{2}kT$

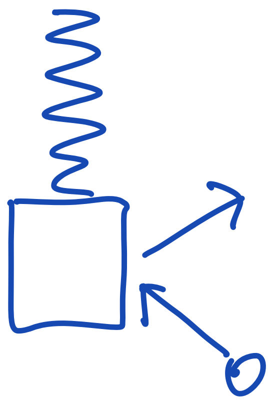
add to E: vibration  N_2 "spring const" → $\frac{1}{2}k\Delta x^2$

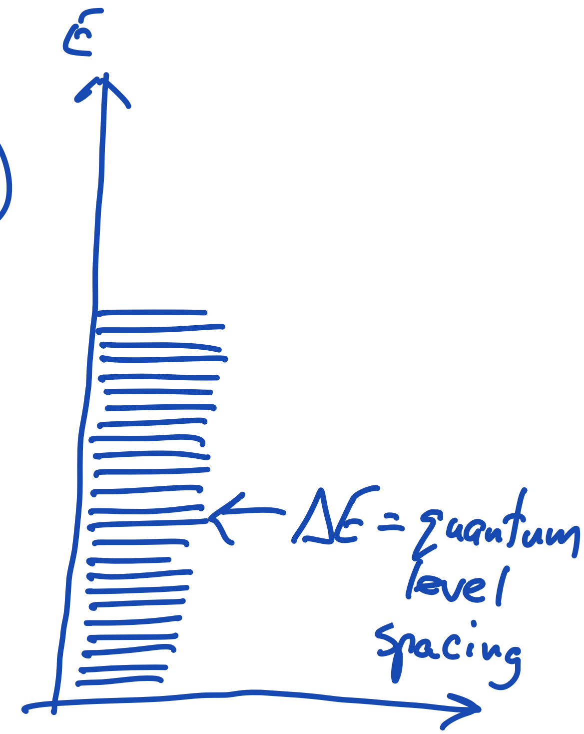
rotate:  $2 \times \frac{1}{2} I\omega^2$

Equipartition Theorem is CLASSICAL

Requires: $kT \gg \Delta E$

1D harmonic oscillator: $E_{osc} = \hbar\omega\left(n + \tfrac{1}{2}\right)$





if $kT \gg \Delta E$ the ΔE steps don't matter.

air molecule with $E \sim kT$ can change E_{osc} by $\sim kT$

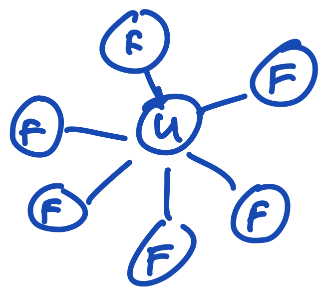
$kT \lesssim \Delta E$: ΔE matters, energy steps important, not continuous energy. NOT CLASSICAL. E.P.T. does not apply

$kT \gg \Delta E$: can't see energy steps, energy look classical. E.P.T. OK

U-235 ← fissionable

U-238 ← not fissionable

Gaseous Diffusion: separate U-235 by exploiting fact that UF_6 (235) molecules move faster than UF_6 (238)



Uranium Hexafluoride, UF_6, a gas

U-238:

$$v_{RMS} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.31\ J/mol\cdot K)(300\ K)}{0.352\ kg/mol}} = 145.8\ \frac{m}{s}$$

molar mass = mass per mol

$$v_{RMS}(UF_6, 235) = 146.4\ \frac{m}{s}$$

U-238 is only 0.6 m/s slower!

☆ Should be 19 ☆

$M_F = 16\ g/mol$

$M_{U\text{-}235} = 235\ g/mol$

$M(UF_6, 235) = 349\ g/mole$

$M(UF_6, 238) = 352\ g/mol$