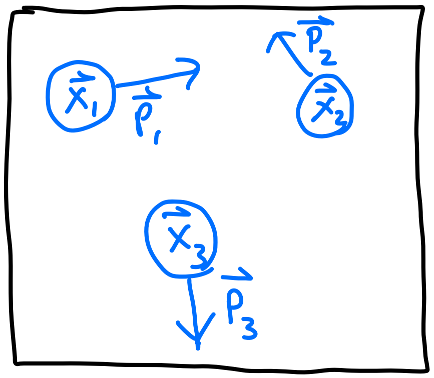


Microstate : microscopic specification of every degree of freedom

$(\vec{x}_1, \vec{x}_2, \dots \vec{x}_N, \vec{p}_1, \vec{p}_2, \dots \vec{p}_N)$

$3 \times 2 \times N$ numbers!

x, y, z   x, p   # of particles

macrostate : P, V, T, ...

For a given macrostate there are MANY microstates.

We need to know how many microstate for a given macrostate.

We'll come back to the ideal gas, but start with something easier.

Binary Models

binary digits 0, 1

random walk L, R steps

coin toss H, T

alloy Zn, Cu atoms

* spin $\frac{1}{2}$ * $\uparrow, \downarrow$

Paramagnet N spin-$\frac{1}{2}$ particles with $N = N_\uparrow + N_\downarrow$

electron has magnetic moment $\mu_B \simeq 5.788 \times 10^{-5}$ eV/T = Bohr magneton

two states, $\uparrow, \downarrow$ so $\mu_e = \pm 5.788 \times 10^{-5}$ eV/T

Spin-$\frac{1}{2}$ paramagnet $\quad N = N_\uparrow + N_\downarrow$

microstates	macrostates (μ/μ_B)
$\uparrow\uparrow\uparrow$	3
$\uparrow\uparrow\downarrow$	1
$\uparrow\downarrow\uparrow$	1
$\uparrow\downarrow\downarrow$	-1
$\downarrow\uparrow\uparrow$	1
$\downarrow\uparrow\downarrow$	-1
$\downarrow\downarrow\uparrow$	-1
$\downarrow\downarrow\downarrow$	-3

# microstates	# macrostates
$= 2^N$	$= 2(N-1)$
	(N odd)

$$\mu = \mu_B(N_\uparrow - N_\downarrow)$$

macrostates (μ/μ_B)	# microstates	
3	1	$\uparrow\uparrow\uparrow$
1	3	$\downarrow\uparrow\uparrow$, $\uparrow\downarrow\uparrow$, $\uparrow\uparrow\downarrow$
-1	3	$\uparrow\downarrow\downarrow$, $\downarrow\uparrow\downarrow$, $\downarrow\downarrow\uparrow$
-3	1	$\downarrow\downarrow\downarrow$

$$(\uparrow + \downarrow)^3 = \uparrow\uparrow\uparrow + 3(\uparrow\uparrow\downarrow) + 3(\downarrow\downarrow\uparrow) + \downarrow\downarrow\downarrow$$

N spins with numbers on them : 1 2 3 4 5 6 7....

$$N = N_\uparrow + N_\downarrow$$

Put them in N slots

N choices	N−1 choices	N−2 choices	...	2 choices	1 choice
1	3	2		4	7

$N!$ different configurations

$N_\uparrow!$ different configs have $\uparrow$ in same slots

$N_\downarrow!$ different configs have $\downarrow$ in same slots

\# of microstates with $N_\uparrow$

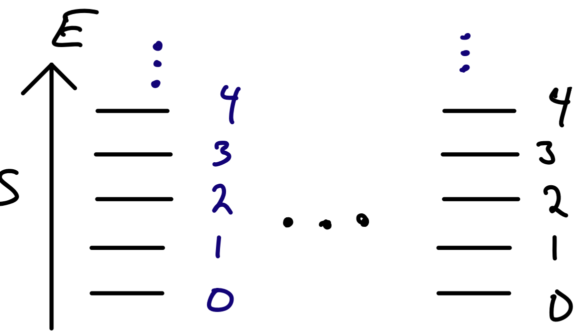
$$\Omega(N, N_\uparrow) = \binom{N}{N_\uparrow} \quad \frac{N!}{N_\uparrow!\; N_\downarrow!}$$

$$(\uparrow + \downarrow)^N = \uparrow\uparrow\ldots\uparrow + (\;)\uparrow\uparrow\ldots\uparrow\downarrow$$

"N, choose $N_\uparrow$"

Einstein Solid

N identical quantum harmonic oscillators



\# of quanta in each oscillator $\quad n_1 \quad \cdots \quad n_N$

each oscillator $\quad E_n = hf(n + \cancel{1/2})$

constant, set $\rightarrow 0$

$$E = E_{n_1} + E_{n_2} + \cdots E_{n_N} = hf(\underbrace{n_1 + \cdots + n_N}_{=q}) + \cancelto{0}{N\tfrac{1}{2}hf}$$

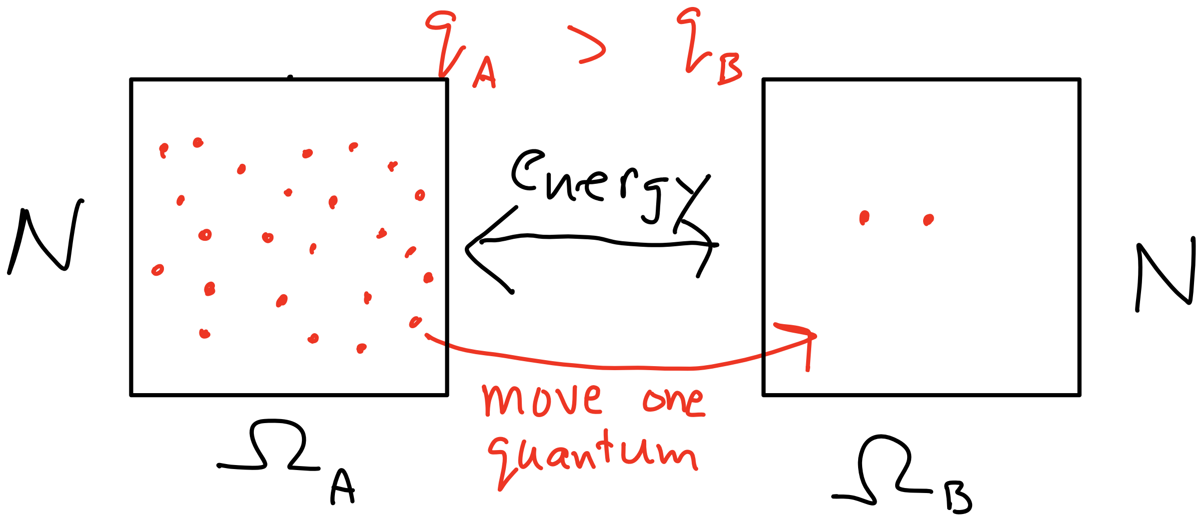
$= q \leftarrow$ macrostate

\# of microstates

multiplicity function

$$\Omega(N, q) = \binom{q+N-1}{q} = \frac{(q+N-1)!}{q!\,(N-1)!}$$

\# of oscillators $\quad$ \# of quanta



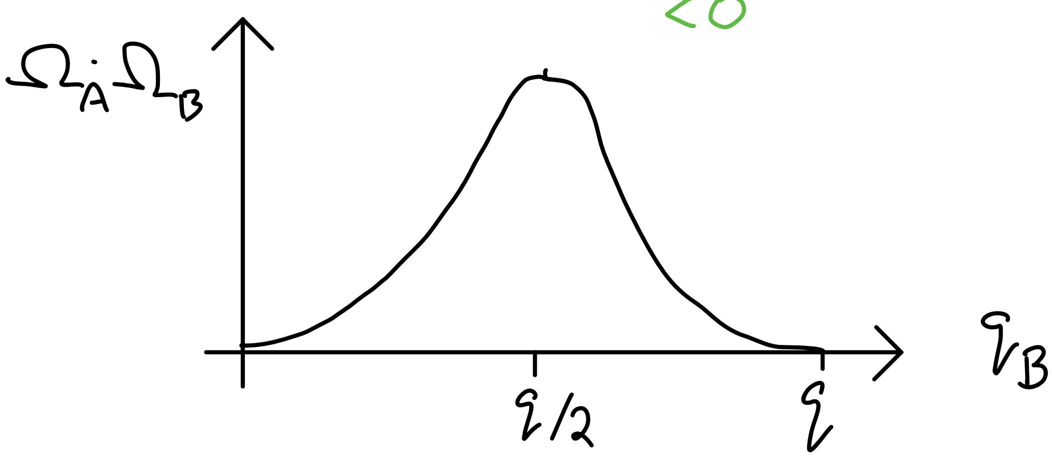
$q_A + q_B = q_{tot} = \text{const}$

$$\Omega_A \Omega_B \xrightarrow{\text{move one}} \left[\frac{(q_A+N-1)!}{q_A!\,(N-1)!}\;\frac{q_A}{q_A+N-1}\right]\left[\frac{q_B+1+N-1}{q_B+1}\;\frac{(q_B+N-1)!}{q_B!\,(N-1)!}\right]$$

$$\frac{(q_A q_B + N q_A)}{(q_A q_B + N q_A) + \underbrace{(q_B - q_A + 1)(N-1)}_{<0}} > 1$$

as long as $q_A > 0$

$\Omega_{tot} = \Omega_A \Omega_B$ keeps getting multiplied by factors > 1 until $q_A = q_B$, then factors < 1



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