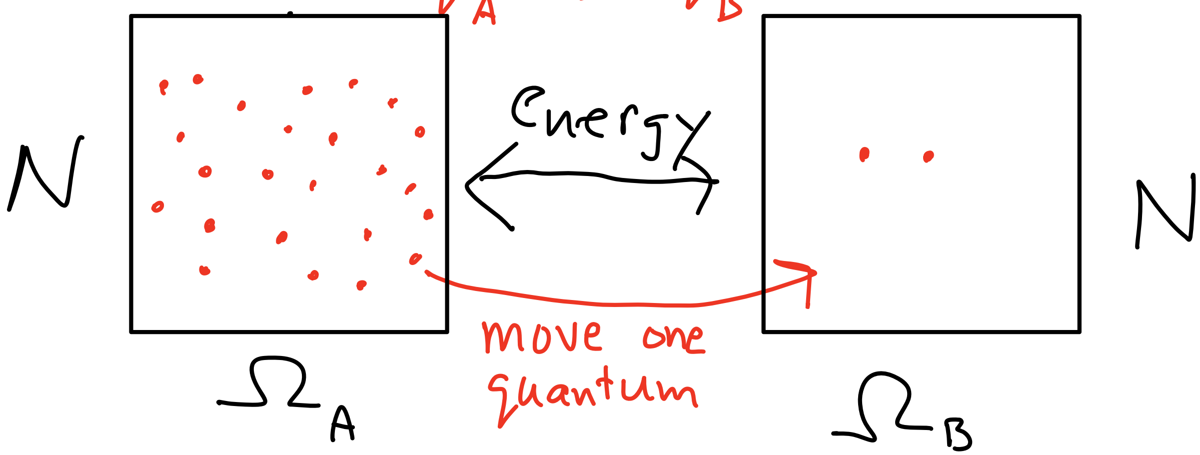
$q_A > q_B$

$q_A + q_B = q_{tot} = \text{const}$



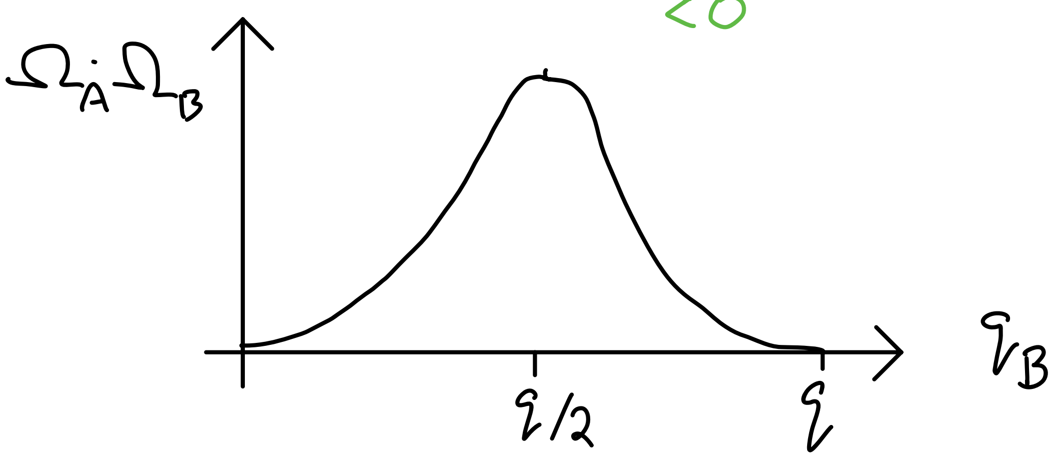
move one

$$\Omega_A \Omega_B \rightarrow \left[\frac{(q_A+N-1)!}{q_A!\,(N-1)!}\;\frac{q_A}{q_A+N-1}\right]\left[\frac{q_B+1+N-1}{q_B+1}\;\frac{(q_B+N-1)!}{q_B!\,(N-1)!}\right]$$

$$\frac{(q_A q_B + N q_A)}{(q_A q_B + N q_A) + \underbrace{(q_B - q_A + 1)(N-1)}_{<0}} > 1$$

as long as $q_A > 0$

$\Omega_{tot} = \Omega_A \Omega_B$ keeps getting multiplied by factors > 1 until $q_A = q_B$, then factors < 1

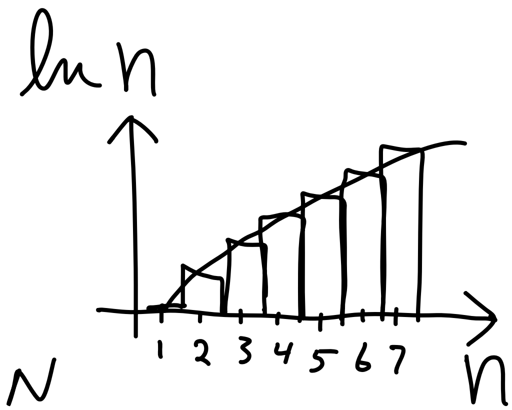


B

Stirling's Approximation

$$N! \approx \left(\frac{N}{e}\right)^N \left(\sqrt{2\pi N}\right) \qquad N >> 1$$

$$\ln N! \approx N \ln N - N + O(\ln N)$$



$$\ln N! = \ln(1 \cdot 2 \cdot \ldots \cdot N) = \sum_{n=1}^{N} \ln n \approx \int_1^N \ln n \, dn$$

$$\approx n \ln n - n \Big|_1^N$$

$$\approx N \ln N - N - (1 \cdot \ln 1 - 1)$$

$$\approx N \ln N - N$$

Einstein Solid, Large N, q

$$\ln \Omega = \ln \frac{(q+N-1)!}{q!\,(N-1)!} = \ln (q+N-1)! - \ln q! - \ln (N-1)!$$

$$\cong (q+N-1)\ln(q+N-1) - (q+N-1)$$

$$-(q)\ln(q) + (q)$$

$$-(N-1)\ln(N-1) + (N-1) \rightarrow 0$$

$N, q \gg 1$ so drop 1's

$$\cong (q+N)\underbrace{\ln(q+N)}_{\ln q(1+N/q)} - q\ln q - N\ln N \leftarrow \text{just } N, q \gg 1$$

$$\ln q + \ln(1+N/q)$$

$$\approx \ln q + N/q \qquad q \gg N, \text{ high } T \qquad \ln(1+\epsilon) \approx \epsilon$$

$$\boxed{\ln\Omega \cong N \ln q/N + N} + N^2/q \quad \text{ignore for } q \gg N$$

Einstein Solid large N, q

$$\ln \Omega_{Einstein} \approx N \ln q/N + N \quad (q >> N \text{ high } T)$$

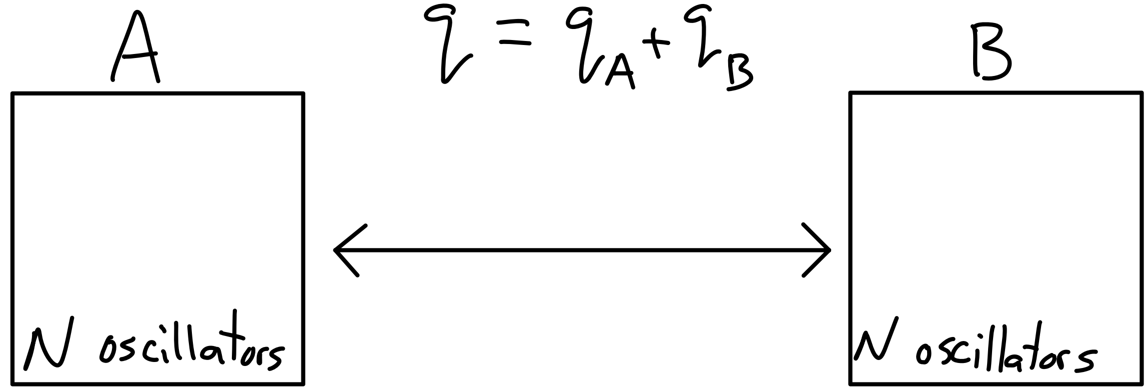
$$e^{\ln \Omega} \cong e^{N \ln \frac{q}{N} + N} = e^{\ln \left(\frac{q}{N}\right)^N + N}$$

$$= \left(\frac{q}{N}\right)^N e^N = \left(\frac{e \cdot q}{N}\right)^N$$

$$\Omega_{Einstein} \approx \left(\frac{e q}{N}\right)^N$$

for $q >> N$
(high T)

$$q = q_A + q_B$$



high temp:

$$\Omega_A \approx \left(\frac{e q_A}{N}\right)^N \qquad \Omega_B \approx \left(\frac{e q_B}{N}\right)^N$$

$$\Omega = \Omega_A \cdot \Omega_B \cong \left(\frac{e}{N}\right)^{2N} \left[q_A q_B \right]^N$$

let $q_{A,B} = \frac{q}{2} \pm x$ deviation from even distribution

$$\Omega \cong \left(\frac{e}{N}\right)^{2N} \left[\left(\frac{q}{2} + x\right)\left(\frac{q}{2} - x\right) \right]^N$$

$$\cong \left(\frac{e}{N}\right)^{2N} \left[\left(\frac{q}{2}\right)^2 - x^2 \right]^N$$

A | $q_A = \frac{q}{2} + x$, N oscillators

$\longleftarrow \; q = q_A + q_B \; \longrightarrow$

B | $q_B = \frac{q}{2} - x$, N oscillators

$$\Omega \simeq \left(\frac{e}{N}\right)^{2N}\left[\left(\frac{q}{2}\right)^2 - x^2\right]^N$$

$$\ln\left[\left(\frac{q}{2}\right)^2 - x^2\right]^N = N\ln\left[\qquad\right]$$

$$= N\ln\left[\left(\frac{q}{2}\right)^2\left(1 - \frac{2x^2}{q^2}\right)\right]$$

$$= N\ln\left(\frac{q}{2}\right)^2 + N\ln\left(1 - \frac{4x^2}{q^2}\right)$$

$$\simeq \ln\left(\frac{q}{2}\right)^{2N} + N\cdot\left(-4x^2/q^2\right)$$

$\ln 1+\epsilon \simeq \epsilon$

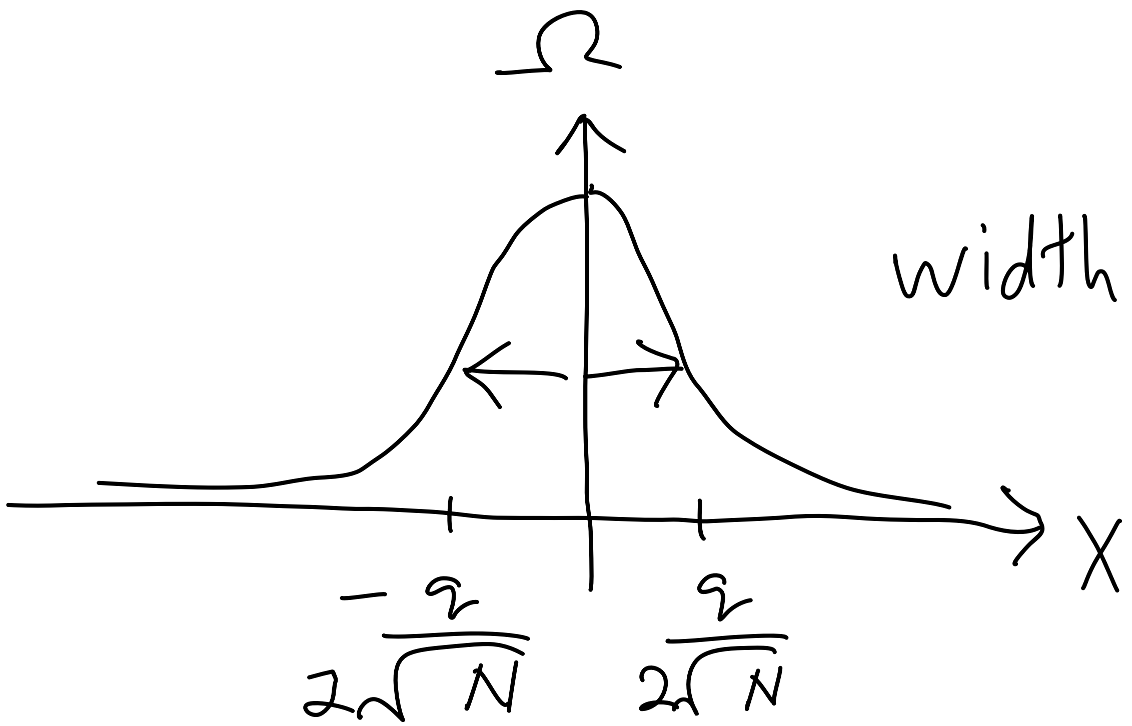
$$\Omega = e^{\ln\Omega} = e^{\ln\left(\frac{e}{N}\right)^{2N} + \ln[\;]^N} = \left(\frac{e}{N}\right)^{2N} e^{\ln\left(\frac{q}{2}\right)^{2N} - 4Nx^2/q^2}$$

$$\Omega = \left(\frac{e}{N}\right)^{2N} e^{\ln\left(\frac{q}{2}\right)^{2N} - 4N x^2/q^2}$$

$$= \left(\frac{eq}{2N}\right)^{2N} e^{-\frac{4N}{q^2} x^2}$$

falls to e^{-1} when

$x = q/2\sqrt{N}$



width $= q/\sqrt{N}$