Binary Systems
$(N = N_\uparrow + N_\downarrow)$

$$\Omega(N_\uparrow) = \frac{N!}{N_\uparrow!\, N_\downarrow!}$$

Einstein Solid
(N oscillators, q quanta)

$$\Omega(N, q) = \frac{(q+N-1)!}{q!\,(N-1)!}$$

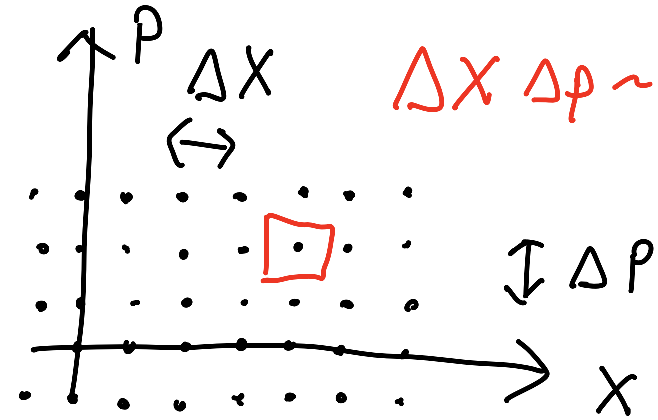
Ideal Gas

$$\Omega(U, V, N) = ?$$

continuous ?

Counting Continuous States

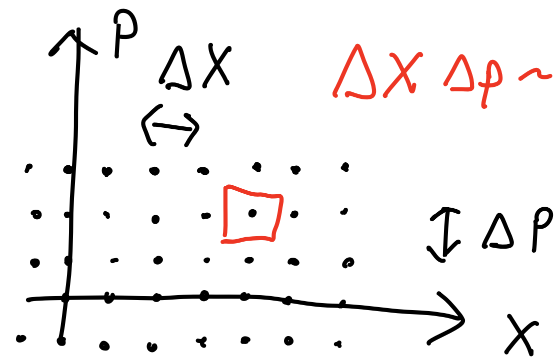
1D : particle with p, x



$\Delta x \, \Delta p \sim h$

Counting Continuous States

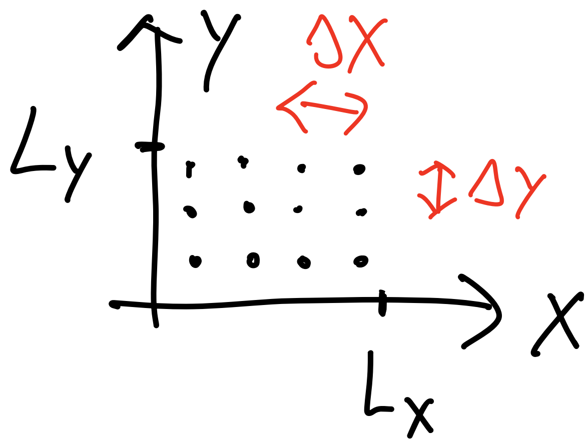
1D : particle with p, X



$\Delta x \Delta p \sim h$

$\Delta x \Delta p_x = h \qquad \Delta y \Delta p_y = h$

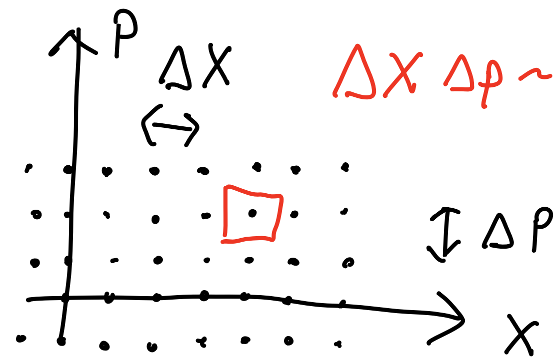
2D : Box $L_x \times L_y$



$$\Omega_{xy} = \frac{L_x L_y}{\Delta x \Delta y}$$

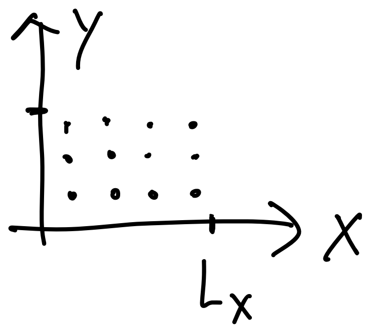
Counting Continuous States

1D : particle with p, x



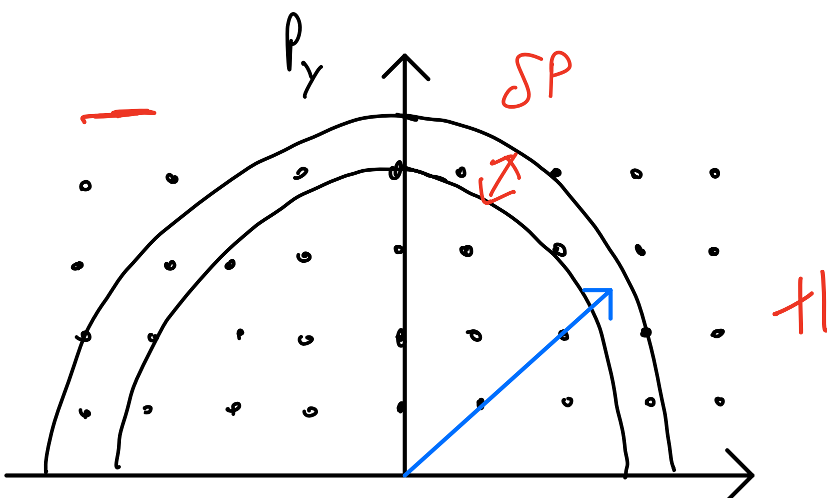
$\Delta x \, \Delta p \sim h$

2D : Box $L_x \times L_y$



$$\Omega_{xy} = \frac{L_x L_y}{\Delta x \, \Delta y}$$

$$\Omega = \frac{V_p}{\Delta P_x \Delta P_y} \cdot \frac{V}{\Delta x \Delta y} = \frac{V_p V}{h^2}$$



$$\frac{p_x^2}{2m} + \frac{p_y^2}{2m} = U$$

"radius" $= \sqrt{2mU}$

thickness circumference

$$\Omega_p = \frac{\delta P \; 2\pi\sqrt{2mU}}{\Delta P_x \; \Delta P_y}$$

3D, 1 particle

area of $p_x^2 + p_y^2 + p_z^2 = 2mU$

$$\Omega = \frac{V_p V}{h^3} = \frac{\delta P \, 4\pi \left(\sqrt{2mU}\right)^2 \cdot V}{h^3}$$

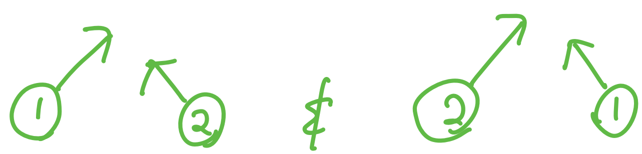
3D, N particles

"area" of $p_{1x}^2 + p_{1y}^2 + p_{1z}^2 + p_{2x}^2 + \dots p_{Nz}^2 = 2mU$

$$\Omega_N = \frac{\delta P \, A_p \cdot V^N}{N! \, h^{3N}}$$

$$\text{area hypersphere} = \frac{2\pi^{d/2}}{\left(\frac{d}{2}-1\right)!} r^{d-1}$$

$$n! = \Gamma(n+1) \qquad \tfrac{1}{2}! = \sqrt{\pi}/2$$



are the same state, but counted twice.

N particles overcounted by N!

$$\Omega_N = \frac{V^N}{N!} \frac{1}{h^{3N}} \frac{\pi^{3N/2}}{(3N/2)!} \left(\sqrt{2mU}\right)^{3N} = f(N) V^N U^{3N/2}$$

Entropy

$$S = k \ln \Omega$$

entropies ADD

$$\Omega_{tot} = \Omega_1 \cdot \Omega_2$$

$$S_{tot} = k \ln(\Omega_1 \cdot \Omega_2) = k \ln \Omega_1 + k \ln \Omega_2 = S_1 + S_2$$

At equilibrium the system will be in the macrostate with the largest Ω $\rightarrow$ largest S or...

Second Law of Thermodynamics

S tends to increase (never decreases)

$$
\begin{aligned}
\frac{S}{k} &= \ln\left(\frac{V^N}{N!}\,\frac{1}{h^{3N}}\,\frac{\pi^{3N/2}}{(3N/2)!}\,(2mU)^{3N/2}\right) \\
&\approx \ln\left(\left(\frac{2\pi mU}{h^2}\right)^{3N/2}\right) + N\ln V - N\ln N + N - \frac{3N}{2}\ln\frac{3N}{2} + \frac{3N}{2} \\
&\approx N\left[\ln\left(\left(\frac{2\pi mU}{h^2}\right)^{3/2}\right) + \ln\frac{V}{N} + 1 - \frac{3}{2}\ln\frac{3N}{2} + \frac{3}{2}\right] \\
&\approx N\left[\ln\left(\left(\frac{2\pi mU}{h^2}\,\frac{2}{3N}\right)^{3/2}\right) + \ln\frac{V}{N} + \frac{5}{2}\right] \\
&\approx N\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m}{3h^2}\,\frac{U}{N}\right)^{3/2}\right) + \frac{5}{2}\right]
\end{aligned}
$$

Sakur-Tetrode equation

$$\frac{S}{k} = N\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m}{3h^2}\frac{U}{N}\right)^{3/2}\right) + \frac{5}{2}\right]$$