3.10

<u>Paramagnetism</u> $\left(\mu_B = \left|\frac{e\hbar}{2m_e}\right|\right)$

spin 1/2 with B

$E_\uparrow = -\mu B$

$E_\downarrow = +\mu B$

$E = -\vec{\mu}\cdot\vec{B}$

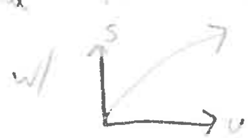
$N = N_\uparrow + N_\downarrow$

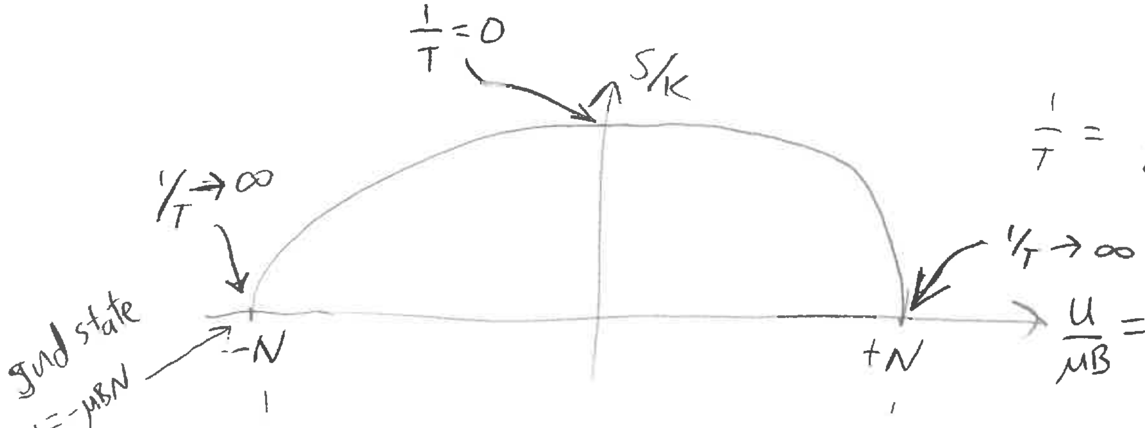
$$U = -\mu B(N_\uparrow - N_\downarrow) = \mu B(N - 2N_\uparrow)$$

magnetization: $M = \mu(N_\uparrow - N_\downarrow) = -\dfrac{U}{B}$ $\quad(E = -\vec{\mu}\cdot\vec{B})$

$$\Omega(N_\uparrow) = \binom{N}{N_\uparrow} = \frac{N!}{N_\uparrow!\,N_\downarrow!}$$

c.f. "normal" systems w/

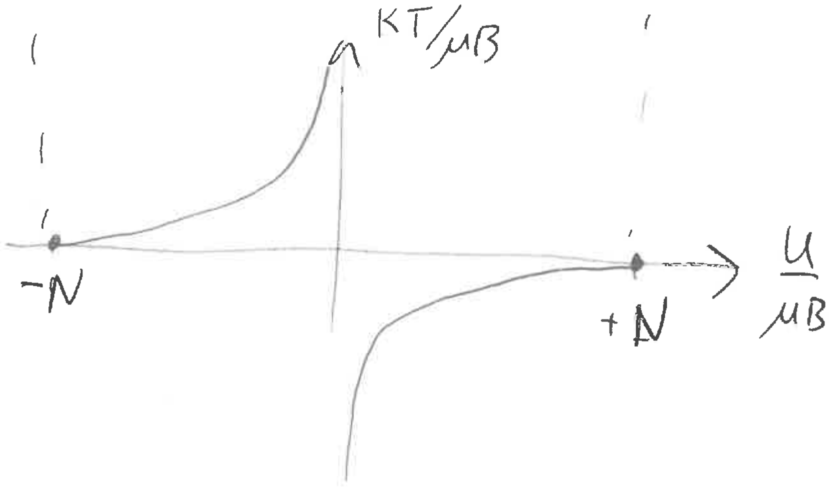




$\dfrac{1}{T} = \dfrac{\partial S}{\partial U}$

From Numerical Table

$T = \dfrac{\Delta U}{\Delta S}$



$C = \dfrac{\partial U}{\partial T}$

3.11

analytic

$$S/k = \ln \frac{N!}{N_\uparrow ! N_\downarrow !} \qquad \text{use } N_\downarrow = N - N_\uparrow$$

$$\ln n \approx n \ln n - n$$

$$= \left[N \ln N - N_\uparrow \ln N_\uparrow - (N - N_\uparrow) \ln (N - N_\uparrow) \right] \quad (3.28)$$

$$(3.29) \quad \frac{1}{T} = \left(\frac{\partial S}{\partial U} \right)_{N,B} = \frac{\partial N_\uparrow}{\partial U} \frac{\partial S}{\partial N_\uparrow} = \frac{-1}{2\mu B} \frac{\partial S}{\partial N_\uparrow} \qquad U = \mu B (N - 2N_\uparrow)$$

Next page

$\vdots$

$$= \frac{+k}{2\mu B} \ln \left(\frac{N - U/\mu B}{N + U/\mu B} \right)$$

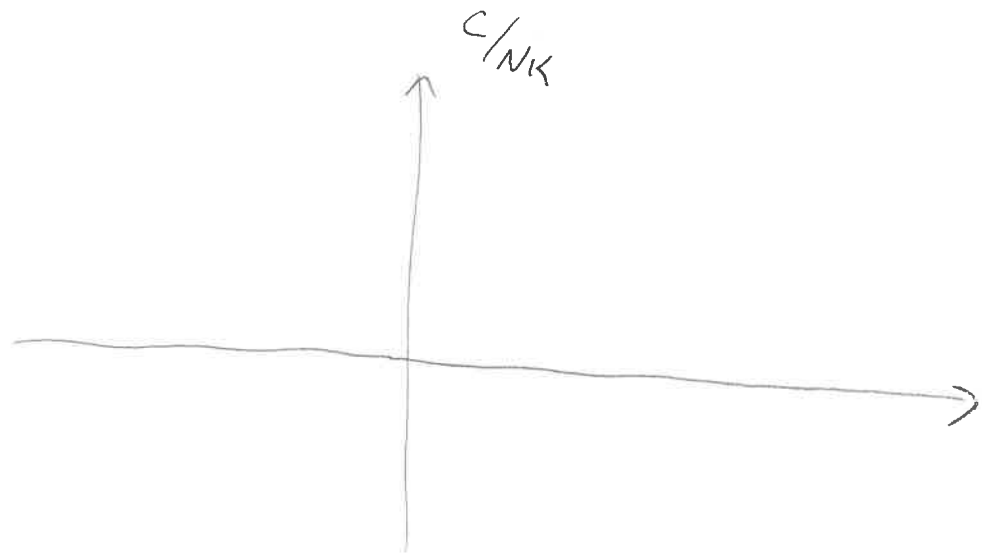
$\vdots$

$$U = N\mu B \left(\frac{1 - e^{2\mu B/kT}}{1 + e^{2\mu B/kT}} \right) = -N\mu B \tanh \frac{\mu B}{kT}$$

$$\sinh x = \frac{e^x - e^{-x}}{2} = \frac{\sin ix}{i} \qquad \cosh x = \frac{e^x + e^{-x}}{2} = \cos ix \qquad \tanh x = \frac{\sinh x}{\cosh x} = \frac{1}{i} \tan ix$$

$$\frac{d}{dx} \sinh x = \cosh x \qquad \frac{d}{dx} \cosh x = \sinh x$$

$$\cosh^2 x - \sinh^2 x = 1$$


C/NK

3.11 A

$$S = k \ln \frac{N!}{N_\uparrow ! \, N_\downarrow !} \cong k \left[N \ln N - N_\uparrow \ln N_\uparrow - (N - N_\uparrow) \ln (N - N_\uparrow) \right]$$

$$\frac{1}{T} = \left(\partial_u S \right)_{N,B} = \frac{\partial N_\uparrow}{\partial u} \frac{\partial S}{\partial N_\uparrow} =$$

$$= \frac{-1}{2\mu B} k \, \partial_{N_\uparrow} [\ ]$$

$$= \frac{-k}{2\mu B} \ln \frac{N_\uparrow}{N - N_\uparrow}$$

$$N_\uparrow = \frac{N}{2} - \frac{U}{2\mu B}$$

$$\exp \left\{ \frac{2\mu B}{k} \frac{1}{T} = \frac{k}{2\mu B} \ln \frac{\frac{N}{2} - U/2\mu B}{\frac{N}{2} - \frac{U}{2\mu B}} \right\}$$

$$U = N\mu B \frac{1 - e^{2\mu B/kT}}{1 + e^{2\mu B/kT}} \times \frac{e^{-\mu B/kT}}{e^{-\mu B/kT}}$$

$$\boxed{U = -N\mu B \tanh \frac{\mu B}{kT}}$$

3.12

$$\frac{1}{T} = -\frac{k}{2\mu B}\left\{\partial_{N_\uparrow}\left[N\ln N - N_\uparrow \ln N_\uparrow - (N-N_\uparrow)\ln(N-N_\uparrow)\right]\right\}$$

$$= -\frac{K}{2\mu B}\left\{-\ln N_\uparrow - \frac{N_\uparrow}{N_\uparrow} + \ln(N-N_\uparrow) + \frac{N-N_\uparrow}{N-N_\uparrow}\right\}$$

$$= \frac{K}{2\mu B}\ln\frac{N_\uparrow}{N-N_\uparrow}$$

but $U = \mu B(N - 2N_\uparrow)$

$$N_\uparrow = \frac{N}{2} - \frac{U}{2\mu B}$$

$$\exp\left\{\frac{1}{T} = \frac{K}{2\mu B}\ln\frac{\frac{N}{2} - U/2\mu B}{\frac{N}{2} + U/2\mu B}\right\}$$

$$\exp\left(\frac{2\mu B}{KT}\right) = \frac{N - U/\mu B}{N + U/\mu B} \rightarrow (N + U/\mu B)e^{\frac{2\mu B}{KT}} = N - U/\mu B$$

$$\frac{U}{\mu B}\left(1 + e^{2\mu B/KT}\right) = N\left(1 - e^{2\mu B/KT}\right)$$

$$\begin{pmatrix} \cosh^2 x - \sinh^2 x = 1 \\ \partial_x \sinh x = \cosh x \\ \partial_x \cosh x = \sinh x \\ \cosh x = \frac{e^x + e^{-x}}{2} \\ \sinh x = \frac{e^x - e^{-x}}{2} \end{pmatrix}$$

$$\rightarrow U = N\mu B\,\frac{1 - e^{2\mu B/KT}}{1 + e^{2\mu B/KT}}$$

mult by $\frac{e^{-\mu B/KT}}{e^{-\mu B/KT}}$

$$= -N\mu B\tanh\frac{\mu B}{KT}$$

$$\frac{d}{dx}\tanh x = \frac{d}{dx}\frac{\sinh x}{\cosh x} = \frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x} = \frac{1}{\cosh^2 x}$$

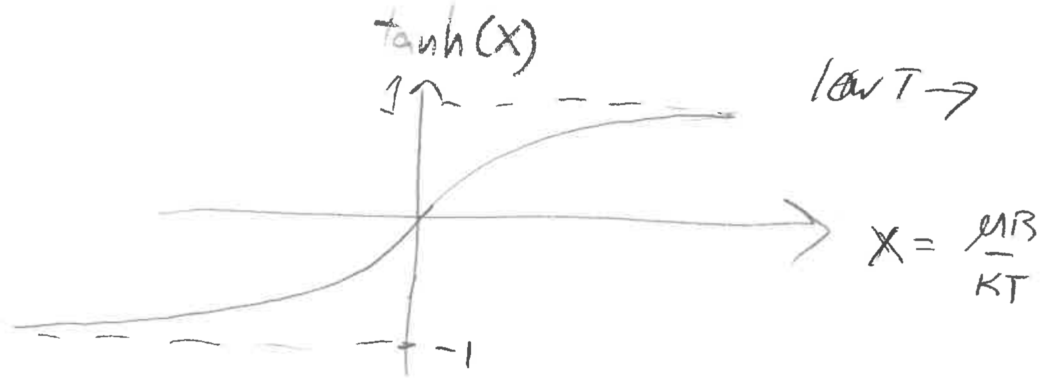
$$\cancel{\rightarrow \frac{dU}{dT} = -N\mu B\frac{\partial}{\partial T}\tanh\frac{\mu B}{KT} = -N\mu B}$$

F→

3.13

$= \mu(N_\uparrow - N_\downarrow)$

$M = -\frac{U}{B} \rightarrow M = N\mu \tanh \frac{\mu B}{KT}$ $\quad U = -MB$



$$C_B = \left(\frac{\partial U}{\partial T}\right)_{N,B} = NK \frac{(\mu B/KT)^2}{\cosh^2(\mu B/KT)}$$

$\frac{d}{dx}\tanh x = \frac{1}{\cosh^2 x}$

Magnetic moment: $\mu_e = -\mu_B$ $\quad \mu_B = \left|\frac{e\hbar}{2m_e}\right| = 5.788 \times 10^{-5} \frac{eV}{T}$

proton, $\mu \sim \frac{1}{2000}\mu_B$

small μ or high T $\quad \mu B/KT \ll 1$

$$\rightarrow \tanh x \approx x \left(-\frac{x^3}{3}\right)$$

$\rightarrow M \approx \frac{N\mu^2 B}{KT}$ $\quad M \propto 1/T \rightarrow$ Curie's law

gives $C_B \propto 1/T^2$