Problem Set 1

August 26, 2023

1 problem 1.1

Part (a)

We can derive the formulas for converting temperatures between Fahrenheit (F) and Celsius (C) using the given facts:

$$32\,^{\circ}\mathrm{F} = 0\,^{\circ}\mathrm{C}$$
$$212\,^{\circ}\mathrm{F} = 100\,^{\circ}\mathrm{C}$$

We'll create a linear equation relating Fahrenheit and Celsius using the formula:

$$C = m \cdot F + b$$

Using the two given points to find the values of m and b:
1. From the freezing point:

$$0 = m \cdot 32 + b$$
$$b = -32 \cdot m$$

2. From the boiling point:

$$100 = m \cdot 212 + b$$
$$100 = m \cdot 212 - 32 \cdot m$$
$$100 = 180m$$

Solving for m:

$$m = \frac{100}{180} = \frac{5}{9}$$

Substituting back into our expression for b:

$$b = -32 \cdot \frac{5}{9}$$

The formula for converting Fahrenheit to Celsius is:

$$C = \frac{5}{9} \cdot (F - 32)$$

To convert Celsius to Fahrenheit, we can invert the relationship:

$$F = \frac{9}{5} \cdot C + 32$$

So the conversion formulas are:

- Fahrenheit to Celsius: $C = \frac{5}{9} \cdot (F - 32)$
- Celsius to Fahrenheit: $F = \frac{9}{5} \cdot C + 32$

Part (b)

Absolute zero is the lowest possible temperature, at which the thermodynamic temperature is 0 Kelvin. To find the equivalent in Fahrenheit, we can convert 0 Kelvin to Celsius using:

$$C = K - 273.15 \tag{1}$$

And then convert to Fahrenheit:

$$F = \frac{9}{5} \cdot (-273.15) + 32 \approx -459.67\,^{\circ}\mathrm{F} \tag{2}$$

Grading Rubric

- 3 points: Correctly deriving the formula to convert from Fahrenheit to Celsius.
- 3 points: Correctly deriving the formula to convert from Celsius to Fahrenheit.
- 4 points: Correctly identifying absolute zero on the Fahrenheit scale.

Total: 10 points

2 problem 1.8a

Solution:

For a solid, we define the linear thermal expansion coefficient, α, as the fractional increase in length per degree. For steel, $\alpha = 1.1 \times 10^{-5}\,\mathrm{K}^{-1}$. To estimate the total variation in length of a 1 km steel bridge between a cold winter night and a hot summer day in Iowa, we can use the formula for linear expansion:

$$\Delta L = \alpha L \Delta T \tag{3}$$

Assuming a typical temperature variation in Iowa of 70 K, we have:

$$\begin{aligned}\Delta L &= 1.1 \times 10^{-5} \times 1000\,\mathrm{m} \times 70\,\mathrm{K} \\ &\approx 0.77\,\mathrm{m}\end{aligned}$$

The total variation in length of the bridge is approximately 0.77 m.

Grading Rubric (Total 10 points):

- **3 points:** Correct use of the linear thermal expansion coefficient, including understanding its meaning and units.
- **4 points:** Correct application of the formula for linear expansion and substitution of given values.
- **3 points:** Correct calculation and final answer, with appropriate units.

3 problem 1.11

Solution

Rooms A and B are the same size, so they have the same volume. The number of moles of air in each room can be given by the ideal gas law, $n = \frac{PV}{RT}$, where P is the pressure, V is the volume, R is the universal gas constant, and T is the temperature in kelvin.

Since the rooms are connected by an open door, the pressure in both rooms is the same, and the volume V is also the same for both rooms. However, Room A is warmer, so the temperature $T_A > T_B$.

Substituting these values into the equation for the number of moles, we find:

$$n_A = \frac{PV}{RT_A}$$
$$n_B = \frac{PV}{RT_B}$$

Since $T_B < T_A$, it follows that $n_B > n_A$. So, Room B has more moles, and thus contains the greater mass of air (assuming the air in each room has the same composition).

Grading Rubric

- 3 points: Explaining that the volume and pressure in both rooms are the same.
- 3 points: Applying the ideal gas law to find expressions for the mass of air in both rooms.
- 4 points: Correctly identifying that Room B contains the greater mass of air and explaining why.

Total: 10 points

4 problem 1.19

Solution:

In a gas at thermal equilibrium, all molecules, regardless of their mass, have the same average translational kinetic energy given by:

$$\frac{1}{2}mv^2 = \frac{3}{2}kT \tag{4}$$

By rearranging the equation, we have:

$$v = \sqrt{\frac{3kT}{m}} \tag{5}$$

Since hydrogen molecules (H_2) are lighter than oxygen molecules (O_2), they will have a higher average speed at the same temperature. The mass of an H_2 molecule is approximately 2 amu, while the mass of an O_2 molecule is approximately 32 amu. So the ratio of the average speeds is given by:

$$\frac{v_{H_2}}{v_{O_2}} = \sqrt{\frac{m_{O_2}}{m_{H_2}}} = \sqrt{\frac{32}{2}} \approx 4 \tag{6}$$

Hydrogen molecules are moving, on average, 4 times faster than oxygen molecules.

Grading Rubric:

- Correct usage of kinetic energy formula: 3 points
- Correct derivation of speed formula: 3 points
- Correct calculation of the mass ratio: 2 points
- Correct conclusion (Hydrogen molecules are moving 4 times faster): 2 points

Total: 10 points

5 problem 1.21

Solution:

Given the mass of hailstones $m = 2\,\text{g} = 2 \times 10^{-3}\,\text{kg}$, their speed $v = 15\,\text{m/s}$, and the area of the window $A = 0.5\,\text{m}^2$, we can first find the force exerted by each hailstone by using the conservation of momentum.

Since the hailstones strike at a 45° angle, only a component of their momentum will be normal to the window. Thus, the change in momentum for each hailstone is given by:

$$\Delta p = 2mv\cos 45^\circ = 2 \times 2 \times 10^{-3}\,\text{kg} \times 15\,\text{m/s} \times \frac{1}{\sqrt{2}} \approx 0.0424\,\text{kg}\cdot\text{m/s} \tag{7}$$

The total force on the window, considering 30 hailstones hitting the window per second, is:

$$F = 30 \times \Delta p \approx 1.273\,\text{N} \tag{8}$$

The average pressure on the window is then:

$$P = \frac{F}{A} \approx \frac{1.26}{0.5}\,\mathrm{Pa} \approx 2.546\,\mathrm{Pa} \tag{9}$$

Compared to the atmospheric pressure (approximately 101325 Pa), the pressure exerted by the hailstones is significantly (2.5×10^{-5} times) lower.

Grading Rubric:

- Correct calculation of the change in momentum for each hailstone: 2 points
- Correct calculation of the total force on the window: 3 points
- Correct calculation of the average pressure on the window: 3 points
- Correct comparison to the atmospheric pressure: 2 points

Total: 10 points