

# problem set 2 solution

10 points total

1.23 He has 3 degrees of freedom (translation) 2 points:  $f = 3$

$$\Rightarrow U = 3N \frac{1}{2} kT = \frac{3}{2} PV$$

3 points numerical U

for  $P = 10^5 \text{ N/m}^2$  &  $V = 10^{-3} \text{ m}^3$ ,  $PV = 100 \text{ J} \Rightarrow U = 150 \text{ J}$

2 points:  $f = 5$ 

for air there are 5 degrees of freedom (3 translation & 2 rotation)  $\Rightarrow U = \frac{5}{2} PV = 250 \text{ J}$

3 points: numerical U

Schroeder

5 points total

1.29

The  $5^{\circ}\text{C}$  increase in temperature requires  
an energy input of  $\Delta U = 1 \frac{\text{cal}}{\text{g}\cdot^{\circ}\text{C}} \times 5^{\circ}\text{C} \times 200\text{g}$

$$= 10^3 \text{ cal}$$

5 points:  $\Delta U = W+Q$  means you can't tell how  
energy is attributed to  $W$  and  $Q$

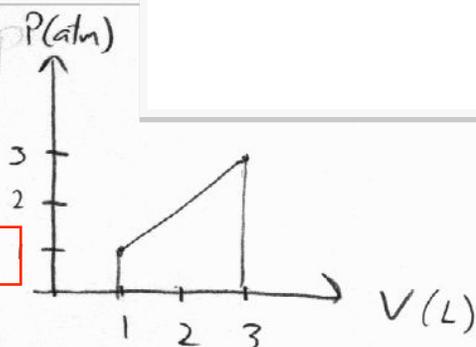
But  $\Delta U = Q+W$ . There's no way to  
know if  $\Delta U$  was from heat, work, or both.

12 points total

1.31

a)

2 points



$$b) W = -\text{Area} = -\left(10^5 \text{ Pa} \times 2 \times 10^{-3} \text{ m}^3\right) - \frac{1}{2} \left(2 \times 10^5 \text{ Pa} \times 2 \times 10^{-3} \text{ m}^3\right)$$

3 points

$$= -400 \text{ J} < 0 \text{ so work done by gas on environment}$$

$$c) \text{ He atom has 3 DOF, } U = \frac{3}{2} NKT = \frac{3}{2} PV$$

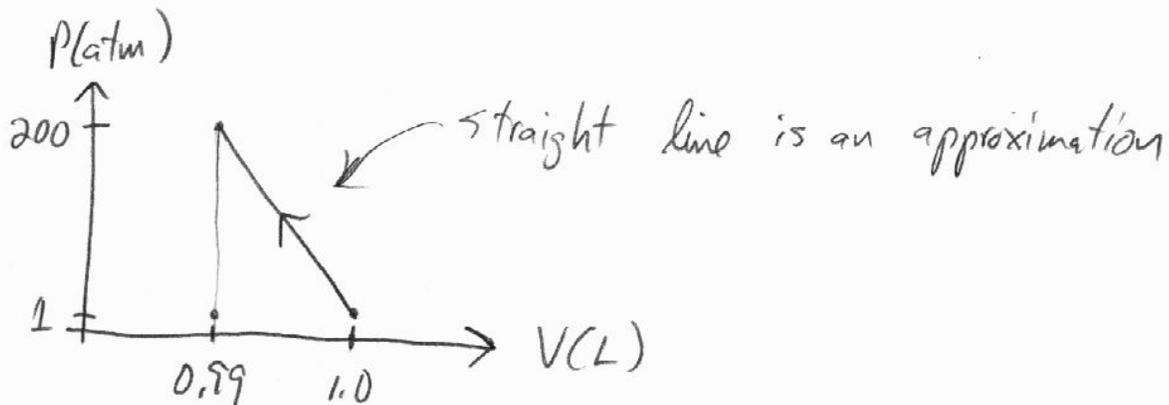
3 points

$$\Delta U = \frac{3}{2} [P_f V_f - P_i V_i] = \frac{3}{2} [3 \text{ atm} \cdot 3 \text{ L} - 1 \text{ atm} \cdot 1 \text{ L}]$$

$$= 12 \text{ L} \cdot \text{atm} = 1200 \text{ J}$$

1.31 (cont.) d)  $Q = \Delta U - W = 1200\text{J} - (-400\text{J}) = 1600\text{J}$   
 2 points this the heat entering the gas

e) To cause an increase in  $P$  &  $T$  as the gas expands requires heat input.  
 2 points



$$W = \int -P dV = -(1\text{atm})(-0.01\text{L}) - \frac{1}{2}(199\text{atm})(-0.01\text{L})$$

$$= -1.005\text{atm}\cdot\text{L} = 100\text{J}$$

3 points

$$\text{or } W \approx -\bar{P} \cdot \Delta V \approx -(100\text{atm})(-0.01\text{L}) = 1\text{atm}\cdot\text{L} = 100\text{J}$$

2 points This isn't much - the same as lifting 10 kg by 1m  
 That's because the water isn't very compressible,  
 So  $\Delta V$  is small.

1.36 a)  $P_f V_f^\gamma = P_i V_i^\gamma \rightarrow V_f = V_i \left(\frac{P_i}{P_f}\right)^{1/\gamma} = (1\text{L}) \left(\frac{1\text{atm}}{7\text{atm}}\right)^{5/7} = 0.25\text{L}$   
 10 points total  
 3 points

$$\gamma = \frac{f+2}{f} = \frac{7}{5} \quad \text{for air @ room temp}$$

1.36 (cont.) b)  $P(V) = C/V^\gamma = P_i V_i^\gamma / V^\gamma$

4 points

$$\begin{aligned} \rightarrow W &= -\int P dV = -P_i V_i^\gamma \int_{V_i}^{V_f} V^{-\gamma} dV = -P_i V_i^\gamma \left( \frac{V^{1-\gamma}}{1-\gamma} \right) \Big|_{V_i}^{V_f} \\ &= \frac{P_i V_i^\gamma}{1-\gamma} \left( \frac{1}{V_f^{1-\gamma}} - \frac{1}{V_i^{1-\gamma}} \right) = \frac{P_i V_i}{\gamma-1} \left[ \left( \frac{V_i}{V_f} \right)^{\gamma-1} - 1 \right] \\ &= \frac{1 \text{ L} \cdot \text{atm}}{2/5} \left[ 4^{2/5} - 1 \right] = 1.853 \text{ L} \cdot \text{atm} = 185 \text{ L} \end{aligned}$$

3 points

c) using equ. 1.35  $T_f = T_i \left( \frac{V_i}{V_f} \right)^{2/f} = (300 \text{ K}) \left( \frac{1 \text{ L}}{0.25 \text{ L}} \right)^{2/5} \approx 522 \text{ K}$

6 points total

1.38

The bubbles have the same number of molecules & end with the same pressure.  $\Rightarrow V = NKT/P$  will be larger for the bubble with the higher  $T$ .  $U = \frac{f}{2} NKT$ , so higher  $T \rightarrow$  larger  $U$ .

2 points

As bubbles expand they do work and lose energy.

2 points

B absorbs heat to maintain constant  $T$

$\Rightarrow$  B has larger  $U$ , higher  $T$ , larger  $V$ .

2 points

(BTW, what lake has seaweed?)

6 points total

1.49

Ignore the volume of the liquid water which is  $\ll$  volume of the vapor

3 points

$$W = PV = nRT = (1.5 \text{ mol})(8.31 \frac{\text{J}}{\text{mol}\cdot\text{K}})(298 \text{ K})$$

$$\approx 4 \text{ kJ}$$

3 points

out of 286 kJ of heat only 4 kJ is from work done by the atmosphere. The other 282 kJ is from the chemical bonds

1.55

14 points total



$$\text{KE: } U_k = 2 \times \frac{1}{2} m v^2 = m v^2$$

$$\text{PE: } U_p = \frac{G m^2}{2r}$$

a) apply  $F=ma$  to one mass

3 points

$$\left[ \frac{G m^2}{(2r)^2} = m \frac{v^2}{r} \right] \times 2r$$

$$\rightarrow \frac{G m^2}{2r} = 2 m v^2 = 2 U_k$$

$$\rightarrow U_p = -2 U_k$$

2 points

$$b) U_{\text{tot}} = U_k + U_p = U_k - 2U_k = -U_k$$

$\Rightarrow$  increasing  $U_{\text{tot}}$  decreases  $U_k$

increasing  $U_{\text{tot}}$  towards zero is towards  $v=0$  at  $U_{\text{tot}}=0$

$$c) U_{\text{tot}} = -U_k = -\frac{3}{2} N k T, \quad C = \frac{dU_{\text{tot}}}{dT} = -\frac{3}{2} N k$$

3 points

negative!

1.55  
(cont.)

d)  $U_p$  should depend on  $G$ ,  $M$  &  $R$  (radius of star)

$$[G] = \frac{Nm^2}{kg^2}$$

$M$  &  $R$  are in fundamental units.

$$[M] = kg \quad [R] = m$$

We must multiply  $G$  by powers of  $M$  &  $R$  to get energy,  $[E] = \text{Length} \times \text{Force}$

$$\text{Want } [G M^a R^b] = N \cdot m \quad \frac{Nm^2}{kg^2} kg^a m^b = N \cdot m$$

$$\rightarrow a = 2, b = -1$$

3 points

So  $U_p \propto G M^2 / R$  &  $U_p < 0$  to be attractive.

e)  $U_K = \frac{3}{2} NKT$ , using this plus the fact that

$$U_K = -\frac{1}{2} U_p \text{ we get } \frac{3}{2} NKT \approx -\frac{1}{2} \left( -\frac{GM^2}{r} \right) = \frac{2GM^2}{2R}$$

3 points

$$\rightarrow T \approx \frac{GM^2}{3NKR} = \frac{GM}{3KR} \cdot \frac{M}{N}$$

$\underbrace{\frac{M}{N}}$   
mass per particle

1.55  
(cont.)

$$\frac{M}{N} = \text{mass per particle} \approx \frac{1}{2} (m_{\text{proton}} + m_{\text{electron}}) \approx \frac{1}{2} m_{\text{proton}}$$

$$\rightarrow T \approx 4 \times 10^6 \text{ K}$$

This is much higher than the  $T \approx 6000 \text{ K}$  of the Sun's surface. The core of the sun is  $T \approx 10^7 \text{ K}$ , which is closer

2 points