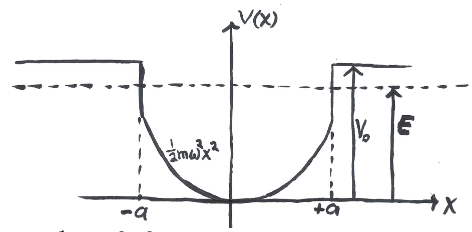
PHYS:5741
Problem Set 7
Due Wednesday, October 20, 2021

October 13, 2021

1. Consider a particle of mass m in a 1D potential $V(x) = \frac{1}{2}m\omega^2 x^2$. Calculate the probability for the particle in its ground state to be detected outside the classically allowed region. (This will be a numerical answer.)

2. A particle of mass m is in a bound state of a modified harmonic oscillator potential

$$V(x) = \begin{cases} \frac{1}{2}m\omega^2 x^2 & \text{if} \quad -a < x < +a \\ V_0 & \text{if} \quad |x| \geq a \end{cases}$$



where $V_0 > \frac{1}{2}m\omega^2 a^2$, and the energy E is bounded by $\frac{1}{2}m\omega^2 a^2 < E < V_0$. Assume the value of the wave function is known at a, that is $\psi(a) = \psi_a$. What is the probability of the particle being measured in the classically forbidden region?

3. Consider the quantum bouncing ball problem, a particle of mass m in a gravitational potential $V(x) = mgx$ with a hard floor at $x = 0$ (i.e. $V(x) = \infty$ for $x < 0$).
a) Find the wave function and energy of the ground state. (Note: You may want to consult dlmf.nist.gov for properties of Airy functions.)
(b) What is the average height of the particle $\langle x \rangle$? (*hint*: the virial theorem.)
(c) Write an expression for the probability of the particle being detected at a height twice its average height or higher.
(d) Evaluate the quantities in parts (b) and (c) numerically for a 1 μg mass, and an an electron, both on the surface of the Earth. (Evaluate numerical integrals using Wolfram alpha, Matlab, etc.)

4. Consider a particle of mass m and energy E incident on rectangular barrier $V(x) = V_0(\theta(x) - \theta(x-a)$, with $E < V_0$.
(a) Calculate the transmission probability.
(b) Assume the barrier has $V_0 = 2$ eV and $a = 0.1$ nm and the incident particle has energy $E = 1$ eV. Numerically evaluate the transmission probabilities for both an electron and a proton.

5. Calculate the energy spectrum of a harmonic oscillator $V(x) = \frac{1}{2}m\omega^2 x^2$ in the semi-classical approximation.