PHYS:4731 Homework #1

Due at the beginning of class, Thursday, September 1, 2022.

1. Show that

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{df}{dr}\right) = \frac{1}{r}\frac{d^2}{dr^2}(rf)$$

2. Prove that $\nabla r = \hat{\mathbf{r}}$ where $r = |\mathbf{r}|$.

3. The Large Plasma Device (LAPD) experiment at UCLA (see `http://plasma.physics.ucla.edu/bapsf/pages/research.html` if you want more information on this experiment) allows for basic plasma physics experiments in a long cylindrical chamber with a strong axial magnetic field. The plasma produced is 19 m long and 75 cm in diameter, with the following parameters: $n = 10^{17}$ m^{-3}, $T_i = T_e = 2 \times 10^4$ K, and $B = 0.1$ T.

   (a) Calculate the (electron) Debye length and electron and ion Larmor radius assuming a plasma of singly-ionized argon.

   (b) Calculate the plasma beta β for this experiment.

   (c) Calculate the plasma parameter N_D and the mean free path for electron-ion collisions λ_m. Would you describe this plasma as collisional, semi-collisional, or collisionless?

   (d) Suppose we wanted to set up a plasma in LAPD with magnetized electrons and unmagnetized ions, but were allowed to change only a single parameter. Which parameter would you change and to what value? Support your answer with a calculation.

4. The plasma in the solar corona has parameters $n = 10^9$ cm^{-3}, $T_i = 2T_e = 100$ eV, and $B = 3$ kG. Note that plasma temperatures are often given in energy units of eV, where the Boltzmann constant has already been included.

   (a) Calculate the (electron) Debye length, electron and ion Larmor radius, and the plasma beta β.

5. The magnetic fields of the planets are often well approximated by dipole fields—at least close to the planet. A dipole field can be represented by a magnetic scalar potential of the form

$$\phi_m = \frac{M\cos\theta}{r^2}$$

   where $r = |\mathbf{r}|$ and the magnetic field is given by $\mathbf{B} = -\nabla\phi_m$.

   Note that ϕ is the azimuthal angle and θ is the polar angle in spherical coordinates.

   (a) Find the vector $\mathbf{B}$ in spherical coordinates and also the magnitude $B = |\mathbf{B}|$.

   (b) Show that $\mathbf{J} = 0$. Hint: Use Ampere's Law.