PHYS:4731 Homework #3

Due at the beginning of class, Thursday, September 15, 2022.

1. Show that the curvature drift

$$\mathbf{V}_c = \frac{v_\parallel^2}{\omega_c B}\frac{\mathbf{R}_c \times \mathbf{B}}{R_c^2}$$

can be written as

$$\mathbf{V}_c = \frac{v_\parallel^2}{\omega_c}\hat{\mathbf{b}} \times (\hat{\mathbf{b}} \cdot \nabla)\hat{\mathbf{b}}$$

You may take the magnetic field to be purely azimuthal $\mathbf{B} = B\hat{\boldsymbol{\phi}}$.
HINT: Convert from cylindrical to Cartesian coordinates, and you may find this expression useful,

$$\frac{\partial\hat{\boldsymbol{\phi}}}{\partial\phi} = \frac{\partial}{\partial\phi}(-\sin\phi\hat{\mathbf{x}} + \cos\phi\hat{\mathbf{y}}) = -\cos\phi\hat{\mathbf{x}} - \sin\phi\hat{\mathbf{y}} = -\hat{\mathbf{r}}$$

2. A 20 keV deuteron in a large mirror fusion device has a pitch angle of 45° at the midplane of the machine, where the magnetic field $B = 0.7$ T. Compute its Larmor radius.

3. The equation for a dipole mangetic field in spherical coordinates is given by

$$\mathbf{B} = \frac{\mu_0 M}{4\pi}\frac{1}{r^3}(2\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\boldsymbol{\theta}})$$

where M is the magnetic moment.

   (a) Show that the equation for a magnetic field line is $r = R\sin^2\theta$, where R is the radius of the magnetic field line at the equator ($\theta = \pi/2$).

   (b) Show that the curvature of the magnetic field line at the equator is $R_C = R/3$.

   (c) Compute the curvature drift of a particle with charge q and parallel velocity $v_\parallel$ at a radial distance R at the equator.

   (d) Compute the ∇B drift of a particle with charge q and perpendicular velocity $v_\perp$ at a radial distance R at the equator.

   (e) Compare the equations for the curvature drift and the ∇B drift at the equator.

4. A particle is trapped in a magnetic mirror field given by

$$B_z = B_0\left[1 + \left(\frac{z}{L}\right)^2\right]$$

and has a total kinetic energy $w = mv^2/2$ and pitch angle α at $z = 0$. Find the oscillation frequency in terms of L, w,and α.