

# PHYS:4731 Homework #8

Due at the beginning of class, Thursday, November 10, 2022.

## 1. Fluid Electron Waves

Assuming that the ions are stationary in a homogeneous, unmagnetized plasma and that the electrons respond to an applied electrostatic wave of the form

$$\phi(x, t) = \phi_1 e^{i(kx - \omega t)},$$

use the electron equations from the Two Fluid Equations (neglecting the drag term)

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{U}_e) = 0$$

$$m_e n_e \left( \frac{\partial \mathbf{U}_e}{\partial t} + \mathbf{U}_e \cdot \nabla \mathbf{U}_e \right) = -\nabla p_e - e n_e (\mathbf{E} + \mathbf{U}_e \times \mathbf{B})$$

$$\frac{d}{dt} \left( \frac{p_e}{n_e^\gamma} \right) = 0$$

along with Poisson's Equation

$$\nabla \cdot \mathbf{E} = \frac{\sum_s n_s q_s}{\epsilon_0}$$

to derive the dispersion relation  $\omega = \omega(k)$ . Hint: Use  $\mathbf{E} = -\nabla\phi$  and you may use the expression for the equilibrium quantities  $p_{0e} = n_{0e} T_{0e}$  (where temperature is expressed in energy units).

- Treating the perturbation as small, write down the linearized set of equations. Hint: Don't forget to use the property of quasineutrality.
- Solve for the dispersion relation  $\omega = \omega(k)$  in terms of the electron plasma frequency  $\omega_{pe}$  and the electron thermal velocity  $v_{te} = \sqrt{2T_{0e}/m_e}$ .
- What is the appropriate adiabatic index  $\gamma$  for these waves if they are very fast (and thus adiabatic) and strictly one-dimensional?
- What is the group velocity for these waves?

## 2. From the general form for linearized Ideal MHD,

$$\omega^2 \mathbf{U}_1 = (c_s^2 + v_A^2)(\mathbf{k} \cdot \mathbf{U}_1) \mathbf{k} - v_A^2 (\hat{\mathbf{b}} \cdot \mathbf{U}_1) (\hat{\mathbf{b}} \cdot \mathbf{k}) \mathbf{k} - v_A^2 (\hat{\mathbf{b}} \cdot \mathbf{k}) (\mathbf{k} \cdot \mathbf{U}_1) \hat{\mathbf{b}} + v_A^2 (\hat{\mathbf{b}} \cdot \mathbf{k})^2 \mathbf{U}_1,$$

solve for the MHD Dispersion Relation in the form  $D(\omega, \mathbf{k}) = 0$  (this is the determinant set equal to zero).

## 3. One-Dimensional Solar Wind Model

Consider a simplified, steady-state (constant in time) model of the solar wind near the equatorial plane in which all quantities depend only on the spherical radius  $r$ . Assume the radial component of the solar wind velocity is a given function  $v_r(r)$ . Hint: Use spherical coordinates, and take  $B_\theta = 0$ .

- Use the MHD continuity equation to derive the mass density  $\rho$  as a function of radius  $r$  in terms of the solar radius  $R_\odot$ , the density at the solar surface  $\rho_\odot$ , and the radial velocity at the solar surface  $v_r(R_\odot)$ .
- Use divergence free condition on the magnetic field to determine the radial component of the magnetic field  $B_r$  as a function of radius  $r$  in terms of the solar radius  $R_\odot$  and the radial field at the solar surface  $B_r(R_\odot)$ . NOTE: This is an expression of the conservation of magnetic flux.
- Use the azimuthal component of the Ideal MHD induction equation to derive an expression for  $B_\phi$  as a function of radius  $r$  in terms of the angular rotational velocity at the solar surface  $\Omega_\odot$ , the radial component of the magnetic field  $B_r$ , radial solar wind velocity  $v_r$ , and the azimuthal solar wind velocity  $v_\phi$ . You may assume that the magnetic field at the solar surface is purely radial  $B_\phi(R_\odot) = 0$ . Hint: Use your results from part (b) to simplify the expression and express the  $v_\phi$  at the solar surface as a function of  $\Omega_\odot$ .