FRONTIERS OF PHYSICS TODAY

PLASMAS

This "fourth state of matter" offers an immense variety of physical phenomena. Applications are tentative, but surprisingly widespread.

HAROLD GRAD

THE TENOR OF OUR TIMES is receptive to a very young science that claims dominion over 99% of the matter in the universe, proposes to fuel a cross-country auto trip with the deuterium from one gallon of sea water, offers to replace the magic of catalysis in polymer chemistry with precise knob turning, promises to alleviate the pollution problem by instant vaporization of waste and garbage, ventures to propel space ships and essays a role in cosmology. Even though these specific future applications of plasma physics are not proven, the potentialities of plasma, the "fourth state" of matter, are difficult to overstate.

Without regard to applications, the wealth of physical phenomena encountered in the plasma state exceeds the variety spanned by substances as diverse as air, water, peanut butter and superfluid helium. I will not presume to give a balanced picture of this explosively developing field. Instead, I present here some of the flavor of the subject through a few topics of personal interest and familiarity, binding it together by an overall evaluation of where we are and where we may be heading; at the conclusion are listed some complementary articles of general interest.

The frontiers of the subject are, in a word, everywhere. Despite a phenomenal growth in theoretical understanding and in experimental control of plasmas, there are almost daily revelations and discoveries of new and unexpected fundamental insights, frequently overturning our most cherished beliefs.

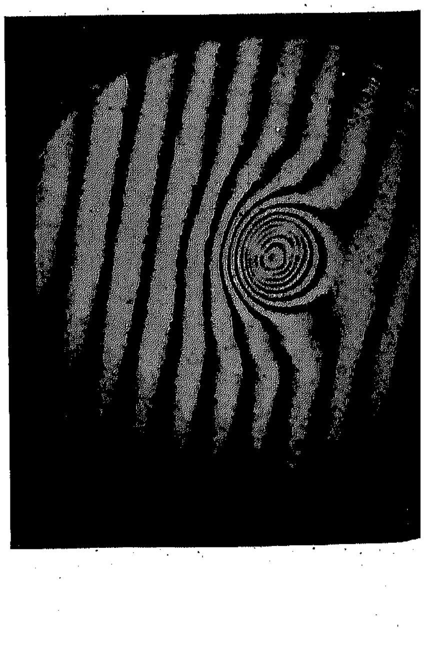
In common with nuclear physics we hear echoes in plasmas; in common with superfluid helium we observe not only second sound but also third, fourth, and more; in common with gas dynamics, we find shock waves and turbulence, both in bewildering variety; in common with crystallography we find anisotropy, but in much more exaggerated form; in common with all other fields, plasmas display waves, but in an unprecedented assortment of types, packets and interactions.

One branch of physics, for example superfluidity, is catalyzed by the discovery of an unexpected natural phenomenon. Another, say fundamental-particle physics, explores unknown territory simply "because it is there" and will uncover unusual phenomena as a matter of course. Plasma physics lies closer, in spirit, to the latter. Unexpected and unfamiliar phenomena are abundant, and each discovery opens a new subfield. Yet no evident single focus unifies the subject other than our desire to discover what we can about ionized and conducting matter. Whether the conceptual unity hoped for in fundamental-particle physics will ever overtake plasma physics is doubtful. Certainly the basic qualitative principles that govern plasma behavior are not yet established. Even so, the initial dust cloud is beginning to settle, goals are taking shape, measurements are becoming reliable (see figure 1), and practical means of answering questions are beginning to emerge.

What is a plasma?

A plasma is any electrically conducting medium whose electrical properties are sufficiently pronounced to react back on an external field. There is no end of materials that fit this description. Plasmas are found in the ionosphere, in the solar wind, within the sun, and on reëntry from space; within the laboratory, we have hot hydrogen plasmas and replicas of the sun, also relatively cool gas discharges and alkali plasmas; other plasmas occur in semiconductors, in polymer chemistry, and in metals, both liquid and solid. These diverse substances are related by many qualitative and even some quantitative features: plasma oscillations, Alfvén waves, the concept of magnetic flux carried with the flow, and so on. Nonetheless, even one of these distinct types of plasmas possesses a vast range of parameters and exhibits an awesome variety of qualitatively different properties.

For example, the major experimental programs that are currently considered



to be directly relevant to the controlled-thermonuclear goal deal with hydrogen plasmas at temperatures ranging over a factor of 10^8 and densities over 10^6 (see table on page 36). The comparison of air with water, which is only 10^3 times as dense, or water with a white dwarf, which is only 10^5 times as dense, or superfluid helium with atmospheric helium, which is only 10^2 as hot, leads us to expect similar large differences in the properties of plasma states separated by so many orders of magnitude.

One of the important plasma parameters is β, the ratio of plasma to magnetic-field energy density. A factor of 10^5 separates the values of β found in hot-plasma research. Thus we can expect all theoretical and experimental problems—orbits, equilibrium, diffusion, stability, injection, heating, impurity control—to be five orders of magnitude apart. Different phenomena dominate plasma behavior in high- and low-β plasmas; the technology, the diagnostic tools, the theoretical models, even the basic qualitative intuitions, are quite distinct.

Plasma parameters

A primitive but important clue to the qualitative types of phenomena that are likely to be encountered is given by the values of key dimensionless parameters. As a fluid, air behaves more like water, at similar Mach and Reynolds numbers, than slowly moving air behaves like hypersonic air. Similarly, macroscopic MHD (see box on page 37) theory may be adequate for very different solar and laboratory plasmas in comparable *scaled* parameter ranges. However, instead of two basic parameters as in classical dissipative fluid dynamics (Mach and Reynolds), two in ordinary kinetic theory (Mach and Knudsen) and two in ideal MHD (Mach and Alfvén) we have *seven or more* in standard, fully ionized, plasma physics.

Crudely subdividing the range of each parameter into small, medium and large, we can expect $3^2 = 9$ qualitatively different regions to cover fluid dynamics (potential, boundary-layer, hypersonic, turbulent flow, and so on), and $3^7 = 2187$ regions to cover plasma physics in a comparably crude way. Entirely different physical phenomena will arise depending on the relative

LASER INTERFEROGRAMS of Scylla IV, showing plasma compression and loss out of the ends, at 2.4, 3.6, 4.9 and 6.1 microsec. The number of fringes is proportional to density. —FIG. 1

values of lengths such as the electron radius, collision cross section, mean distance between electrons, Debye length, Larmor radius, mean free path. Different phenomena will also arise that depend on the frequencies obtained by combining these lengths with thermal speed, or Alfvén speed, or speed of light, not to speak of interference and resonances with each other and with independent externally imposed geometrical lengths, excitation frequencies, and speeds. The high dimensionality of this parameter space is the key without which we cannot begin to understand the structure of plasma physics. Our goal is not to find *one* theory of plasma behavior but to find very many theories of the behavior of many different plasmas.

Typical Hot Plasma Parameters

Device	*Density (ions/cm³)*	*Ion temperature (KeV or 10⁷K)*	*Confinement time*	β
2X (Lawrence Radiation Laboratory) Mirror with well	5×10^{13}	8.0	1 millisec	0.1
T3 (USSR) Tokomak	5×10^{13}	0.5	20 millisec	0.002
Scylla IV (Los Alamos) Theta pinch	4×10^{16}	5.0	3 microsec	0.8
Focus (Los Alamos) Coaxial gun*	2×10^{19}	6.0	0.1 microsec	high
DCX-2 (Oak Ridge) Mirror	5×10^{9}	500	0.5 sec	0.001
Stellarator C (Princeton)	3×10^{13}	0.15	1 millisec	0.0001
Centaur (Culham) Cusp-ended theta pinch	10^{16}	0.3	5 microsec	0.99+

* Volume of plasma is about 0.01 cm³.

Medium versus geometry

A considerable amount is known about shock waves. In ordinary kinetic theory of an unionized gas a single parameter (the shock strength or Mach number) completely determines the profile of a plane shock wave. The corresponding steady plane shock wave in a fully ionized plasma takes *six* dimensionless parameters to specify its profile. In the special case of a weak shock propagating perpendicular to the magnetic field, the profile depends only on β and the Hall factor $\omega\tau$, in addition to the mass ratio $\alpha^2 = m_-/m_+$. One might expect the mass ratio to be an unessential parameter, because it is always small, but the limit as α^2 approaches zero is quite singular. It is most easily surveyed by taking $\beta \approx \alpha^r$ and $\omega\tau \approx \alpha^s$ for a variety of values (positive and negative) of r and s. Each r,s region shown in figure 2 represents qualitatively different behavior; the shock thickness is dominated by a different dissipative mechanism such as ion viscosity, electron Hall heat flow, and so on. The transition regions, combining two or more mechanisms, are more complicated. To obtain all this information must surely have taken scores of man-years of calculation! Fortunately this wealth of physical information, representing the asymptotic solution of a pair of Boltzmann equations (ions and electrons) and Maxwell's equations, is given by an explicit, though very complicated, algebraic formula. Unfortunately we cannot expect other problems to yield explicit solutions of such generality.

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As we have mentioned, the number of parameters for more general (finite strength, oblique), but still classically collision-dominated shock profiles goes up from three to six. Only a very small part of this parameter space has been investigated. More seriously, entirely new dissipative mechanisms, involving a host of instabilities, turbulence and so on, enter with greater shock strength.

The shock problem is posed for an infinite medium with no boundaries or geometrical features; its complexity, in different parameter ranges, arises entirely from intrinsic plasma properties. To isolate plasma from geometrical complications we turn to the opposite extreme of the simplest plasma model, ideal static MHD. There is now only a single plasma parameter, β (instead of up to seven with more realistic models). But as soon as we try to contain the plasma, geometrical complications enter.

For example, consider containment in a stellarator. In its more complex forms this concept may involve separate curved and straight sections, each with a different helical winding, or several superposed helical windings on a common circular axis. The simplest stellarator has a circular axis and a single, symmetric, periodic helical winding; to describe it requires four lengths and three field parameters for a total of at least six dimensionless parameters. But from the limited theory that is available, we find that the simplest MHD model is sufficiently fertile to reveal qualitatively different physical behavior in different corners of the six-dimensional parameter space. Most of this space is still terra incognita.

The shock and stellarator examples just given illustrate plasma and geometrical complexities respectively. Some idea of the possible interplay between physical and geometrical effects in plasmas can be obtained by a glance at classical fluid dynamics where much more is known. One description of a fluid is by a dispersion formula, say $\omega^2 = k^2a^2$ where $a^2 = \partial p/\partial\rho$ is the speed of sound. Hidden in this trivial formula for an ordinary gas are the theory of the organ pipe, lift and drag, all of diffraction theory, and the transition from wave to ray optics. Spatial variation of a^2, through its dependence on density and temperature, introduces refraction, transonic flows, shocks, implosions and explosions, all sorts of waves (gravity, ship, and tidal), breakers and bores, wakes, cavitation, bubbles, and so on. Viscosity complicates the dispersion

GLOSSARY

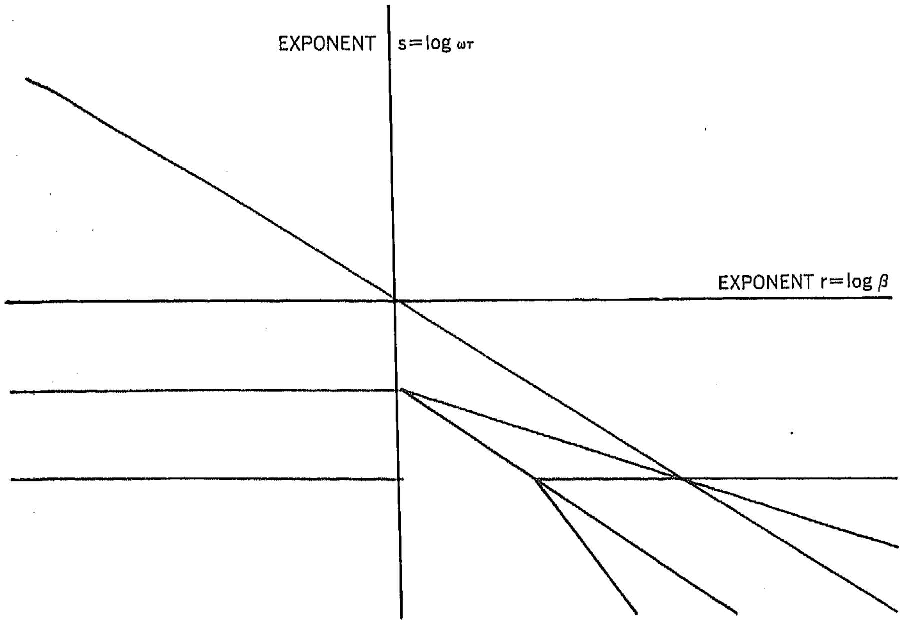
MHD—ideal, nondissipative, macroscopic magnetofluid dynamics
guiding center—small Larmor radius orbit (and collective plasma) approximation
$\beta=8\pi p/H^2$—ratio of plasma to magnetic pressure (or enegy)
z-pinch—cylindrical plasma column with I_z and B_θ
θ-pinch—cylindrical plasma column with I_θ and B_z ("Pinch" originally referred to a transient; now it refers also to static equilibria.)
Q-machine—alkali plasma (originally hoped to be "quiescent")
Tokomak—toroidal z-pinch; flux surfaces formed by plasma current
stellarator—toroidal; flux surfaces formed by external windings, usually helical
multipole—usually toroidal configuration with internal conductors, either supported or levitated
banana—drift surface (see figure 9b)
loss cone—part of phase space from which an orbit will eventually escape
Bohm diffusion, D_B—an arbitrary unit, $\frac{ckT}{16eB}$.

formula only slightly, but it introduces an assortment of boundary layers, sedimentation, and all of meteorology and oceanography. We see that *almost all* the interesting qualitative physical phenomena in a classical fluid are not visible in the dispersion formula and are found only in finite geometries with boundaries.

In an infinite homogeneous plasma the formula that describes propagation of small-amplitude plane waves, corresponding to $\omega^2 = k^2a^2$, has been studied extensively but by no means exhaustively. It is a transcendental relation involving the ion and electron velocity-distribution functions, and it exhibits an infinite number of dispersive and anisotropic modes as well as many continua. In principle each plasma mode could ramify as widely as all of classical fluid dynamics in a real geometry. Taking fluid dynamics as our model, we expect that *most of the basic qualitative plasma phenomena will be discovered only as nonlinear and finite geometry effects,* not directly visible in the dispersion formula. Only in the simplest plasma models, such as MHD, guiding center, and magnetoionic theory, is there an appreciable corpus of nonlinear- and finite-geometry plasma effects. Some geometrical effects appear as rather direct generalizations of classical effects; an example is the Fresnel zones, which are essential to the description of excitation and detection of ion-acoustic waves. Other effects such as coupling of different linear modes through boundary conditions or through variable density of the medium are more peculiar to plasmas.

We should not leave the impression that plasmas are always more complex than their neutral counterparts. As a possible counterexample we point to fluid turbulence, which is a strongly nonlinear and essentially three-dimensional phenomenon, only slightly related to fluid instability. We can compare it with the motion of a bouncing ball on a cobblestone street and the unrelated facts that the top of a stone is unstable and a pot hole stable. Plasma, with its greater variety of waves and interactions and spectral complexity, allows an entirely different type of *weak turbulence* in which nonlinearity can be handled as a quasilinear perturbation. We also have strong plasma turbulence, which is likely to remain essentially empirical.

To return to the question "What is a plasma?", we can only say that we are just beginning to find out.



DISSIPATION MECHANISMS in a weak shock. Each region represents a different dissipation mechanism as a function of the values of β and $\omega\tau$ relative to the mass ratio. From P. N. Hu, Phys. Fluids 9, 89 (1966). —**FIG. 2**

To catch a hot plasma

Although nature is always exceedingly complex, physics gains its strength precisely by rejecting complexities as they occur in nature in order to study selected, isolated "basic" phenomena. "The unreasonable effectiveness of mathematics in the natural sciences"[1] results from natural selection of isolated phenomena—both experimental and theoretical—as the subject matter of science. The basic goal of experimental plasma physics is the construction of experiments, each of which isolates an individual phenomenon, in enough variety to cover qualitatively the entire field. This is a long-term project, but the multiplicity of effects is not the most serious road block. Before we can study a plasma, we have to catch one. For a hot plasma, this stipulation conflicts with the best scientific sequence. To create a hot plasma and keep it away from the walls long enough for study requires complex experimental procedures and complicated geometries that conflict with the desire to isolate individual phenomena. In a contained hot plasma the scientific problems are presented all at once rather than in sequence. Analysis of complex systems is commonplace in engineering, but *not* when the individual phenomena have not yet been scientifically explored.

The relatively high degree of understanding of mirror plasma, compared with toroidal plasmas, is probably a result of the dominance, in mirrors, of a single identifiable physical mechanism. The mirror configuration is characterized by extreme anisotropy and a pronounced loss cone (see figure 3). The two principal containment problems are scattering into the loss cone and plasma instability. Both the basic scattering loss mechanism and the basic loss-cone instability can be studied analytically in an infinite homogeneous medium without boundaries, and therefore with a high degree of theoretical reliability. "Finite-geometry" complications can be adjoined afterwards as relatively minor peturbations of the basic phenomena that do not drastically change the qualitative picture.

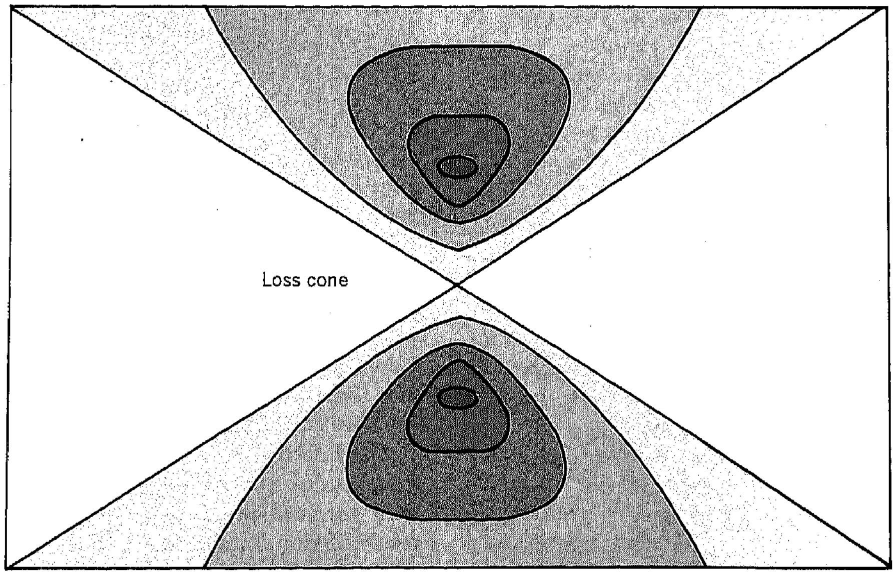
Only recently have we discovered that toroidal systems, in addition to their own peculiarly toroidal difficulties, possess most of the problems of mirror machines. Loss cones emerge in many forms in a toroidal system, but less obtrusively than in a mirror. Many classes of mirroring (trapped) orbits appear with all their attendant problems, but they do not dominate. In all but the simplest toroidal geometries, anisotropy appears in an essential and complex way, but again in a weakened form. In even the geometrically simplest closed configuration (Tokomak), the field topology enters significantly. It appears more than likely that the reason we do not yet understand the limitations of toroidal confinement is that there are so many comparable competing effects, not that a single elusive effect remains to be discovered. Thus far every simple mechanism has been proved to be inadequate. Synergistic combinations are beginning to be explored. Empirically the simplest configuration (Tokomak) is also the most successful. Perhaps recent recognition of the complexity of the toroidal-containment problem will turn out to be the single most important step towards its ultimate resolution.

The rapid growth of technology and empirical experience in containing hot plasmas (see table on page 36) has also made itself felt in accelerating the discovery of basic physical phenomena. For example, in a situation that is not atypical of containment experiments, the plasma found in the DCX-2 apparatus is quite different from what is injected, and it is contained by incidental fine structure in the applied mirror field. But the plasma is quite uniform and has served as an excellent medium for basic studies of finite Larmor radius and extremely anisotropic effects (including, ultimately, the discovery of the mechanism for the origin of the plasma).

There has also been a large recent development of relatively low budget, nonthermonuclear plasma experiments insensitive to wall isolation. From the point of view of basic physics, the two classes of plasmas are sufficiently distinct to require pursuit of both.

One example of a basic phenomenon not intimately tied to containment is the plasma echo (to be discussed later). Another example is the interaction of plasma oscillations and optical emissions. The fluctuating electron density modulates light emission at the plasma frequency. This effect, first observed in the ionosphere, can be used for atomic measurements of excited-state lifetimes or for plasma diagnostics.

The relatively well developed field of alkali plasmas is, in some respects, a bridge between the physics of contained and noncontained plasmas.



DENSITY CONTOURS in velocity space for a typical mirror-machine loss-cone distribution. Darker color represents greater density. —FIG. 3

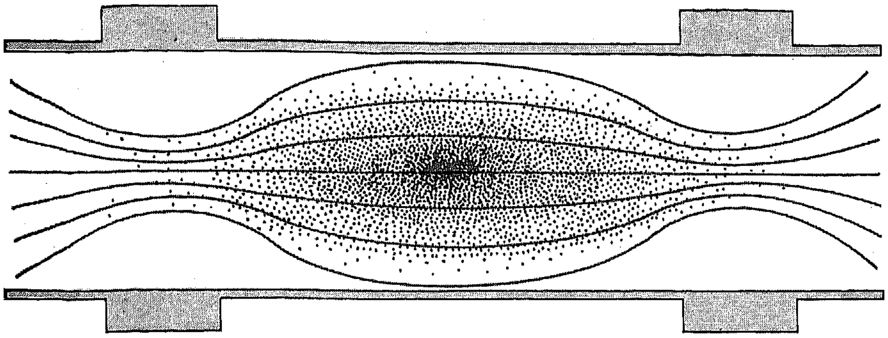
Where is the frontier?

The goal in plasma physics, as in the study of any fluid such as air, water, or liquid helium, is to understand and to control it—to pump, to compress, to heat and extract energy, to propagate waves, to measure and, above all, to keep it from leaking.

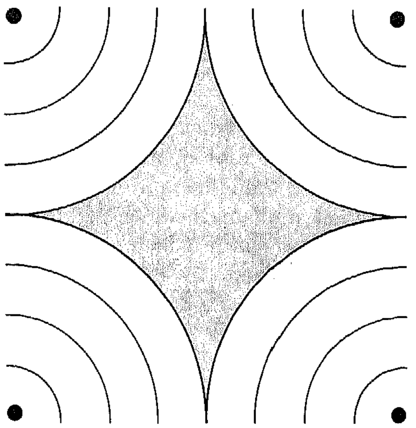
In view of the ramification of the subject, it is not surprising to find that progress is not uniform over all of plasma physics. At the frontier, open questions range from determination of an equation of state in one type of plasma to highly specialized effects dominated by details of the geometry and distribution function in another. In highly condensed plasmas, such as very high-pressure alkali (classical) or solid-state (degenerate-electron) plasmas, elementary thermodynamic and transport properties are the immediate problem, both experimentally and theoretically. In moderately dense θ-pinches, macroscopic equilibrium and stability questions on a microsecond time scale are the most urgent present concern. In some well documented mirror-contained hot plasmas the spectrum of identified phenomena is much broader, and we have consistent theoretical and experimental information about large classes of waves and instabilities in strongly non-Maxwellian plasmas on a relatively long time scale. In at least one case, measurements provide an essentially complete ion-distribution function in velocity and physical space.

The one feature common to all experimental areas is the impressive improvement in reliability and flexibility of diagnostic methods. By pushing the state of the art in x-ray techniques, in charge-exchange neutral measurements, in Thomson scattering and in laser holography, selective tools are being developed to cover wide ranges of density and temperature.

In problems where theory and experiment make an attempt to converge on an isolated phenomenon (for example, the plasma echo, which is approximately one dimensional in an infinite homogeneous medium), there is very good agreement between the two. In containment geometries, where experimental flexibility is severely



MIRROR-MACHINE configuration, typical of open-ended devices. —FIG. 4



CUSP GEOMETRY with opposed Helmholtz coils. This is the prototype magnetic-well geometry. —FIG. 5

limited, the agreement is not nearly as good.

The factors that limit currently operating mirror machines and θ-pinches are fairly well understood, but scaling to new parameters is not at all certain. In stellarators and Tokomaks even the present limiting factors have not been identified, and scaling is unknown. Multipoles lie somewhere in between.

The hallmark of a clean physics experiment, isolation of a specific phenomenon, is just as much a necessity for an effective theory. Even in pure theory it is not easy to isolate an effect by fiat, just by adding or dropping a term. As in experiment, the long-term goal of plasma theory is to find and develop an arsenal of models and diagnostic techniques that are capable of separating out different effects. The development of sophisticated theoretical diagnostics has lagged somewhat behind that of experiment. This lag may be because the tradition that valid experimental results require great care is not quite so widespread in theoretical work.

Qualitative concepts

Because of the enormous complexity of plasma physics, rough qualitative models take on more than their usual importance. Where do the qualitative, intuitive concepts that bind a field arise? If history is any guide, they do not come from synthesis of masses of experimental or theoretical data; nor do they come from use of crude theoretical analyses of complicated problems (such as an analysis of a containment configuration). Rather they arise from solutions of simple problems that turn out to be more accurate and more representative of the general case than one could reasonably expect.

It is illuminating to consider fluid dynamics as a prototype of a well developed subject. The simplest fluid model, incompressible irrotational flow, is hardly realistic. But every aerodynamicist expends considerable effort to develop a strong intuition about this nonexistent fluid. He describes actual flows as deviations from this ideal (in boundary layers, shocks, and so on). Without a precise knowledge of these deviations, potential flow has very little value; with this knowledge, it is priceless. Without the aid of the ideal theory, the more exact viscous theory would also have very little value, simply because of its complexity. Despite great advances in theory and in computing capability, we still solve the full Navier–Stokes equations only in elementary geometries. Fluid dynamics thus exhibits a complex, symbiotic relation among its subtheories, with the whole greater than the sum of its parts.

The ideal plasma concept of "frozen" flux, carried with the plasma, has a similar significance. Although it is almost always a poor approximation, one can hardly carry on a sensible discussion of plasma containment or motion without this concept as the starting point.

On the other hand, qualitative descriptions do not always help. A simple illustration of the pitfalls of semantic analogy is given by the diamagnetic properties of a plasma. This concept clearly requires quantitative modification, because a plasma is a rather complex substance. But the extent of this modification is surprising.

The simplest evidence for the diamagnetic effect is the current carried by the circular orbit of a charged particle in a uniform magnetic field. Similarly, in a nonuniform but unidirectional field, the macroscopic equilibrium pressure balance, $p + B^2/2 =$ constant, indicates that the plasma depresses the field strength. In a more complex geometry the orbits become complicated, and the static pressure balance is anisotropic. The guiding-center approximation to the orbits is closely tied to the diamagnetic image, because it assigns to each particle a constant (negative) magnetic moment. The plasma current perpendicular to B is the sum of a definitely diamagnetic contribution from the magnetic-moment density and a current arising from the drift of guiding centers across the field. The latter component, frequently called the "diamagnetic drift," can easily have a paramagnetic sign. When paramagnetic it can even dominate the contribution of the magnetic moment and create a locally paramagnetic region in the plasma.

One can be more precise with a special class of guiding-center equilibria that yields an exact mathematical analog of a classical nonlinear magnetic medium (B is a function of H). For this anisotropic equilibrium, the pressure components, $p_\parallel$ and $p_\perp$, are constant on $|B|$ contours. Taking $\mu \equiv B/H$ as the definition of permeability, we find the criterion for a paramagnetic region, $\mu > 1$, to be $dp_\parallel/dB > 0$ (or $p_\parallel > p_\perp$). The alternative definition, $\mu \equiv dB/dH$, yields $\mu > 1$ wherever $dp_\perp/dB < 0$. These criteria are not intuitively evident. But with either definition, locally paramagnetic and diamagnetic regions can be found easily.

More striking than the existence of local paramagnetic regions is the possibility of a fully self-consistent plasma equilibrium that is *globally* paramagnetic (for example, in a simple mirror or cusp field, figures 4 and 5). In other words, the inductance of the external coil is increased by introducing plasma. Because plasma current along **B** complicates the interpretation of the diamagnetic effect, I have given only examples in which $J_{\|} = 0$.

The special case quoted, in which the plasma can be unambiguously identified as diamagnetic or paramagnetic, is also one in which stability is easily determined. In contrast to the classical result that a diamagnetic solid is stable in a well, each of the four combinations, diamagnetic or paramagnetic plasma in a well or on a hill, can be either stable or unstable.

One plain conclusion is that, in competition between the elegance and simplicity of a concept and the complexity possible in a plasma, complexity can usually be expected to triumph.

Psychological roadblocks

Another example of an appealing but somewhat specious qualitative concept is that of a *magnetic well.* Basically, we expect a plasma (diamagnetic!) to be stable in a well. The original magnetic-well formulation (1955), for a plasma with no internal magnetic field, separated at a sharp surface from a vacuum field, gave the necessary and sufficient condition for MHD stability that the magnitude of B increase everywhere from the plasma surface. An immediate consequence was that no plasma with a smooth boundary, mirror or toroidal, can be stably contained; only the cusped geometries (for example, figure 5) are stable. This qualitative stability principle was dramatically demonstrated experimentally, in 1960, by applying cusped coils to the very unstable pinch.

A much more significant experiment from the point of view of thermonuclear confinement, was M. S. Ioffe's in 1962; he showed that cusped coils reduced fluctuations and improved containment in a mirror. But, although this was a landmark from the point of view of containment, the physics was (and is) not clear. The mirror-contained plasma is strongly anisotropic, and its boundary is not a flux surface. The mechanism for the initial fluctuations and high loss rate has not been definitely identified. Nor do we know the reason for the improvement after application of the well, because its imposition has implications for several classes of micro as well as macro stability and also for the equilibrium drift-surface topology.

Among the theoretical instabilities affected by well-like field configuration are interchange, drift, trapped particle, resistive, local and modified negative mass. Each one depends on a different magnetic-field criterion. In addition, there are several related (but different) well-like properties of the field that have a bearing on the containment of individual orbits and phase mixing of plasma imperfections rather than on any collective property. And finally there are cases where application of a magnetic well is detrimental for containment.

To summarize, the magnetic well may be ten of the most important plasma-containment concepts, but it is not just one! It is one thing to synthesize and coördinate; it is another to obliterate essential differences.

Another example of a non-concept is the term "Bohm diffusion." As a diffusion coefficient, the Bohm value is the product of thermal speed and Larmor radius. The Bohm time can also be obtained as the length of time required for a drifting ion or electron to pass once around a minor circumference. As the term is used, Bohm diffusion does not refer to a phenomenon or to a mechanism but to a natural plasma time scale that can arise in many ways, both collective and noncollective. There are easily a dozen different physical mechanisms that can give rise to loss rates comparable to Bohm. Their semantic synthesis into a single concept is artificial and a degradation of information.

Echoes, shocks, and phase mixing

Dissipation appears in a time-reversible theory in the guise of phase mixing; analytically, it is recognized as a continuous spectrum. The basic point is that any finite or infinite discrete sum, $\Sigma a_n \exp(i\omega_n t)$, oscillates indefinitely; an integral, $\int a(\omega) \exp\,(i\omega t)\, d\omega$, can, however, decay. The most important qualitative feature of a continuous spectrum is that it preserves much of the information fed into the system by initial and boundary data and gives rise to much more complex phenomena than a discrete normal mode, which is primarily a property of the medium. The fact that "Landau damping" is not universally given by Landau's formula and that the wave preserves initial and boundary data has long been recognized theoretically and has recently come to the fore with experimental observations of echoes and various "ballistic" or free-flow effects.

Phase mixing with collisionless damping is not restricted to kinetic models but is also found in macroscopic theory of Alfvén waves and in cold plasma and magnetoionic theory.

In ordinary air, a wall oscillating at a fixed frequency ω gives rise to a disturbance $\exp[i\omega(t-x/v)]$. Integrating over a Maxwellian velocity distribution gives a signal that damps approximately as $\exp[-(\omega x)^{2/3}]$. This collisionless decay has been experimentally confirmed for high frequency waves in argon. The damping is reduced at lower frequencies, and when the wavelength exceeds the mean free path and collective behavior dominates over ballistic, the wave eventually approaches an undamped ordinary sound wave.

Exactly the same phenomenon holds in a plasma, except that the collective effect of the charge-separation field enters much more strongly than that of collisions—at the Debye length instead of at the much larger mean free path. Landau damping describes the electrostatic modification of free-flow collisionless damping and is valid for wavelengths not smaller and not too much larger than the Debye length.

Only within recent years has more accurate theory delimited, and very careful experiment been able to confirm Landau's more than 20-year-old formula. At the same time, "non-Landau" damping effects, such as echoes and ballistic effects in ion-acoustic and other waves, are also being observed.

To obtain a spatial echo, two parallel grids are excited at different frequencies. Any nonlinear coupling of the two disturbances $\exp[i\omega(t-x/v)]$ and $\exp[i\omega'(t-x'/v)]$ will produce a signal $\exp[i(\omega-\omega')t + i(\omega'x'-\omega x)/v]$. The phase mixing disappears and the modulated signal is regenerated as an echo at a position such that $\omega'x'-\omega x = 0$.

Space-resolved electron-plasma echoes have been observed, as have time-resolved ion-acoustic echoes produced by asynchronism from plasma gradients. The magnitude of the echo is being used as a sensitive measure of the collisional dissipation and of the dissipation resulting from externally imposed noise between initiation and echo, as in the nuclear-magnetic-resonance effect.

Collisionless shocks are also a manifestation of phase mixing, but in a more complex nonlinear version. In most laboratory shocks that are identified as collisionless, the dissipation is presently attributed to an instability or to turbulence. In some of the more elaborate theories, a structure involving two or more distinct instabilities in sequence is invoked. For example, a steep wavefront with large electron current density induces a two-stream instability. The instability produces thermalization only in the direction of the current; the resultant unstable anisotropic distribution induces further thermalization.

In principle, neither instability nor turbulence is needed to effect irreversibility in a collisionless model. The earliest collisionless shock models, antedating experiment, were laminar; they involved phase mixing of ion orbits and "collisions" of particles with the electric and magnetic fields.

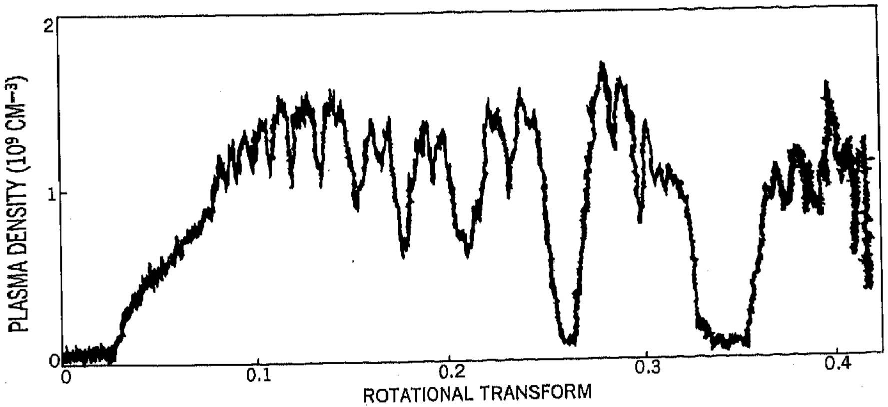
Although the first reliable collisionless shock measurements were of the earth's bow shock, laboratory experiments have recently become quite reliable in several collisionless regimes. Present understanding is largely empirical, based on the introduction of ad hoc "anomalous" collision frequencies into a theoretical calculation to achieve an experimental fit. Definite identification of irreversibility mechanisms remains open, because the most reliable theory is for weak shocks whereas most experimental data is for strong shocks. The particular value of a shock wave in heating a plasma is that the plasma itself chooses which is the most efficient irreversible mechanism under the given circumstances.

Factors in containment

In the physics of hot plasmas the problem we must face first is containment. This problem can be split into a study of orbits, equilibrium, stability and diffusion. They are all interrelated; in particular, knowledge of particle orbits is used everywhere. But only the most primitive approximations of the highly developed orbit theory can be used quantitatively in the more difficult self-consistent plasma application. Nevertheless, we shall see that even the most sophisticated orbit results give very important *qualitative* information about all these subjects.

The logical sequence for a study of containment is that given above: first individual orbits, then self-consistent equilibrium, then stability and diffusion. In particular, poor containment can follow as easily from orbit and equilibrium considerations as from instability. This prescription has been moderately well followed in mirrors, taking into account the dominant role of the loss cone. Although the emphasis in toroidal investigations has long been on microinstability (in particular, drift and resistive), it has, somewhat belatedly, become clear that these mechanisms should defer in priority to the more basic questions of orbits, self-consistent equilibrium, and macrostability, all of which are inadequately understood. Especially on the present time scale of high-beta experiments, it is quite unlikely that microphenomena are important.

The distinction between "macro" and "micro" instability is not a question of the theoretical model but a distinction between an instability that moves the plasma bodily to the wall and one that exhibits small-scale fluctuations. The containment effect of the latter is usually described as "enhanced" diffusion. Either type of instability can be tolerable or catastrophic, depending on the time scale. The basic time of growth of a small-amplitude disturbance is rarely a measure of the importance of the instability. The two-stream instability, for example, exhibits extremely fast growth but is self-limiting, saturating at a low amplitude of fluctuation that preserves the velocity profile at a marginally unstable shape.



PLASMA DENSITY in the Wendelstein $l=2$ stellarator as a function of the rotational transform. There are strong resonances where the field lines close after 3, 4, 5 . . . 15 . . . times the long way round the torus. —FIG. 6

Exactly the same distinction should be made between a localized failure of microequilibrium and global macrodisequilibrium. For example, it was the impossibility of gross pressure balance in the simplest toroidal field that led to the invention of the original figure-eight stellarator. But there are also strictly local failures in maintaining a self-consistent plasma-field equilibrium. These failures can lead to anomalously high currents, irreducible fluctuations propagating as Alfvén waves, and enhanced losses.

Moreover, even in macroscopic MHD stability theory, it is only by making distinct separation between local and global instability that we can establish some points of contact between theory and experiment.

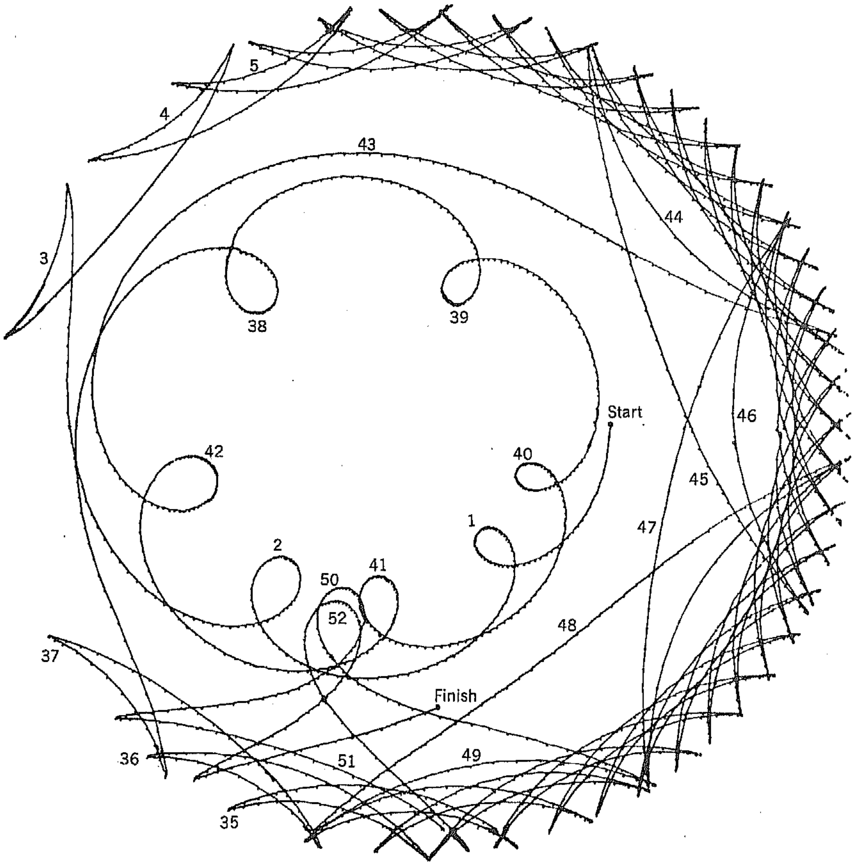
The one property that is unique to toroidal containment is *closure* (as distinguished from curvature, which can be mimicked in open systems). Magnetic lines carry all sorts of information: electrostatic potential, Alfvén waves, guiding-center orbits, and so on. In an open system, information is exchanged with the outside world in both directions along a magnetic line, whether intentionally or not. In a toroidal system, the information remains inside; there are specific plasma complaints that are neither sensed nor easily remedied. An example is resonance effects in magnetic surfaces and particle orbits, which have long been known from experience with accelerators. But recently *collective* plasma closure effects have been predicted and also observed. Figure 6 shows the observed dependence of equilibrium plasma density on rotational transform in a carefully designed stellarator. Distinct peaks are found for fields with resonances up to the 15th order (that is, a field line closes after 15 circuits the long way round the torus). Because the mean free path is less than one circuit of the torus, this observation can not be an orbit effect. A possible explanation of this effect is in terms of microequilibrium. Selective mathematical diagnostic methods that may allow comparison with experiment are gradually being developed.

Instability has had a much more intensive development than equilibrium. The proliferation over the years of theoretical microinstabilities is itself characteristics of an explosive instability. There are recent signs of saturation. This can not be ascribed to the hypothesis that most instabilities are already known, because as we have already pointed out, only an infinitesimal part of the totality of gross qualitative plasma phenomena has yet been examined. What is a more likely explanation of the microinstability slowdown is discouragement, as only a small fraction of the list of theoretical instabilities has been identified experimentally.

Equilibrium

The difficulty of attaining plasma equilibrium can be seen by a glance at figure 7, which shows a typical particle orbit in a stellarator magnetic field. As an indication of how an equilibrium configuration might appear, recall that a fixed value of the distribution function must be assigned to each orbit in phase space (this requirement is prior to any strictures of self-consistency). It is clear that any equilibrium distribution function is very complicated, to say the least. More careful study shows that it is, in many cases, mathematically impossible. Even a Maxwell demon could not inject the plasma correctly. In a real plasma we must expect to find a certain irreducible level of fluctuations —independent of any question of stability. When we add self-consistency, we find that the number of special situations that allow time-independent solutions is even more restricted.

There is, of course, no reason other than mathematical convenience to look for stationary states. But without this convenience, the whole of conventional stability theory, based on perturbations about an assumed equilibrium, evaporates! When faced with the collapse of a theory one usually argues that something has been left out—finite Larmor radius, Debye radius, resistivity, and so on. But further study in this case shows that the only chance of resolving the crisis in containment theory lies in using cruder rather than more sophisticated models.



PARTICLE ORBITS in a stellarator field, calculated by computer. The numbers are provided only to facilitate following the orbit sequence. From H. Gibson, J. B. Taylor, Phys. Fluids *10,* 2653 (1967). —FIG. 7

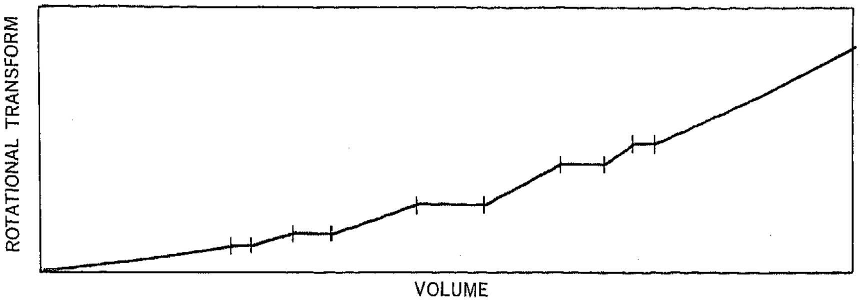
For some mathematical purposes it is appropriate to consider rational numbers as negligible, "of measure zero" compared to irrationals. But for many purposes the rationals must be considered on a par with the irrationals. For example, in a stellarator field with shear (variable rotational transform), the volume occupied by rational transform is finite. The variation of rotational transform would look qualitatively as shown in figure 8, where the flat stretches, of constant rational transform, occupy finite regions. Within these regions of constant transform, the magnetic field exhibits islands, ergodic regions, and all sorts of pathology. (The magnetic field is as smooth as you like—the pathology enters only in answer to the delicate question of what happens to a magnetic line if it is followed forever.)

With axial symmetry the magnetic field exhibits no such pathology. There are no gaps in the flux surfaces, and the rotational transform varies smoothly. But particle orbits (which can also be assigned a rotational transform) will, even in the case of axial symmetry, behave as in figure 8. In other words any time-independent equilibrium, in which a constant value of the distribution function must be assigned to each orbit, will be pathological.

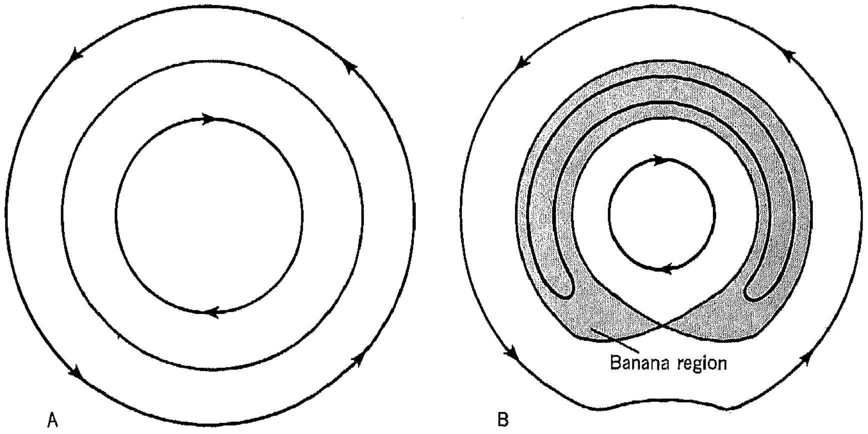
Returning to an asymmetric geometry (such as a stellarator), although a field line can be very complex (for example, ergodic) in a flat region, it is contained forever between two legitimate flux surfaces. This is not true of orbits in the asymmetric geometry. There are everywhere dense (possibly thin) loss cones from which particles can escape, given enough time. A true equilibrium distribution would have the value zero on this complex distributed loss cone. The same is true of any asymmetric mirror machine.

In some cases the pathological regions can be estimated to be very thin. In this case, the additional requirement of self-consistency of plasma currents with magnetic field turns out to be an independent source of pathology, namely very high current density in a new set of "flat" regions.

There is a large body of theory concerning equilibrium, frequently presenting "proofs" of the existence of equilibria to all order in general geometries. Interpretation of such formal analytic results needs great care. They have the valuable property of being blind to certain complicated phenomena. But they remain blind whether the neglected phenomena are negligible or dominant! We need a careful mix of naive and sophisticated calculations to extract the most useful information—for example, how long does it take for an approximate equilibrium to break up? Some slight progress is being made here.



ROTATIONAL TRANSFORM shown schematically as a function of plasma volume. In principle the flat portions occur at each rational value. —FIG. 8



DRIFT CURVES in a mirror machine. The direction of drift reverses as the turning point moves out. The zero-drift curve in a symmetric field (a) opens up into a "banana" region with asymmetry (b). —FIG. 9

More phase mixing

Most of the orbit and equilibrium pathology disappears when we use the guiding-center orbit approximation in a mirror machine. Moreover, phase mixing can sometimes be relied upon to correct imperfections of symmetry in the injected plasma. For example, consider an axially symmetric mirror machine. Because of field gradients, a guiding-center orbit will drift around a flux surface in a time comparable to the Bohm diffusion time. It is easy to verify that the sense of the drift reverses for particles that mirror close to the center and those that almost spill over. Phase-plane shear (variable drift speed, see figure 9a) provides phase mixing so that, if we ignore collective effects, any asymmetric plasma injection will be corrected after a number of drift periods. (More precisely, for low-energy nondrifting orbits, there is a stationary phase point, indicating poor mixing in this part of phase space.)

If axial symmetry is disturbed, the degenerate zero velocity curve in figure 9a will split, to form a drift pattern like that in figure 9b. The contours in figure 9a are level lines of a volcano, which, in figure 9b, lies on a slope. Although the shear is not zero, it is small throughout the banana region, and we can expect local errors in plasma symmetry to die away relatively slowly. During this process, local fluctuations would be observed. This poorly mixing region of phase space is the same one that is influential in creating drift instabilities, but the present phenomenon is quite distinct—in particular, it is noncollective. It is interesting to note that there are some magnetic-well configurations that do not exhibit drift reversal and banana regions.

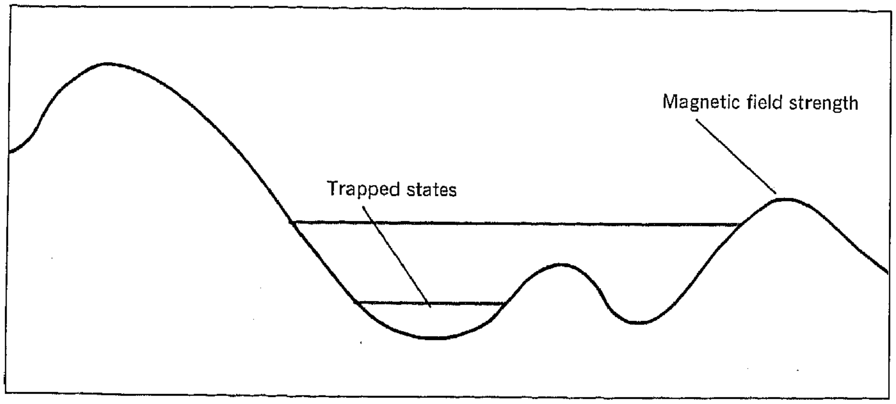
In a mirror machine with multiple mirrors, and in most toroidal devices, there will be several trapped states, as in figure 10, each generating its own family of drift surfaces. These drift surfaces have no relation to the flux surfaces, even for vanishing Larmor radius. Since a local maximum of B will vary from one magnetic line to another, a drifting particle can spill and change its trapped state (see figure 11). This change produces a random walk among the drift "surfaces," which turn out not to be surfaces at all, but to cover a finite volume of phase space ergodically. Equilibration through phase mixing will probably be slow in such regions. An easy estimate shows that drift surfaces or drift volumes that touch a wall or a loss cone give a direct loss rate comparable to Bohm, without the intervention of scattering or fluctuations.

Diffusion

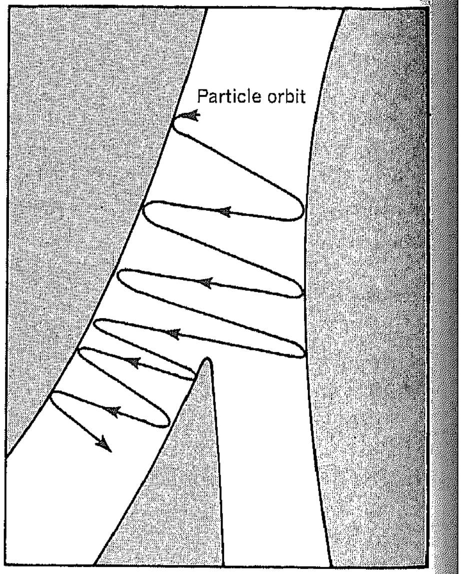
Diffusion is a very general term. It describes a variety of dissipative mechanisms that allow violation of the elementary, perfectly conducting concept of a plasma whose elements remain fixed to given magnetic lines or flux surfaces.

In the formulation as a random walk, diffusion of particles is not self-consistent. With an ambipolar calculation, it becomes slightly self-consistent. But to make it fully self-consistent involves at least all the complexities of the self-consistent-equilibrium problem that we have briefly outlined.

The diffusion problem is both qualitatively and quantitatively different in different ranges of the collision frequency ν. If ν is large, we have a random walk of particles, localized in physical space. With ν smaller than the Larmor frequency, there is a random walk of guiding centers. If ν becomes smaller than the "bounce" frequency of reflection between mirrors, collisions induce a random walk from magnetic line to line. Still smaller ν, comparable to the drift frequency around the machine, yields a random walk of drift surfaces (qualitatively similar to the noncollisional random walk induced by changes in the trapped state, figures 10, 11). The first two cases are considered to be classical because they can be treated macroscopically, with a plasma resistivity. Diffusion among magnetic lines or drift surfaces is frequently termed "anomalous," or nonclassical although



TRAPPED STATES. **Particles in different energy states can be trapped in different mirrors in a complex mirror machine.** **—FIG. 10**



CHANGE OF TRAPPED STATE for a drifting particle. The choice of second state is essentially random. —FIG. 11

it is a consequence of purely classical orbits and Coulomb scattering.

But even the completely macroscopic resistive model of diffusion (small mean free path) can exhibit anomalies. Macroscopic plasma diffusion across a field is a complex interaction between two more basic types of diffusion. In a mixture of neutral gases we have a diffusion coefficient, D_0. Diffusion of a magnetic field through a conducting solid is described by a coefficient $D_M = 1/\mu_0\sigma$ (σ is the conductivity). The elementary kinetic-theory formula, $\sigma \approx e^2D_0/kT$, suggests that D_0 and D_M are essentially reciprocals! For a plasma in a simple magnetic field, the competition between these two effects at different rates gives (at low β) the classical diffusion coefficient $D_c \approx \beta D_M$; (the competition is fierce, and the combined diffusion equation that results is unconventional, with a nonlinear decay as $1/t$ rather than exponential as in simple diffusion).

In more complex geometries we expect coupling between these two basic rates. This coupling is only partially accounted for in the standard theory by the Pfirsch-Schluter factor. Because this factor diverges under the same circumstances that lead to difficulties with microequilibrium, we can not consider the macroscopic "self-consistent" theory to be definitive.

Some rough estimates have been made of a slightly self-consistent model with realistic guiding-center orbits. A model that combines such drift surface (banana) diffusion with realistic self-consistency (as in the macroscopic theory) appears far in the future.

Trends

Originating in discharge physics and astrophysics, spurred mainly by the controlled thermonuclear program in the past 15 years, plasma physics is now branching into many new directions, meanwhile developing into a recognized academic discipline.

We can grasp the significance of the field of plasma physics only in the context of its enormous phenomenological variety and—especially for hot plasmas—experimental difficulty. Growth, measured both in achievable experimental plasma parameters and in depth of understanding, is either fabulous, when compared to the state of the art a few years ago, or negligible, when compared to what visibly remains to be done. Observable trends toward simplicity in plasma experiments, toward simpler theoretical models and at the same time away from simplistic theoretical explanations are both in the right direction.

Both experimental and theoretical techniques are becoming more specific and more precise, more quantitative and more professional. A portent is the recent start, internationally, of serious engineering studies of hypothetical operating thermonuclear reactors. The significance is not that we can see a target date, but that we can imagine being caught short. The complexity of plasma phenomena implies a concomitant large variety of options; with some ingenuity, success is not in doubt. But the time scale is not easily estimated, because the scientific and technological problems that must be solved are not yet fully formulated.

Plasma physics is, in a sense, the union of three classical fields—fluid dynamics, kinetic theory, electromagnetic theory. Although classical, these are fields that have all seen profound advances in the past 20 years. In the specialities of nonlinear waves and instabilities, there has been a gratifying infusion into plasma physics of ideas from electrical engineering. We should hope and expect that in a subject as vast as plasma physics, similar profit will ensue from interaction with these and other scientific disciplines with their divergent backgrounds, techniques and viewpoints.

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I am grateful for the unstinting assistance and criticism of Albert A. Blank in preparing this manuscript, and also to Harold Weitzner, Herman Postma, Robert Hirsch, Nathan Marcuvitz, Norman Lazar and Raul Stern for their valuable suggestions.

This work was supported by the US Atomic Energy Commission under contract AT-(30-1) 1480.

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