PHYS:7729 Homework #7

Due at the beginning of class, Thursday, April 6, 2023.

1. (16 pts) Consider the problem of the counter-streaming Cauchy distribution,

$$F_0(v_z) = \frac{C}{2\pi}\left[\frac{1}{C^2 + (v_z - V)^2} + \frac{1}{C^2 + (v_z + V)^2}\right].$$

Sketch the Nyquist diagrams for mapping the $\gamma = 0$ contour in the complex p-plane onto the complex D-plane for the following cases:

(a) $C > V$.

(b) $C = V$.

(c) $C < V$.

2. (24 pts) Ion Acoustic Waves and the Current-Driven Ion Acoustic Instability: A plasma consists of a Maxwellian distribution of ions at temperature T_i and electrons at temperature T_e. The average velocity of the ions is zero and the electrons are drifting with an average velocity U_e.

(a) Make an appropriate modification of the derivation for the linear dispersion relation for electrostatic waves (which includes ion acoustic waves) involving the plasma dispersion function to take into account the drifting electrons. Your answer should be presented in the usual form $D(\mathbf{k}, p) = 0$, and be sure to specify your definition of the arguments of the plasma dispersion function $Z(\xi_s)$.

(b) Using the low-phase velocity approximation for the electrons and the high-phase velocity approximation for the ions, obtain an approximate expression for the real frequency of the ion acoustic mode. You may assume that $k\lambda_{De} \ll 1$.

(c) Show that the growth rate is approximately

$$\frac{\gamma}{\omega} = -\sqrt{\frac{\pi}{8}}\left[\sqrt{\frac{m_e}{m_i}}\left(1 - \frac{U_e}{C_i}\right) + \left(\frac{T_e}{T_i}\right)^{3/2}\exp\left(-\frac{T_e}{2T_i}\right)\right]$$

where the ion acoustic speed is given by

$$C_i = \sqrt{\frac{T_e}{m_i}}$$

(d) Show that the instability condition for the current-driven ion acoustic mode is

$$|U_e| > C_i\left[1 + \sqrt{\frac{m_i}{m_e}}\left(\frac{T_e}{T_i}\right)^{3/2}\exp\left(-\frac{T_e}{2T_i}\right)\right]$$