

# 29:278 Homework #6

Suggested Reading: Read KR95 Chapter 9 (p.227–284)

Due at the beginning of class, Thursday, March 13, 2014.

## 1. Magnetic Field Due to a Ring of Current

Consider a circular current loop of radius  $R_0$  centered about the origin of a cylindrical coordinate system with current  $I_0$  in the  $-\hat{\phi}$  direction. The current density for this current loop may be expressed as

$$\mathbf{j}(\mathbf{r}) = -I_0\delta(r - R_0)\delta(z)\hat{\phi}.$$

- Use the Biot-Savart Law to compute the magnetic field at the origin  $\mathbf{r} = 0$  due to this current loop. Do not forget to include the vector direction in the answer for  $\mathbf{B}(0)$ .
- The dipole moment for this current loop is  $\boldsymbol{\mu} = -I_0A\hat{\mathbf{z}}$ , where the loop encircles the area  $A$ . Use the Biot-Savart Law to compute the far-field limit of the magnetic field due to a dipole at a position in the plane of the loop far away from the loop,  $r \gg R_0$ ,

$$\mathbf{B}(r, \phi, 0) = \frac{-\mu_0}{4\pi} \frac{\boldsymbol{\mu}}{r^3}$$

## 2. Radius of the Plasmasphere

For cold plasma that has evaporated from the ionosphere, the convection of the plasma in the inner magnetosphere is governed by a balance of the convection electrostatic potential and the corotation electrostatic potential,

$$\Phi_{tot} = -E_c r \sin \phi - \frac{\Omega_E B_E R_E^3}{r}.$$

The stagnation point in the convective flow occurs where the electric field due to this potential is zero,  $\mathbf{E} = -\nabla\Phi_{tot} = 0$ .

- Compute the formula for radius of the stagnation point in terms of  $\Omega_E$ ,  $B_E$ ,  $R_E$ , and the convective electric field  $E_c$ . In which direction is the stagnation point from the Earth, noon, dusk, midnight, or dawn?
- For a convective electric field of magnitude  $E_c = 2$  mV/m, compute the  $L$  value of the stagnation point, which gives an estimate of the radius of the plasmopause, the outer boundary of the plasmasphere.

## 3. Partial Ring Current

For hot plasma with energies  $W \gtrsim 10$  keV, the convection of the plasma is governed by a balance of the convection electrostatic potential and the effective potential associated with the azimuthal drift due to the  $\nabla B$  and curvature drifts,

$$\Phi_{tot} = -E_c r \sin \phi + \frac{\mu B_E R_E^3}{qr^3},$$

where the magnetic moment of the charged particle is  $\mu = mv_{\perp}^2/2B$  and, for simplicity, we have assumed equatorial trapped particles with a pitch angle of  $\alpha = 90^\circ$ .

- Compute the formula for radius of the stagnation point in terms of the perpendicular kinetic energy  $W_{\perp} = mv_{\perp}^2/2$  and the convective electric field  $E_c$ . In which direction is the stagnation point from the Earth, noon, dusk, midnight, or dawn?  
HINT: When taking the gradient of the potential, the magnetic moment—which is the first adiabatic invariant—is constant.
- For particles with energy  $W_{\perp} = 20$  keV and a convective electric field of magnitude  $E_c = 1$  mV/m, compute the  $L$  value of the stagnation point. This gives an estimation of the radius at which the ring current transitions to the partial ring current for particles of this energy.