29:293 Homework #4

Reading: Required: Read GB Chapter 10 (p.391–411)
Optional: Read BS Chapter 10, Sec 10.1–10.2 (p.376–388)

Due at the beginning of class, Tuesday, March 3, 2015.

1. In his 1959 paper, Buneman analyzed the maximum growth rate of a cold electron beam streaming at velocity V through an equal number density of ions initially at rest. The dispersion relation for this system is given by

$$D(\mathbf{k}, \omega) = 1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pe}^2}{(kV - \omega)^2} = 0.$$

   (a) If we assume that $\omega_{pi}^2 \ll \omega^2$, show that the dispersion relation can be written

$$kV = \omega + \omega_{pe} + \frac{1}{2}\frac{\omega_{pe}\omega_{pi}^2}{\omega^2}.$$

   (b) Show that the above dispersion relation has a solution for real k of the form

$$\omega = (\omega_{pe}\omega_{pi}^2 \cos\theta)^{1/3} e^{i\theta}$$

   where θ is any arbitrary angle.

   (c) Show that the imaginary part of ω reaches a maximum when $\theta = \pi/3$.

   (d) Show that, when $\theta = \pi/3$,

$$\gamma = \omega_{pe}\left(\frac{m_e}{2m_i}\right)^{1/3}\frac{\sqrt{3}}{2}$$

   and

$$\omega_r = \omega_{pe}\left(\frac{m_e}{2m_i}\right)^{1/3}\frac{1}{2}.$$

   (e) Is assumption (a) satisfied?

2. Consider the problem of the counter-streaming Cauchy distribution,

$$F_0(v_z) = \frac{C}{2\pi}\left[\frac{1}{C^2 + (v_z - V)^2} + \frac{1}{C^2 + (v_z + V)^2}\right].$$

   (a) Sketch the distribution for the choice $C = V$.

   (b) Apply Gardner's Theorem to show that stability is guaranteed for $C > \sqrt{3}V$.

   (c) Apply the Penrose Criterion to show that this equilibrium is stable for $C > V$ and unstable for $C < V$.