29:194 Homework #5

Due at the beginning of class, Friday, October 4, 2024.

1. Show that for an electric field of the form

$$\mathbf{E}(\mathbf{x},\tau,t) = \mathbf{E}_0(\mathbf{x},\tau)\cos(\omega t - \mathbf{k}\cdot\mathbf{x})$$

the magnetic field is given by

$$\mathbf{B}(\mathbf{x},\tau,t) = -\frac{1}{\omega}\left\{[\nabla\times\mathbf{E}_0(\mathbf{x},\tau)]\sin(\omega t - \mathbf{k}\cdot\mathbf{x}) - [\mathbf{k}\times\mathbf{E}_0(\mathbf{x},\tau)]\cos(\omega t - \mathbf{k}\cdot\mathbf{x})\right\}$$

2. A mirror machine has a mirror ratio $R_m = 2$. A group of electrons with an isotropic velocity distribution (Maxwellian) is released at the center of the machine. In the absence of collisions, what fraction of these electrons is confined?

3. A singly ionized, 20 eV Argon plasma is confined in a magnetic cusp field such that throughout the volume of the plasma, except for the edges, the magnetic field is zero. The plasma diameter is 1.0 m, and its density is $n = 10^{11}$ m^{-3}. A small, 0.5 cm diameter spherical probe is placed at the center of the plasma and set to a potential of 100 V above the wall potential. At what distance from the probe surface will the measured potential be 1 V?

4. Consider the single particle motion of an electron of charge $q_e = -e$ and mass m_e and an proton of charge $q_e = e$ and mass $m_i = m_p$ that are initially at rest at $\mathbf{x} = (0,0,0)$ in a magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$. An electric field is then turned on at $t = 0$ and increased linearly until time $t_1 = \frac{20\pi m_i}{eB_0}$, at which point the electric field is held constant,

$$\mathbf{E}(t) = \begin{cases} 0 & t < 0 \\ E_0(t/t_1)\hat{\mathbf{y}} & 0 \le t \le t_1 \\ E_0\hat{\mathbf{y}} & t > t_1 \end{cases}$$

If the plasma has a density n_0 (where there are an equal number of electrons and protons), find the total current density as a function of time $\mathbf{j}(t)$ due to the drifts of the different particle species (neglect the current due to the fast Larmor oscillation).

5. NUMERICAL: Polarization Drift

Use the same Matlab m-files `lorentz.m`, `magnetic.m`, `electric.m`, `euler1.m`, `leapfrog2.m`, and `spm.m` as used in HW#4 and the adaptive RK45 method with a specified tolerance RelTol= 1.10×10^{-5}. Specify $\mathbf{B} = (0,0,1)$, $q_i/m_i = 1$, $q_e = -q_i$ and an artificial mass ratio $m_i/m_e = 10$. Specify an electric field that increases with time $\mathbf{E}(t) = E_0 t/t_f\hat{\mathbf{y}}$ with $E_0 = 0.5$ and t_f equal to 10 ion cyclotron periods. NOTE: The `hold on` command can be used to plot a second trace on the same plot; `hold off` turns this off.

   (a) Plot the trajectories over $t = [0, t_f]$, on the same plot, of both the ion and electron each with an initial position $\mathbf{x}_0 = (0,0,0)$ and initial velocity $\mathbf{v}_0 = (-1,0,0)$.

   (b) Why do we not use a realistic mass ratio (for protons) of $m_i/m_e = 1836$ to do this calculation? HINT: Try using $m_i/m_e = 40$.

6. Laser Trapping: A charged particle can be trapped by a spatially varying intense laser field. (Note that there is no background magnetic field in this problem.) Using intereference of several lasers, the electric field near a charged particle is given by

$$\mathbf{E}(\mathbf{x}, t) = E_0[1 + (x/x_0)^2] \sin(\omega t - k_y y)\hat{\mathbf{x}}.$$

Calculate the velocity of the oscillation center U as a function of position x for a particle initially at rest at $t = 0$ at position $\mathbf{x} = (x_0, 0, 0)$. You may assume that the particle velocity v and laser frequency ω satisfy $v \ll \omega/k_y$ and $v/x_0 \ll \omega$.

7. NUMERICAL: Ponderomotive Force

Use the adaptive RK45 method with a specified tolerance RelTol$= 1.0 \times 10^{-5}$. Plot the x position of the particle vs. time t over a time $t = [0, 50]$ for the problem above using $E_0 = 2$, $x_0 = 1$, $\omega = 10$, $k_y = 0.01$, and $q/m = 1$.