PHYS:4731 Plasma Physics I: Lecture #1 - Introduction to Plasmas
What? How? Why?

I. What is a plasma?

A. Definition: There is no good, single definition!

 1. Fourth State of Matter: Solid $\Rightarrow$ Liquid $\Rightarrow$ Gas $\Rightarrow$ Plasma
   Ice   Water   Steam

 2. A gas that is significantly ionized.

 3. Ionized gas that display collective behavior

B. Variety:

 1. Plasma physics concepts are used to describe an enormous
   variety of topics — not just "traditional" plasmas
   Ex: Electrons in a semi-conductor in solid state physics

C. Common:

 1. Most of the matter in the universe exists in the plasma state,
   except "dark matter"

 2. Some say 95-99% of baryonic matter is plasma

D. Applications:

 1. Unfortunately, many plasma courses tend to be a "grab bag" of
   seemingly unrelated topics.

 2. GOAL: I hope to give you a sense of the overall
   framework of plasma physics in this course
   and how different approaches are connected.

 3. I will not always be formal—formality is often a barrier to understanding

 4. There will be a heavy emphasis on skills—both mathematical & numerical

Lect #1 (Continued)

II. How do we describe plasmas?

A. Basic Parameters:

1. A plasma consists of one, or more, ion species and electrons.

2. Intensive variables:
   a. Density n_s
   b. Temperature T_s
   c. Magnetic Field B_0

3. Physical properties
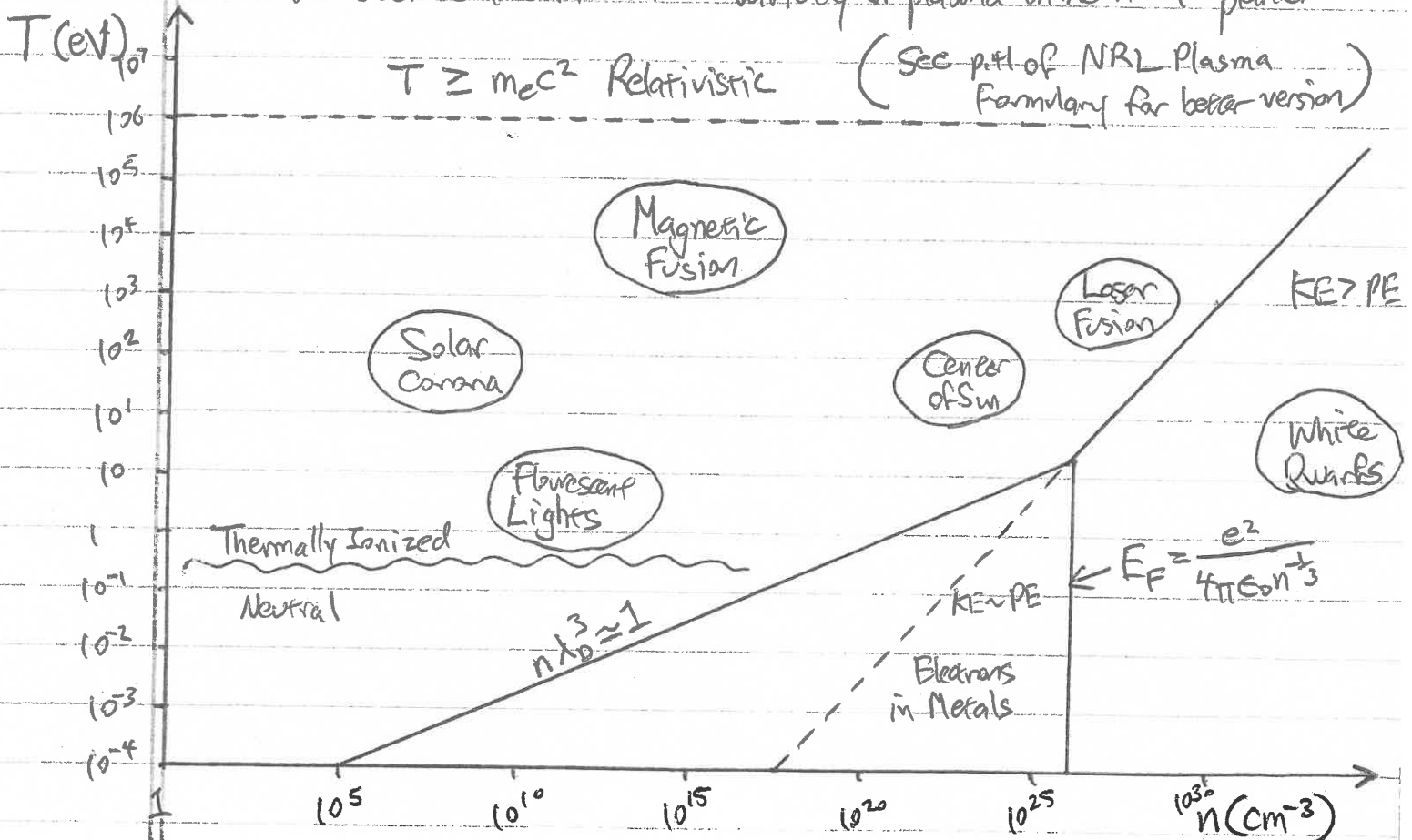   a. mass m_s
   b. charge q_s

<u>NOTE:</u> We often use $m_e/m_i \ll 1$ to simplify a problem.

4. Most plasmas have magnetic fields (although possibly no <u>mean</u> magnetic field) due to currents from charged particles moving within the plasma

   (Ampere's Law: $\nabla \times \underset{\sim}{B} = \mu_0 \underset{\sim}{j} + \epsilon_0 \mu_0 \frac{\partial E}{\partial t}$)

5. We can describe the incredible variety of plasma on the $n-T$ plane.

$T \gtrsim m_e c^2$ Relativistic

(See p.H of NRL Plasma Formulary for better version)

Magnetic Fusion

Solar Corona

Laser Fusion

Center of Sun

$KE > PE$

Fluorescent Lights

White Dwarfs

Thermally Ionized

Neutral

$n\lambda_D^3 \simeq 1$

$KE \sim PE$

$E_F = \frac{e^2}{4\pi\epsilon_0 n^{\frac{1}{3}}}$

Electrons in Metals

Lect #1 (Continued)

II. (Continued)

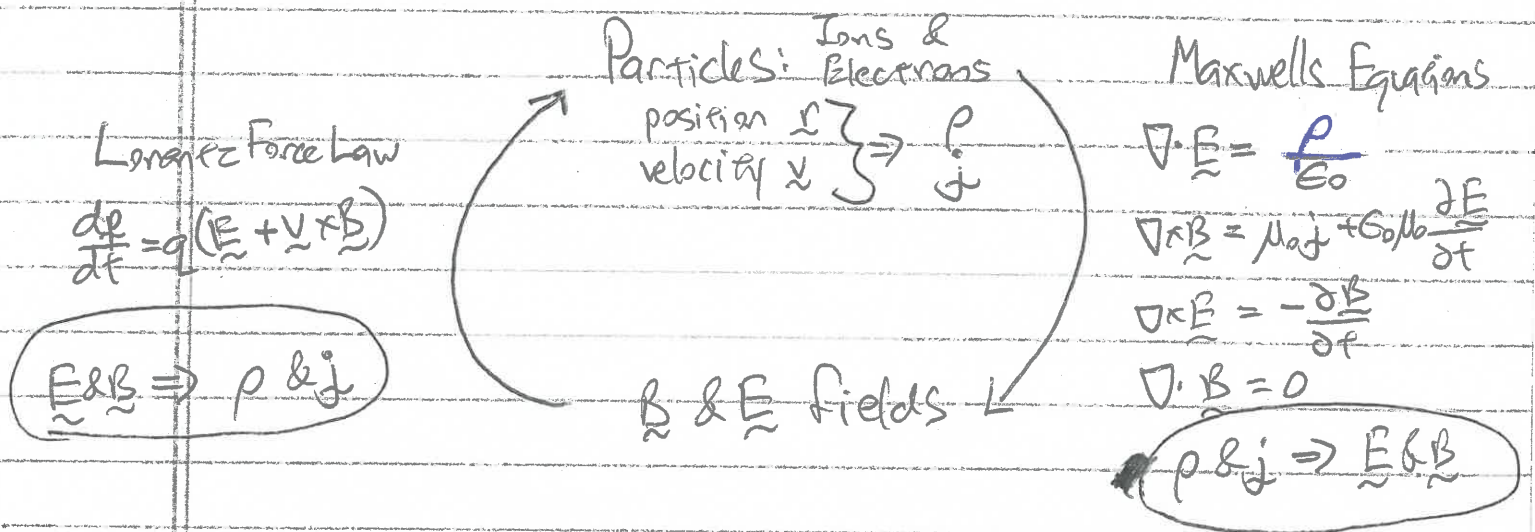   B. Units:

     1. Two major systems: a. SI Units   (mks)

                          b. Gaussian Units (cgs)

     2. We'll be using SI to agree with text (Chen)

III. How do we study the physics of plasmas?

   A. The Basic Picture:

     1. Plasma physics is really nothing more than the study of fluctuating magnetic and electric fields and the motion of charged particles in those fields.

Particles: Ions & Electrons

position $\underset{\sim}{r}$ ρ,

velocity $\underset{\sim}{v}$ $\Rightarrow$ $\underset{\sim}{j}$

Maxwell's Equations

$\nabla \cdot \underset{\sim}{E} = \dfrac{\rho}{\epsilon_0}$

$\nabla \times \underset{\sim}{B} = \mu_0 \underset{\sim}{j} + \epsilon_0 \mu_0 \dfrac{\partial E}{\partial t}$

$\nabla \times \underset{\sim}{E} = -\dfrac{\partial \underset{\sim}{B}}{\partial t}$

$\nabla \cdot \underset{\sim}{B} = 0$

Lorentz Force Law

$\dfrac{d\underset{\sim}{p}}{dt} = q(\underset{\sim}{E} + \underset{\sim}{v} \times \underset{\sim}{B})$

$\boxed{\underset{\sim}{E} \& \underset{\sim}{B} \Rightarrow \rho \& \underset{\sim}{j}}$

$\underset{\sim}{B} \& \underset{\sim}{E}$ fields

$\boxed{\rho \& \underset{\sim}{j} \Rightarrow \underset{\sim}{E} \& \underset{\sim}{B}}$

     2. This coupling is precisely what presents the challenge of plasma physics!

   B. Inconsistent Models: | Single Particle Motion |

     1. Assume we know $\underset{\sim}{E} \& \underset{\sim}{B}$ fields

     2. Determine motion of a particle in these fields

     3. Good approach to develop intuition

     4. Does not account for the collective of other particles' motion (and the resulting $\underset{\sim}{E} \& \underset{\sim}{B}$ fields) on that single particle.
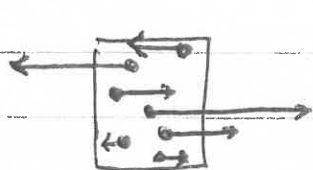
II. (Continued)

C. Consistent Models:

1. [Kinetic Theory:]

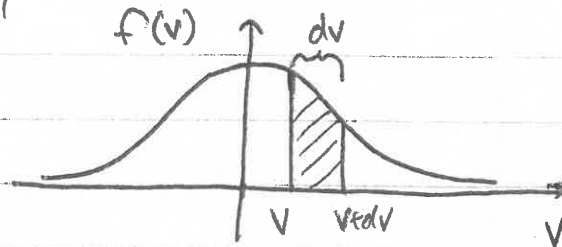a. Statistical theory averages over the motion of many particles
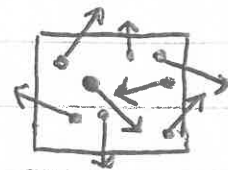
$\Rightarrow$ Distribution function $f(\underline{r}, \underline{v}, t)$

Ex: For a 1-D system, at a single point in space, particle velocities may be distributed as follows:

Here $f(v)$ is number of particles with velocity between v & $v+dv$. We'll discuss this more later.

b. Assuming a Maxwellian distribution (thermal equilibrium) allows some closed form results to be derived.

c. Any more detailed treatment quickly becomes cumbersome, not possible in closed form.

2. [Fluid Theory:]

a. Assumes all particles at a given point move similarly

b. One may take moments of the distribution function:

i) Density $\quad n = \int d^3\underline{v}\, f(\underline{v})$

ii) Fluid velocity $\quad \underline{U} = \dfrac{\int d^3\underline{v}\; \underline{v}\; f(\underline{v})}{\int d^3\underline{v}\; f(\underline{v})}$

etc. ...

c. One must assume some closure to obtain a closed set of fluid equations.

III. C. 2.

    d. [Two Fluid Theory] allows for different behavior between ions and electrons

    e. If you assume that ions and electrons move together, you get a single fluid theory, [Magnetohydrodynamics]   or MHD.

      i) This is the most simple, consistent description of a plasma.
      ii) Probably the most widely used system, especially by "non-plasma physicists"

    f. Even fluid theory is difficult in the complicated, nonlinear situations of most practical interest.

D. Computational Studies:
1. Numerical Solution: Solving a system of equations using the computer.
   Exs. a. Boundary value or eigenvalue problems.
       b. May use ODE integrators, shooting methods, etc.

2. Simulation: (Initial Value codes) Big business/big codes
   a. Particle-in-Cell (particle method)
   b. Fokker-Planck (Continuum method)
   c. Gyrokinetic
   d. Two Fluid / MHD

IV. Why do we study plasma physics?

A. Plasma physics and the methods used in plasma physics contribute significantly to a wide variety of fields:
  1. Stellar and Galactic Astrophysics
  2. Earth's magnetosphere and the heliosphere
  3. Fusion Energy research
    a. Magnetic Confinement
    b. Inertial Confinement (laser fusion)
  4. Advanced Accelerators
  5. Laser-plasma interactions
  6. Industrial processing (especially computer chips)
  7. Solid state, electrons in a metal

V. The NRL Plasma Formulary
  Meet your new best friend!
  A. Most Frequently Used Sections:
  1. Vector Identities   p. 4-5
  2. Differential Operators in Curvilinear Coordinates p. 6-9
  3. Physical Constants (SI) p. 14-15
  4. Fundamental Plasma Parameters p. 28-29

NOTE: I will refer to the Plasma Formulary in the lecture notes as NRL.

  Ex: Use $\underline{A} \times (\underline{B} \times \underline{C}) = \underline{B}(\underline{A} \cdot \underline{C}) - \underline{C}(\underline{A} \cdot \underline{B})$ [NRL, p.4, (2)]

II. Vector Notation Review & Vector Calculus

A. Why? 1. Vector notation simplifies the mathematical notation

      2. You will get lots of practice with vector algebra & calculus.

B. Notation:

1. Under-Tilde denotes vector quantity $\underset{\sim}{B}$

a. In cartesian coordinates $\hat{x}, \hat{y}, \& \hat{z}$,

$$\underset{\sim}{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}$$

2. Unit vectors: $\hat{\underset{\sim}{b}} \equiv \dfrac{\underset{\sim}{B}}{|\underset{\sim}{B}|}$

3. Magnitude: $|\underset{\sim}{B}| = \sqrt{\underset{\sim}{B} \cdot \underset{\sim}{B}} = \sqrt{B_x^2 + B_y^2 + B_z^2}$

4. Tensor: Denoted by double under-tilde $\underset{\approx}{\epsilon} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{pmatrix}$

Vector Calculus:

5. a. $\dfrac{\partial f}{\partial \underset{\sim}{V}} = \dfrac{\partial f}{\partial V_x} \hat{x} + \dfrac{\partial f}{\partial V_y} \hat{y} + \dfrac{\partial f}{\partial V_z} \hat{z}$

b. $\nabla f = \dfrac{\partial f}{\partial \hat{\underset{\sim}{x}}} = \dfrac{\partial f}{\partial x} \hat{x} + \dfrac{\partial f}{\partial y} \hat{y} + \dfrac{\partial f}{\partial z} \hat{z}$

c. Thus $\nabla = \hat{x} \dfrac{\partial}{\partial x} + \hat{y} \dfrac{\partial}{\partial y} + \hat{z} \dfrac{\partial}{\partial z}$

C. Vector Algebra and Calculus Review:

1. Dot Product: $\underset{\sim}{A} \cdot \underset{\sim}{B} = A_x B_x + A_y B_y + A_z B_z$

2. Cross Product: $\underset{\sim}{A} \times \underset{\sim}{B} = (A_y B_z - A_z B_y) \hat{x}$
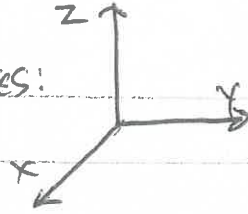
$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \qquad + (A_z B_x - A_x B_z) \hat{y}$$

$$+ (A_x B_y - A_y B_x) \hat{z}$$

Lecture #1 (Continued)

II. C. (Continued)

3. Right-handed coordinates:

$$\hat{x} \times \hat{y} = \hat{z}$$
$$\hat{y} \times \hat{z} = \hat{x}$$
$$\hat{z} \times \hat{x} = \hat{y}$$

4. Integration: $\int d^3 v \, f(\underline{v}) = \int dv_x \int dv_y \int dv_z \, f(\underline{v})$

5. $\underline{v} \cdot \nabla = \left(v_x \hat{x} + v_y \hat{y} + v_z \hat{z} \right) \cdot \left(\hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z} \right) = v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z}$

6. $\underline{v} \times \nabla = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix} = \left(v_y \frac{\partial}{\partial z} - v_z \frac{\partial}{\partial y} \right) \hat{x} + \left(v_z \frac{\partial}{\partial x} - v_x \frac{\partial}{\partial z} \right) \hat{y} + \left(v_x \frac{\partial}{\partial y} - v_y \frac{\partial}{\partial x} \right) \hat{z}$

D. Examples:

1. MHD continuity: $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{v}) = 0$

   a. By NRL p.4 (7), $\nabla \cdot (\rho \underline{v}) = \rho \nabla \cdot \underline{v} + (\underline{v} \cdot \nabla) \rho$, so

$$\boxed{ \frac{\partial \rho}{\partial t} + (\underline{v} \cdot \nabla) \rho = -\rho \nabla \cdot \underline{v} }$$

2. MHD momentum: $\rho \frac{\partial \underline{v}}{\partial t} + (\underline{v} \cdot \nabla) \underline{v} = -\nabla p + \underline{j} \times \underline{B}$

   a. We also have $\underline{j} = \frac{1}{\mu_0} \nabla \times \underline{B}$ (Ampere's Law, displacement current dropped)

   b. Rewrite $\underline{j} \times \underline{B}$ term in terms of only $\underline{B}$:

$$\underline{j} \times \underline{B} = \frac{1}{\mu_0} (\nabla \times \underline{B}) \times \underline{B} = -\frac{1}{\mu_0} \underline{B} \times (\nabla \times \underline{B})$$

   From NRL p.4 (12) with $\underline{B} = \underline{A} \Rightarrow \nabla (\underline{B} \cdot \underline{B}) = 2 \underline{B} \times (\nabla \times \underline{B}) + 2 (\underline{B} \cdot \nabla) \underline{B}$

   So $\underline{B} \times (\nabla \times \underline{B}) = \frac{1}{2} \nabla (\underline{B} \cdot \underline{B}) - (\underline{B} \cdot \nabla) \underline{B}$

Thus $\boxed{ \underline{j} \times \underline{B} = -\frac{\nabla B^2}{2\mu_0} + \frac{(\underline{B} \cdot \nabla) \underline{B}}{\mu_0} }$