PHYS:5905 Homework #2

Please submit your solutions as a single PDF file with answers to the questions asked.
Please complete required problems before lecture on Thursday, January 24, 2019.

1. (Required) **Larmor Motion in constant, uniform magnetic field with zero electric field**

   (a) Use Matlab (or any other language you prefer) to code a single particle motion solver for motion in a constant magnetic field using forward Euler finite differencing in time.

   (b) Equations:

$$\frac{d\mathbf{x}}{dt} = \mathbf{v} \tag{1}$$

$$\frac{d\mathbf{v}}{dt} = \frac{q}{m}(\mathbf{v} \times \mathbf{B}) \tag{2}$$

   (c) Parameters: Simulate the problem over the time interval $0 \leq t \leq T = 20\pi$, with charge $q = 1$, mass $m = 1$, and constant uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$ with $B_0 = 1$. Take the initial position to be $\mathbf{x}_0 = (0, 1, 0)$ and the initial velocity to be $\mathbf{v}_0 = (1, 0, 0)$.
   NOTE: We will discuss how to deal with dimensional quantities in Lecture # 3, but for now let us just take these numerical values as dimensionless.

   (d) Take a minimum of 1000 timesteps in the simulation time T.

   (e) (Return) Create two output plots of $\mathbf{x}(t)$:
      i. Plot the Trajectory of the particle in the (x, y) plane.
      ii. Plot the Position x as a function of Time t.

   Note that, for both of these plots, you should plot the numerical solution along with the analytical solution.

   (f) (Return) Compute the error in the position at $t = 20\pi$ from the analytical solution as a function of the number of timesteps taken N. Plot the error over the a minimum range $1000 \leq N \leq 1000000$ using an appropriate choice to visualize the results. What is the slope of the resulting error plot?

2. (Required) **$\mathbf{E} \times \mathbf{B}$ drift in a constant, uniform magnetic and perpendicular electric field**

   (a) Modify the code for the first problem to allow also for a constant, uniform perpendicular electric field.

   (b) Equations:

$$\frac{d\mathbf{x}}{dt} = \mathbf{v} \tag{3}$$

$$\frac{d\mathbf{v}}{dt} = \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{4}$$

   (c) Take the same parameters as the Larmor Motion problem above, adding the non-zero electric field $\mathbf{E} = E_0\hat{\mathbf{y}}$ with $E_0 = 0.1$.

   (d) (Return) Create two output plots of $\mathbf{x}(t)$:
      i. Plot the Trajectory of the particle in the (x, y) plane.
      ii. Plot the Position x as a function of Time t.

   Note that, for both of these plots, you should plot the numerical solution along with the analytical solution.

   (e) (Return) Compute the error in the position at $t = 20\pi$ from the analytical solution as a function of the number of timesteps taken N. Plot the error over the a minimum range $1000 \leq N \leq 1000000$ using an appropriate choice to visualize the results. What is the slope of the resulting error plot? Interpret these results relative to the comparable result in the previous Larmor Motion problem.

   (f) (Optional) Can you think of a plot that will illustrate the difference between the error results in these two problems? Create the plot and explain your reasoning.