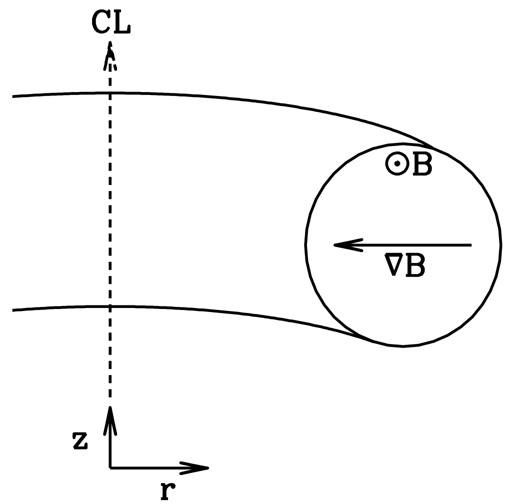
PHYS:7730 Homework #1

Suggested Reading:	Review GB17 Chapter 2 (p. 4–18)
	Review GB17 Chapter 3, Sec 3.1–3.8 (p. 22–64)
Optional Reading:	Review BS03 Chapter 1 (p. 1–11)
	Review B06 Chapter 1 (p. 1–25)
	Review BS03 Chapter 2, Sec 2.1–2.15 (p. 12–43)

Due at the beginning of class, Thursday, January 27, 2022.

1. Consider the confinement of ions and electrons in a purely toroidal magnetic field from a single-particle motion point of view. The toroidal geometry is shown in cross section in the figure below.



The lowest order motion is the Larmor motion about a toroidal magnetic field line which closes on itself. Consider the effect of drifts due to the topology of the magnetic field.

(a) In what direction do the ions drift due to ∇B? In what direction do the electrons drift due to ∇B?

(b) In what directions do the ions and electrons drift due to the curvature of the magnetic field?

(c) The drift of ions and electrons leads to a polarization charge ρ_q in the plasma. In what direction is the electric field produced by this polarization charge? What is the direction of the resulting $\mathbf{E} \times \mathbf{B}$ drift?

(d) Comment on the effect these drifts have on the confinement of ions and electrons in a purely toroidal magnetic field.

2. A cylindrical column of plasma rotates around its central axis (as though it were a rigid solid) at an angular velocity ω_0. A constant uniform magnetic field $\mathbf{B}$ is present parallel to the axis of rotation.

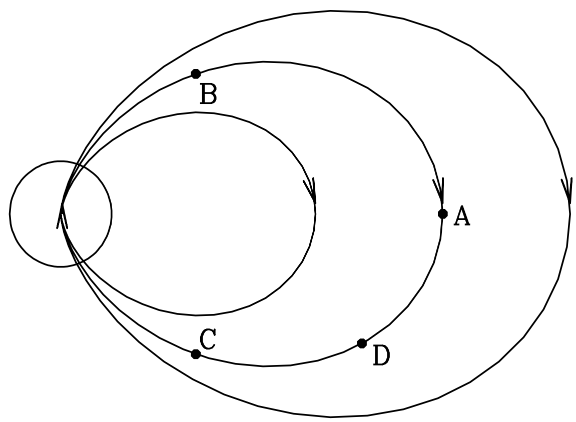
(a) Assuming that the rigid rotation can be described by $\mathbf{v}_E = (\omega_0 \hat{\mathbf{z}}) \times \mathbf{r}$, where $\mathbf{v}_E = \mathbf{E} \times \mathbf{B}/B^2$, compute the electric field $\mathbf{E}$ in the plasma column. Use cylindrical (r, ϕ, z) coordinates.

(b) Is there a polarization charge $\rho_q = \epsilon_0 \nabla \cdot \mathbf{E}$ associated with this electric field? If so, how does ρ_q depend on the distance from the central axis?

(c) Find the electrostatic potential Φ such that $\mathbf{E} = -\nabla \Phi$.

3. The Earth's magnetosphere can be roughly modeled as a dipole magnetic field. For such a magnetosphere, describe the three types of periodic motion associated with charged particles trapped in the magnetosphere, providing a sketch for each.

4. A charged particle is trapped within a planetary dipole field as shown below. At point A, the particle has $v_\perp = v_\parallel = v_0$. Its turning points are at points B and C. At point D, how do the velocities $v_\perp$ and $v_\parallel$ compare to v_0 ($<$, $>$, or $=$)?



5. Sketch the trajectory in the x-y plane for an ion initially at rest in a uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$ when an electric field $\mathbf{E} = E\hat{\mathbf{y}}$ is applied with the time variation as given in the plot below. You may assume that the timescale of the variation of the electric field is long compared to the timescale of the ion gyromotion.