PHYS:7730 Homework #4

Reading: Sridhar and Goldreich, ApJ **432**:612 (1994)
Toward a Theory of Interstellar Turbulence. I. Weak Alfvenic Turbulence

Due at the beginning of class, Thursday, February 17, 2022.

1. **Turbulence in a Coffee Mug**
The Reynolds number at a given scale L is defined by

$$\mathrm{Re} \equiv \frac{LU}{\nu},$$

where L is the scale length, U is the velocity of the fluid flow at that scale, and ν is the kinematic viscosity of the fluid. The Reynolds number of a given turbulent flow is typically computed with $L = L_0$ and $U = U_0$, where L_0 is the driving scale length and U_0 is the driving velocity at that driving scale.

   (a) For a fixed driving scale velocity U_0, how does the Reynolds number depend on the scale L. Express your answer in terms of L, L_0, U_0, and ν.

   (b) Compute the Reynolds number in a hot cup of coffee: Consider driving turbulence in a coffee mug with diameter $L_0 = 10$ cm, stirring with a velocity U_0 such that the spoon circles the coffee cup once in 2 s. The kinematic viscosity of water at 90°C is $\nu = 0.326 \times 10^{-6}$ m^2/s. Compute the Reynolds number at the driving scale.

   (c) Is the flow in this hot coffee mug expected to be turbulent or laminar?

   (d) Compute the viscous scale in a hot cup of coffee: The viscous scale l_ν is determined by the scale at which the Reynolds number falls to Re = 1. For the same case of driving as in (b) above, compute the viscous scale, expressing your answer in mm.

   (e) Starting from when you begin to stir your coffee, estimate the time it takes for the turbulent cascade to reach the viscous scale. HINT: You may assume the flow energy at scale l is transferred to scale $l/2$ in the eddy turnaround time at scale l.

2. **Weak MHD Turbulence in the Near-Earth Solar Wind**
Consider a weak MHD turbulent cascade driven in the solar wind, with isotropic flucations driven with normalized parallel and perpendicular wavenumbers $k_{\parallel 0}\rho_i = k_{\perp 0}\rho_i = 10^{-5}$, where the ion Larmor radius is $\rho_i = 3 \times 10^6$ cm. The background magnetic field has amplitude $B_0 = 10^{-4}$ G, and the amplitude of the driven Alfvénic fluctuations is $\delta B_{\perp 0} = 10^{-5}$ G.

   (a) Assuming a weak MHD turbulent cascade (as defined by Sridhar and Goldreich, 1994), at what normalized perpendicular wavenumber $k_\perp \rho_i$ does the turbulent cascade transition from weak turbulence to strong turbulence?

   (b) What is the anisotropy $k_\parallel/k_\perp$ at this transition to strong turbulence?

   (c) What is the ratio of the amplitude of the turbulent fluctuations at this transition scale to the background magnetic field?