PHYS:7730 Advanced Plasma Physics

Lecture #1: Introduction, Characteristic Scales in a Plasma

What is a plasma?

I. Overall Framework of Plasma Physics

A.

Particles: Ions & Electrons

position, $\underset{\sim}{x}_s$ }
velocity, $\underset{\sim}{v}_s$ } $\Rightarrow$ ρ_q, $\underset{\sim}{j}$

Maxwell's Equations:

$$\nabla \cdot \underset{\sim}{E} = \frac{\rho_q}{\epsilon_0}$$

$$\nabla \times \underset{\sim}{E} = -\frac{\partial \underset{\sim}{B}}{\partial t}$$

$$\nabla \times \underset{\sim}{B} = \mu_0 \underset{\sim}{j} + \mu_0 \epsilon_0 \frac{\partial \underset{\sim}{E}}{\partial t}$$

$$\nabla \cdot \underset{\sim}{B} = 0$$

Electromagnetic Fields:

$\underset{\sim}{B}$, $\underset{\sim}{E}$

Lorentz Force Law:

$$m_s \frac{d\underset{\sim}{v}_s}{dt} = q_s (\underset{\sim}{E} + \underset{\sim}{v}_s \times \underset{\sim}{B})$$

The coupling of the particle motion & electromagnetic fields presents the challenge of plasma physics!

B. Theoretical Description of Plasma Physics

1. Inconsistent Models:

   a. $\boxed{\text{Single Particle Motion}}$: - Determine particle motion in known $\underset{\sim}{E}$ & $\underset{\sim}{B}$.
      - Good for developing intuition

Lecture #1 (Continued) Howes ②

I.B. (Continued)

2. Consistent Models:

a. $\boxed{\text{Kinetic Theory}}$: - Statistical theory averages over motion of many particles

$\Rightarrow$ Distribution function $f_s(\underline{x}, \underline{v}, t)$

- Most complete common description of plasmas
- Extremely challenging to obtain analytical results
- We won't tackle much kinetic plasma physics in this course $\Rightarrow$ Take 029:194 & 029:293.

b. $\boxed{\text{Two-Fluid Theory}}$: - Evolves moments of the distribution function

Ions & electrons

- Density: $n_s = \int d^3\underline{v}\, f_s(\underline{x}, \underline{v}, t)$
- Fluid velocity, $\underline{U}_s = \dfrac{\int d^3\underline{v}\, \underline{v} f_s(\underline{x}, \underline{v}, t)}{n_s}$

- Must assume a closure (Equation of State) to obtain closed set of equations
- Allows for different behavior of ions and electrons

c. $\boxed{\text{Magnetohydrodynamics (MHD)}}$: - Single fluid theory is simplest consistent model.

- Probably the most widely used system in space physics & astrophysics.
- We will focus on MHD models in this course

C. References to Texts:

I will often give references to chaps, sections, or figures in our texts:

1. [KR95] Kivelson & Russell, Introduction to Space Physics, Cambridge Univ Press: Cambridge, 1995

2. [S92] Shu, The Physics of Astrophysics, Volume II: Gas Dynamics, University Science Books: Sausalito, CA, 1992.

II. Vector Notation Review & Vector Calculus

A. Why?

1. Vector notation simplifies the mathematical notation
2. You will get lots of practice with vector algebra & calculus.

B. Notation:

1. Under-tilde denotes vector quantity $\underset{\sim}{B}$

a. In cartesian coordinates $\underset{\sim}{\hat{x}}$, $\underset{\sim}{\hat{y}}$, & $\underset{\sim}{\hat{z}}$,

$$\underset{\sim}{B} = B_x \underset{\sim}{\hat{x}} + B_y \underset{\sim}{\hat{y}} + B_z \underset{\sim}{\hat{z}}$$

2. Unit vectors: $\underset{\sim}{\hat{b}} \equiv \frac{\underset{\sim}{B}}{|\underset{\sim}{B}|}$

3. Magnitude: $|\underset{\sim}{B}| = \sqrt{\underset{\sim}{B} \cdot \underset{\sim}{B}} = \sqrt{B_x^2 + B_y^2 + B_z^2}$

4. Tensor: Denoted by double under-tilde $\underset{\approx}{\epsilon} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{pmatrix}$

Vector Calculus:

5.

a. $$\frac{\partial f}{\partial \underset{\sim}{v}} = \frac{\partial f}{\partial v_x} \underset{\sim}{\hat{x}} + \frac{\partial f}{\partial v_y} \underset{\sim}{\hat{y}} + \frac{\partial f}{\partial v_z} \underset{\sim}{\hat{z}}$$

b. $$\nabla f = \frac{\partial f}{\partial \underset{\sim}{x}} = \frac{\partial f}{\partial x} \underset{\sim}{\hat{x}} + \frac{\partial f}{\partial y} \underset{\sim}{\hat{y}} + \frac{\partial f}{\partial z} \underset{\sim}{\hat{z}}$$

c. Thus $$\nabla = \underset{\sim}{\hat{x}} \frac{\partial}{\partial x} + \underset{\sim}{\hat{y}} \frac{\partial}{\partial y} + \underset{\sim}{\hat{z}} \frac{\partial}{\partial z}$$

C. Vector Algebra and Calculus Review:

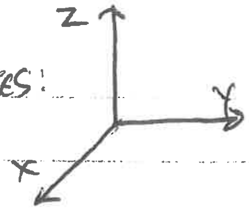
1. Dot Product: $\underset{\sim}{A} \cdot \underset{\sim}{B} = A_x B_x + A_y B_y + A_z B_z$

2. Cross Product:

$$\begin{vmatrix} \underset{\sim}{\hat{x}} & \underset{\sim}{\hat{y}} & \underset{\sim}{\hat{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \qquad \underset{\sim}{A} \times \underset{\sim}{B} = (A_y B_z - A_z B_y)\underset{\sim}{\hat{x}} + (A_x B_z - A_z B_x)\underset{\sim}{\hat{y}} + (A_x B_y - A_y B_x)\underset{\sim}{\hat{z}}$$

II. C. (Continued)

3. Right-handed coordinates:



$$\hat{\mathbf{x}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}}$$
$$\hat{\mathbf{y}} \times \hat{\mathbf{z}} = \hat{\mathbf{x}}$$
$$\hat{\mathbf{z}} \times \hat{\mathbf{x}} = \hat{\mathbf{y}}$$

4. Integration: $\int d^3\mathbf{v}\, f(\mathbf{v}) \equiv \int dv_x \int dv_y \int dv_z\, f(\mathbf{v})$

5. $\mathbf{v}\cdot\nabla = (v_x \hat{\mathbf{x}} + v_y \hat{\mathbf{y}} + v_z \hat{\mathbf{z}})\cdot\left(\hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}\right) = v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z}$

6. $\mathbf{v}\times\nabla = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ v_x & v_y & v_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix} = \left(v_y \frac{\partial}{\partial z} - v_z \frac{\partial}{\partial y}\right)\hat{\mathbf{x}} + \left(v_z \frac{\partial}{\partial x} - v_x \frac{\partial}{\partial z}\right)\hat{\mathbf{y}} + \left(v_x \frac{\partial}{\partial y} - v_y \frac{\partial}{\partial x}\right)\hat{\mathbf{z}}$

D. Examples:

1. MHD continuity: $\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho \mathbf{v}) = 0$

a. By NRL p. 4 (7), $\nabla\cdot(\rho\mathbf{v}) = \rho\nabla\cdot\mathbf{v} + (\mathbf{v}\cdot\nabla)\rho$, so

$$\boxed{\frac{\partial \rho}{\partial t} + (\mathbf{v}\cdot\nabla)\rho = -\rho\nabla\cdot\mathbf{v}}$$

2. MHD momentum: $\rho\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla p + \mathbf{j}\times\mathbf{B}$

a. We also have $\mathbf{j} = \frac{1}{\mu_0}\nabla\times\mathbf{B}$ (Ampere's Law, displacement current dropped)

b. Rewrite $\mathbf{j}\times\mathbf{B}$ term in terms of only $\mathbf{B}$:

$$\mathbf{j}\times\mathbf{B} = \frac{1}{\mu_0}(\nabla\times\mathbf{B})\times\mathbf{B} = -\frac{1}{\mu_0}\mathbf{B}\times(\nabla\times\mathbf{B})$$

From NRL p. 4 (12) with $\mathbf{B} = \mathbf{A} \Rightarrow \nabla(\mathbf{B}\cdot\mathbf{B}) = 2\,\mathbf{B}\times(\nabla\times\mathbf{B}) + 2(\mathbf{B}\cdot\nabla)\mathbf{B}$

So $\mathbf{B}\times(\nabla\times\mathbf{B}) = \frac{1}{2}\nabla(\mathbf{B}\cdot\mathbf{B}) - (\mathbf{B}\cdot\nabla)\mathbf{B}$

Thus $\boxed{\mathbf{j}\times\mathbf{B} = -\frac{\nabla B^2}{2\mu_0} + \frac{(\mathbf{B}\cdot\nabla)\mathbf{B}}{\mu_0}}$

III. Characteristic Scales in a Plasma

A. Basic Parameters of a Plasma

1. Plasma consists of one, or more, ion species and electrons
2. Intensive variables: a. Density, n_s
   b. Temperature, T_s
   c. Magnetic Field, $\underline{B}_0$
3. Physical properties: a. mass, m_s
   b. charge, q_s

B. Units:

1. Two Major Systems: a. SI units (mks) (Kivelson & Russell, 1995)
   b. Gaussian units (cgs) (Shu, 1992)
2. I will do my best to be consistent, but may switch units from one lecture to the next based on units common to particular problems.

C. Length Scales: (Ordered from largest to smallest)

1. System size, L: Typical scale of system under investigation
   a. Earth's Magnetosphere: i. Magnetopause, $L \sim 10\, R_E$
   iii. Bowshock, $L \sim 15\, R_E$ $R_E = 6371$ km
   iv. Magnetotail, $L \gtrsim 100\, R_E$ (see [KR95, Fig 9.3])
   b. Solar Wind Turbulence: i. Outer Scale, $L \sim 10^{11}$ cm $= 10^6$ km
   c. Accretion Disk (around $M = 1\, M_\odot$ star) i. Radius, $R \sim 10^{10}$ cm
   ii. Height, $H \sim 10^8$ cm
   d. Galaxy Clusters i. Virial radius, $L \sim 1$ Mpc $1\,\mathrm{pc} = 3\times10^{16}$ m
   ii. Central core, $L \sim 100$ kpc

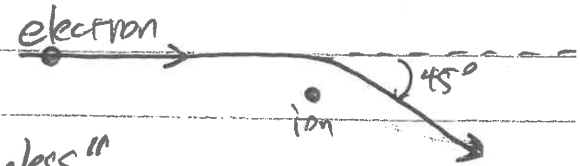
NOTE: What matters for plasma physics is not the absolute scale, but the scale relative to characteristic plasma length scales.

Lecture #1 (Continued)

III. C. (Continued)

2. Mean Free Path, λ_{m}: a. Distance between Coulomb collisions between charged particles.

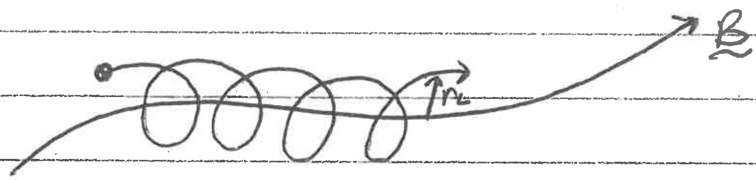
b. A collision is typically defined as a deflection of $\gtrsim 45°$



c. i. If $\lambda_m \gg L$, system is "collisionless".

ii. If $\lambda_m \ll L$, system is collisional.

3. Thermal Larmor radius (gyroradius), r_L:

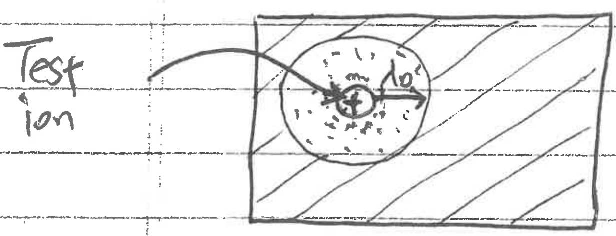


i. $\boxed{r_s \equiv \frac{v_{ts}}{\Omega_s}}$

Species thermal velocity

Species cyclotron frequency

4. Debye Length, λ_D: i. Length scale over which charge imbalance may occur.



ii. Electrons adjust to shield Coulomb field of "test ion"

$\Rightarrow$ Debye Shielding (see notes from 029:194, Lec#2)

iii. Net charge inside sphere of radius λ_D is zero.

5. Particle Separation, $n_0^{-1/3}$: i.) Typical distance between charged particles in plasma.

D. Velocities:

1. Thermal Velocity, v_{ts}: i. Defined $\boxed{v_{ts}^2 \equiv \frac{2T_s}{m_s}}$

ii. NOTE: Boltzmann constant $k = 1.38 \times 10^{-23} \frac{J}{K}$ is absorbed to give T in energy units.

III. D. 1. (Continued)

iii. For protons $\frac{m_i}{m_e} = 1836$, so $\frac{v_{te}}{v_{ti}} = \sqrt{\frac{m_i}{m_e}} \simeq 43$ (when $T_i = T_e$)

2. Alfven velocity, v_A: i. $\boxed{v_A^2 = \frac{B_0^2}{\mu_0 \rho}}$ (SI)

(or) $\boxed{v_A^2 = \frac{B_0^2}{4\pi \rho}}$ (cgs)

ii. Here, ρ is mass density $\left[\frac{M}{L^3}\right]$

iii. This is the characteristic speed of large scale motions ($L \gg r_{Li}$) in a magnetized plasma

3. Sound speed, c_s: i. $\boxed{c_s^2 \equiv \frac{\gamma p}{\rho}}$

ii. Here, γ is the adiabatic index ($\gamma = 5/3$ for monatomic gas)

iii. p is thermal pressure — determined by equation of state (fluid closure)

E. Frequencies (Timescales):

1. Observation Time: τ: i. Associated angular frequency $\omega \sim \frac{2\pi}{\tau}$

ii. If we observe a system for a time τ, we are most sensitive to frequencies $\gtrsim \omega \sim \frac{2\pi}{\tau}$

iii. Dynamics on a slower timescale (lower frequency) will not be apparent.

2. Crossing Time/Frequency, τ_c / ω_c: i. $\tau_c \sim \frac{L}{v_A}$ or $\omega_c \sim \frac{v_A}{L}$

ii. The time it takes for a signal to cross the system at characteristic velocity (here we take Alfven velocity, v_A)

III, E. (Continued)

3. <u>Collision Frequency, ν</u>: i. $\boxed{\nu \equiv \frac{v_{ts}}{\lambda_m}}$

ii. Typical velocity of particles is v_{ts}

iii. Distance between collisions is λ_m

4. <u>Cyclotron Frequency, Ω_s</u>: i. $\Omega_s = \frac{q_s B}{m_s}$ (SI)

(angular)

(or) $\Omega_s = \frac{q_s B}{m_s c}$ (cgs)

ii. Characteristic frequency of gyration of charged particle about magnetic field (non-relativistic)

5. <u>Plasma Frequency, ω_p</u>: i. $\omega_p^2 \equiv \frac{n_0 q_e^2}{\epsilon_0 m_e}$ (SI)

(or) $\omega_p^2 = \frac{4\pi n_0 q_e^2}{m_e}$ (cgs)

ii. Typical frequency of charge imbalance oscillations in a plasma (very rapid, much faster than most space or astrophysical plasma time scales).

iii. An applied electric field with $\omega < \omega_p$ will be shorted out by rapid electron response in plasma

F. <u>Dimensionless Parameters of a Plasma</u>:

1. <u>Plasma Parameter, N_D</u>: i. Number of particles in a Debye sphere

$$N_D = \frac{4\pi}{3} n \lambda_D^3$$

ii. For nearly all space & astrophysical plasmas of interest, $N_D \gg 1$.

$\Rightarrow$ Many particles within a Debye sphere

iii. Often used to define plasma behavior (collective behavior).

III. F. (Continued)

2. Plasma Beta, β: i. $\boxed{\beta \equiv \frac{\text{Thermal Pressure}}{\text{Magnetic Pressure}} = \frac{2\mu_0 n_0 (T_i + T_e)}{B_0^2}}$ (SI)

or $\boxed{\beta = \frac{8\pi n_0 (T_i + T_e)}{B_0^2}}$ (cgs)

iii. Most important parameter affecting plasma behavior

iv. In kinetics, $\boxed{\beta_i \equiv \frac{v_{ti}^2}{v_A^2}}$ In MHD $\boxed{\beta \equiv \frac{c_s^2}{v_A^2}}$

v. Low beta plasmas, $\beta \ll 1$, are magnetically dominated (fusion plasmas, solar corona)

vi. High beta plasmas, $\beta \gg 1$, have a magnetic field that can be highly deformed by plasma motions (black hole accretion disks)

3. Magnetization: i. $r_{Li}/L \ll 1$ Magnetized

ii. $r_{Li}/L \gg 1$ Unmagnetized

4. Collisionality: i. $\lambda_m/L \gg 1$ "Collisionless"

ii. $\lambda_m/L \ll 1$ collisional

G. Summary

1.

Length	Time/Frequency	
Particle spacing, $n_0^{-\frac{1}{3}}$		very small scale
Debye length, λ_D	Plasma Frequency, ω_p	$\Rightarrow$ "microscopic"
Larmor radius, r_L	Cyclotron Frequency, Ω_i	of most interest
Mean free path, λ_m	Collision Frequency, ν	for space &
System size, L	Observation "Frequency", $\frac{1}{T}$	astrophysical plasmas

2. Typical Conditions of Space & Astrophysical Plasmas:

a. Dynamics are quasi-neutral (no net charge imbalance) $L \gg \lambda_D$, $\frac{1}{T} \ll \omega_p$

b. Generally magnetized: $r_{Li}/L \ll 1$

c. Both plasma beta β and collisionality can be large, unity, or small.