Lecture #12: Key Results on Kinetic Turbulence: Driving Scales

Here we cover key results for understanding kinetic turbulence. A good reference is:

Howes, G.G. "Kinetic Turbulence", in Magnetic Fields in Diffuse Media, ed. de Gouveia Dal Pino, E. & Lazarian, A., Springer: Heidelberg (2015).

I. Driving or Energy Containing Scales

A. Powering the Turbulent Cascade

1. Turbulence is generally driven by violent events or instabilities at large scales that generate fluctuations in the magnetic field or plasma flows.

a. Driving Scale: $L \gg \rho_i$

b. Driving wavenumber $k_0 \sim \frac{1}{L}$

2. Isotropic Driving: In the absence of arguments to the contrary, we assume driving is isotropic relative to the magnetic field direction,

$$k_{\parallel 0} \sim k_{\perp 0} \sim k_0$$

3. Define: Isotropic Driving Wavenumber $\boxed{k_0 \rho_i \ll 1}$

Ref: Howes et al. JGR (2008); Howes et al. (2011) PoP. $\Rightarrow 10^{-3} \gtrsim k_0\rho_i \gtrsim 10^{-4}$

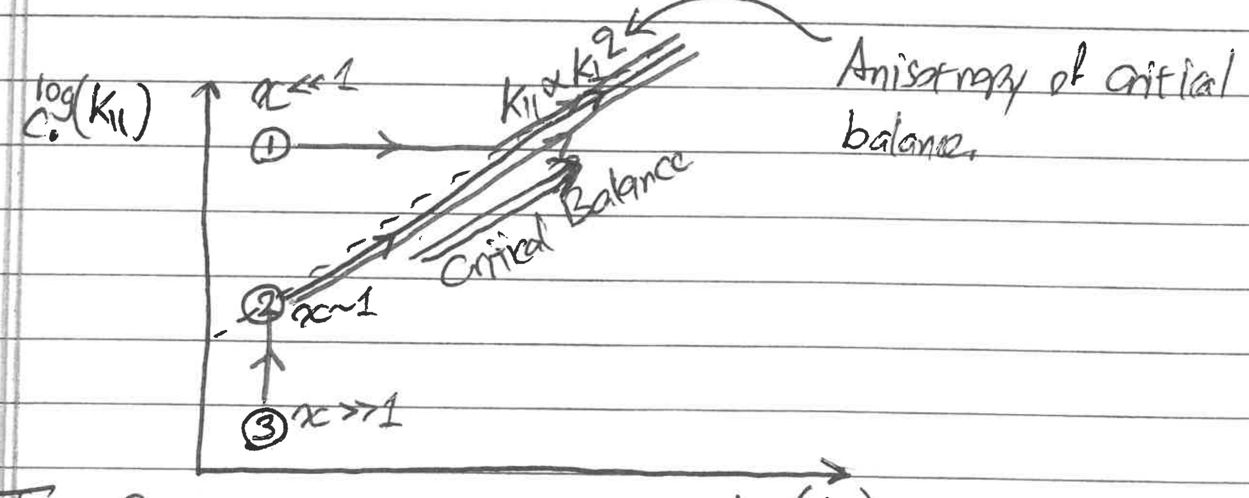
4. Amplitude of Driving:

a. If the MHD approximation is satisfied at L, then MHD turbulence theory will apply
(Sridhar & Goldreich 1994, Goldreich & Sridhar 1995 (GS95)
Galtier et al. 2000, Lithwick & Goldreich 2001, Boldyrev 2006)
(BØ6)

b. Nonlinearity Parameter:

i. $\chi = \dfrac{k_\perp v_k}{k_\parallel v_A}$ (GS95)

ii. $\chi = \dfrac{k_\perp v_k \theta_k}{k_\parallel v_A}$ (BØ6) At L, take $\theta_k \sim 1$ rad. thus both versions are equivalent.



Three Cases:

① Weak Driving $\chi \ll 1$: No parallel cascade, $k_\parallel \propto$ constant
$E_{k_\perp} \propto k_\perp^{-2} \Rightarrow v_\perp \propto k_\perp^{-\frac{1}{2}} \Rightarrow \chi \propto k_\perp^{\frac{1}{2}}$
$\Rightarrow \chi$ increases to 1: $\chi \to 1 \Rightarrow$ Strong turbulence

② Strong (Critically Balanced) Driving, $\chi \sim 1$: GS95 $k_\parallel \propto k_\perp^{2/3}$ $q = +\frac{2}{3}$
BØ6 $k_\parallel \propto k_\perp^{\frac{1}{2}}$ $q = +\frac{1}{2}$

③ Over-Strong Driving $\chi \gg 1$:

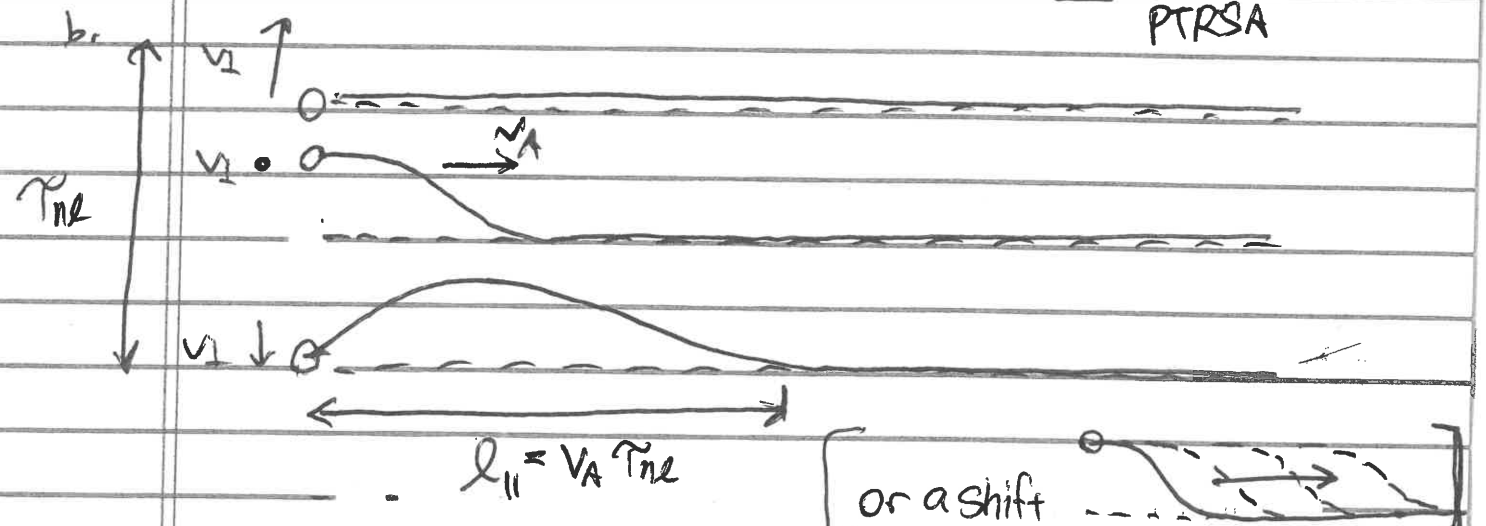
Perpendicular fluctuations ($k_\perp v_\perp \gg k_\parallel v_A$) will quickly become decorrelated along the magnetic field, rapidly increasing $k_\parallel$ until $\chi \to 1$.

$\Rightarrow$ Back to strong, critically balanced turbulence.

5. Why? a. Shake a magnetic field transversely with period $\tau_{nl} \sim \frac{1}{k_\perp v_\perp} \sim \frac{\ell_\perp}{v_\perp}$

Ref: Howes (2015) PTRSA

b.



c. Now, $\ell_\parallel = v_A \left(\frac{\ell_\perp}{v_\perp}\right) \Rightarrow \frac{\ell_\parallel v_\perp}{\ell_\perp v_A} = 1 \Rightarrow \boxed{\frac{k_\perp v_\perp}{k_\parallel v_A} = \chi = 1}$.

d. If you shake the magnetic field more rapidly (higher $v_\perp$) it will simply generate smaller parallel scales.

$\Rightarrow \ell_\parallel$ decreases $\Rightarrow k_\parallel$ increases.

e. Thus uncorrelated driving motions ($\chi \gg 1$) will rapidly decorrelate to reach $\chi = \frac{k_\perp v_\perp}{k_\parallel v_A} = 1$ by increasing $k_\parallel$.

I. (Continued)

B. The MHD Approximation

1. MHD requires the following conditions for fluctuations with scales $\ell_\parallel, \ell_\perp$, and frequency ω

① Non-relativistic: $\frac{v_{ts}}{c} \ll 1$

② Strongly collisional: $\frac{\lambda_{mfp}}{\ell_\parallel} \ll 1$

③ Magnetized, large scale motions: $\frac{\rho_i}{\ell_\perp} \ll 1$

④ Low-frequency: $\frac{\omega}{\Omega_i} \ll 1$

C. Nature of Driven Fluctuations

1. If the MHD approximation is satisfied, the driving fluctuations will be a mixture of:

Alfven waves $\Rightarrow$ incompressible wave

$\left.\begin{array}{l}\text{Slow magnetosonic waves}\\ \text{Fast magnetosonic waves}\end{array}\right\} \Rightarrow$ Compressible waves

Entropy mode $\Rightarrow$ Non-propagating, pressure balanced structures (PBS).

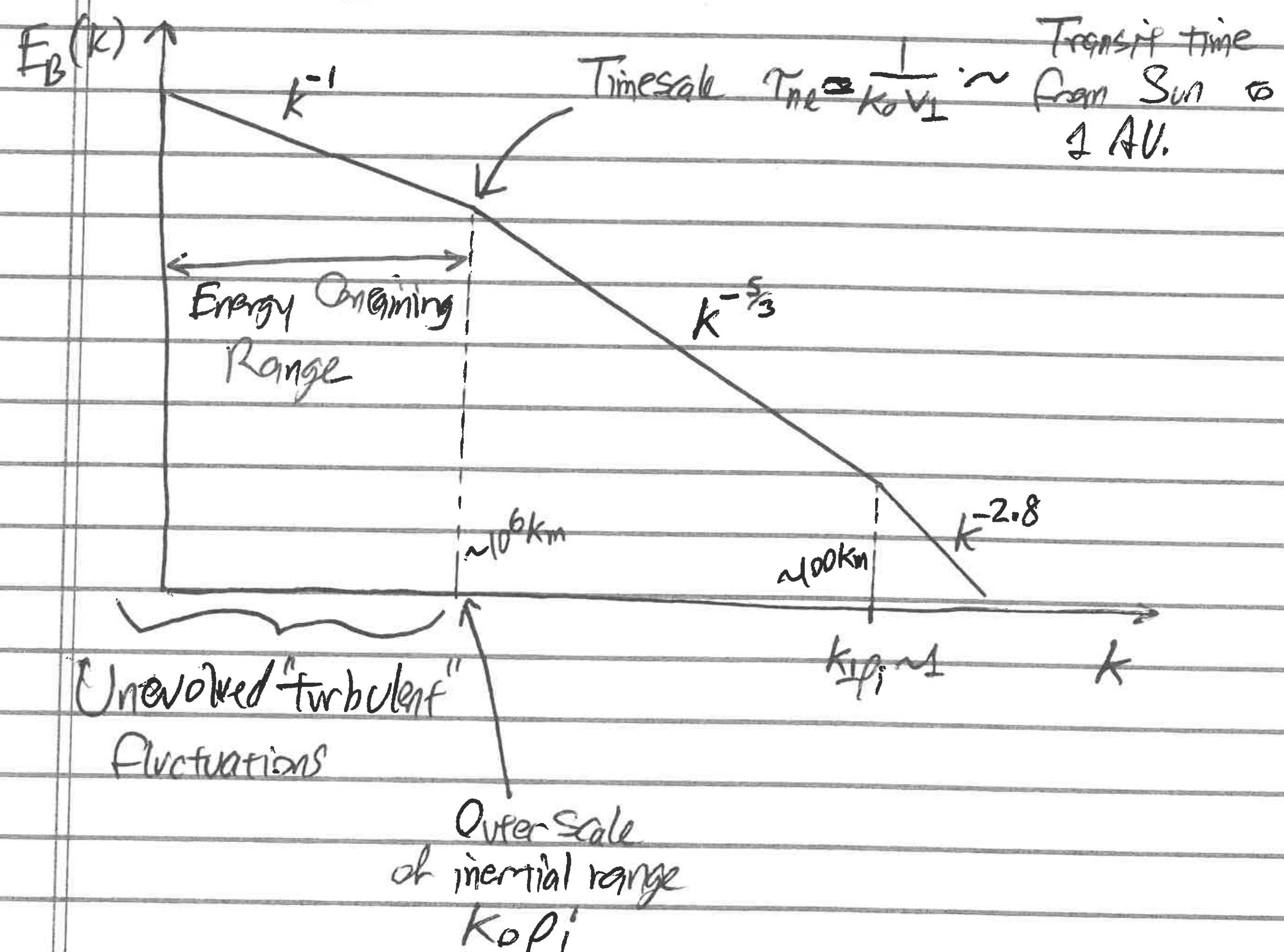
2. Resulting Turbulent Cascade will depend on mixture.

D. Energy Containing Range: The Solar Wind

Refs: Matthaeus et al. (1994); Kiyani et al. (2015) PTRSA

1. In the solar wind at 1 AU, the solar wind magnetic wavenumber spectrum (Using Taylor Hypothesis

$$k = \frac{2\pi f_{sc}}{V_{sw}\cos\theta_{kV}}, \text{ See Lecture \#11: I.B.3}$$)



a. At the largest scales, the fluctuations have not had enough time to interact nonlinearly (and thereby transfer their energy to smaller scales via a turbulent cascade) within their lifetime since leaving the Sun
$\Rightarrow$ Unevolved turbulence.

b. Outer scale $k_0\rho_i$ is where fluctuations have had time to interact,

2. a. Thus, solar wind turbulence is <u>not</u> being actively driven, but instead is a case of decaying turbulence.

b. However, the energy containing range is a reservoir of turbulent energy at large scales

c. As the solar wind flows from the Sun (and therefore continues to evolve), the fluctuations at the small scale (high k) end of the energy containing range have time to interact, and thus feed their energy in the inertial range turbulent cascade.
$\Rightarrow$ Thus $k_{0\rho i}$ decreases in time (or distance R from Sun)

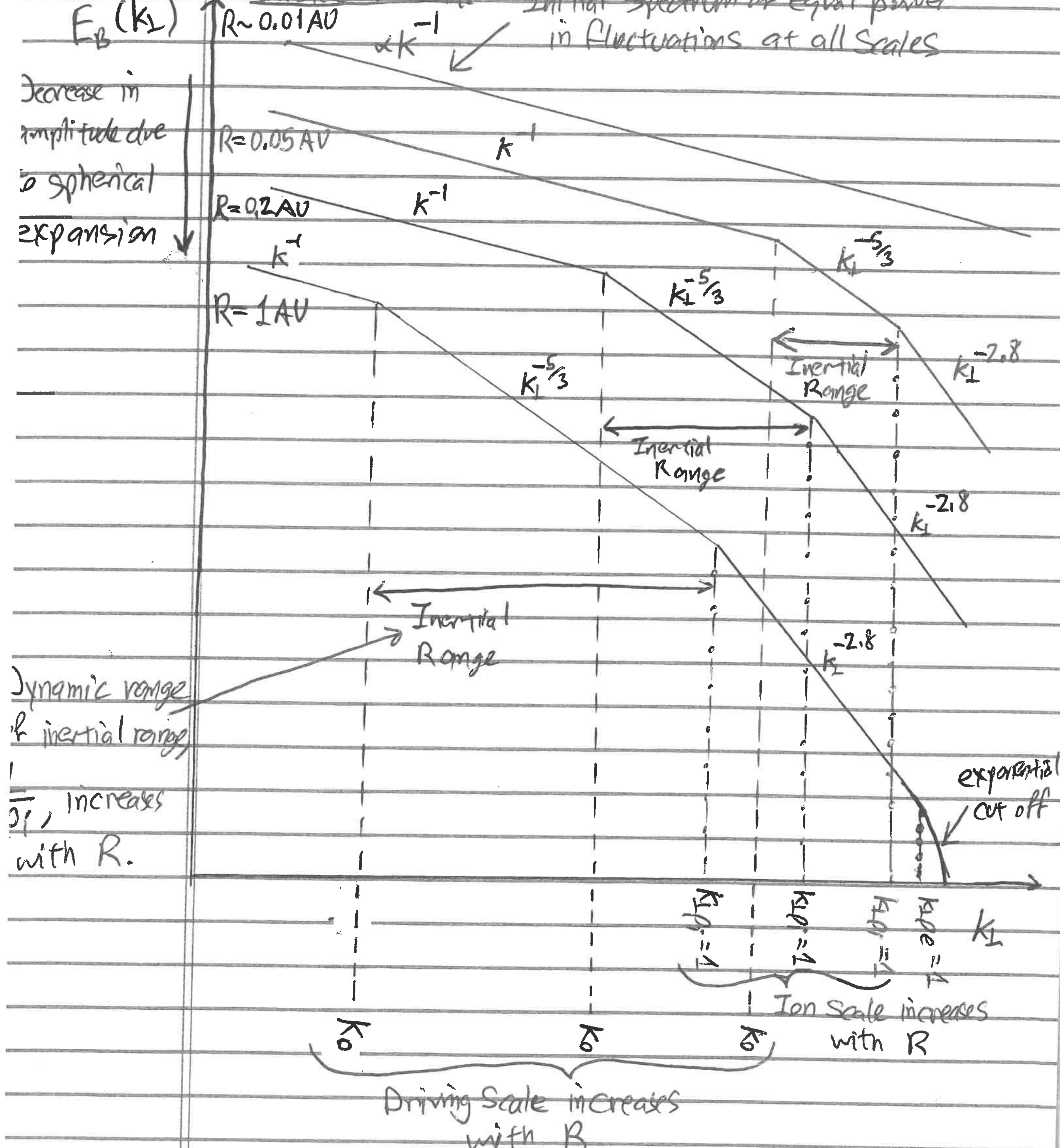
(Roberts, D.A, et al. 1992)

3. Some studies claim solar wind turbulence is driven by Kelvin-Helmholtz instabilities at velocity shears, but the instability growth rates are <u>much</u> too slow at the large scales to drive the turbulence.

4. a. The fluctuations in the energy containing range should provide information about the large-scale conditions of the solar wind when it was launched from the Sun.

b. Fluctuations at wavenumbers above $k_{0\rho i}$ have been processed by strong turbulence interactions

E. Predicted Evolution of Solar Wind Magnetic Energy Spectrum Inside Corona:



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