Howes (1)

Lecture #20: Quasi-Perpendicular Supercritical Shocks

I. Particle Dynamics & Shock Structure

A. Supercriticality & Ion Reflection

1. When a fast magnetosonic shock exceeds the (first) critical Mach number,

$$M_f > M_c$$

fluid dissipation mechanisms (specifically resistivity, viscosity, and heat conduction) are insufficient to achieve the plasma heating & entropy production required by Rankine-Hugoniot jump conditions for conservation of mass, momentum, & energy.

a. Critical Mach Number is a function of θ_{Bn} & β.

$$M_c(\theta_{Bn}, \beta)$$

b. For $\theta_{Bn} = 90°$ & $\beta \ll 1$, $M_c = 2.76$

c. For $\theta_{Bn} < 90°$ or $\beta \gtrsim 1$, M_c is lower

d. Most heliospheric shocks are supercritical.

2. When fluid dissipation within the shock layer is insufficient, the plasma resorts to particle reflection to arrest the incoming flow & heat the plasma.

a. This is particularly important for collisionless shocks.

I.A. (Continued)

3. Ion Reflection has several results:
   a. Reduces net inflow momentum & kinetic energy density
      $\Rightarrow$ effectively reduces Mach number of inflow
   b. Creates an additional obstacle to inflow
   c. May trigger kinetic instabilities

4. The dynamics of reflected particles impact the typical structure of collisionless shocks
   a. Here we will explore the structure of quasi-perpendicular shocks.

B. Quasi-perpendicular vs Quasi-Parallel Classification

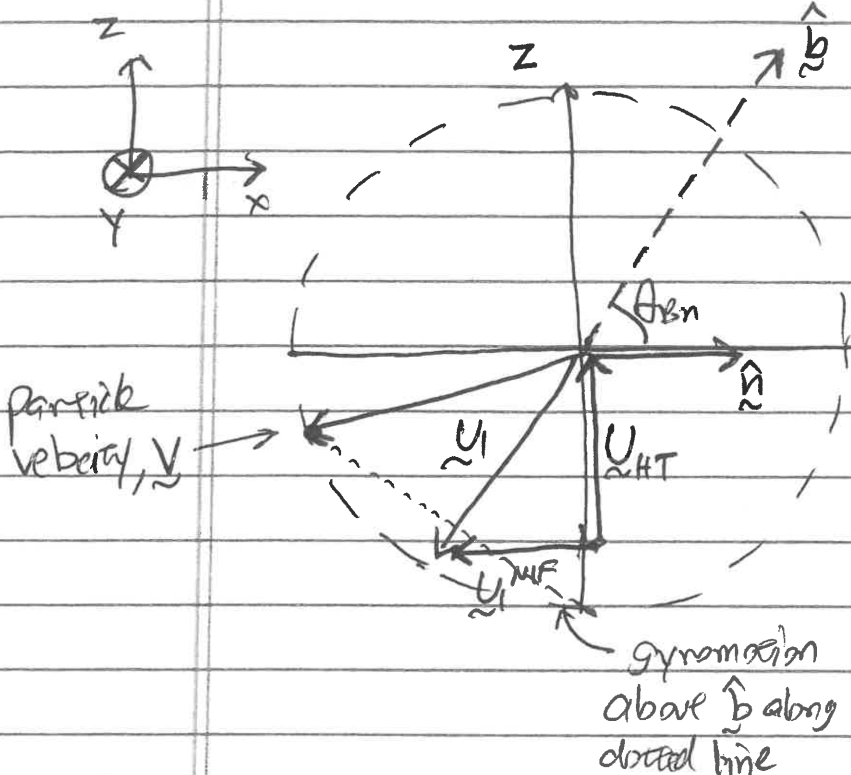
1. The dynamics of reflected ions motivates the classification into
   a. Quasi-perpendicular shocks: $\theta_{Bn} \geq 45°$
   b. Quasi-parallel shocks: $\theta_{Bn} < 45°$

2. The key distinction is whether reflected particles can escape upstream, or eventually pass downstream.
   Ref: Schwartz, Thomsen, & Gosling JGR 88:2039 (1983)

3. The calculation is most easily done in the de Hoffmann-Teller Frame (HTF)
   a. In this frame, the particle guiding centers simply move along the magnetic field.

I. B. (Continued) Howes ③

4. de Hoffmann–Teller Frame (HTF)



a. $$\boxed{\underset{\sim}{U}_1^{NIF} = \underset{\sim}{U}_1 + \underset{\sim}{U}_{HT}}$$

HTF velocity (U_{HT})

Frame transformation velocity.

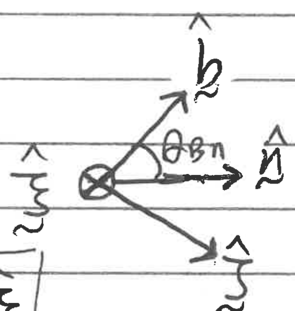
b. In general,

$$\underset{\sim}{U}_{HT} = \frac{\hat{n} \times (\underset{\sim}{U}_1 \times \underset{\sim}{B}_1)}{\hat{n} \cdot \underset{\sim}{B}_1}$$

where $\underset{\sim}{U}_1$ is <u>any</u> upstream flow with $\theta_{Bn} \neq 0$.

c. Define an upstream magnetic field aligned coordinate (FAC) system.

i) $(\hat{b}, \hat{\xi}, \hat{\zeta})$



$$\boxed{\hat{b} = \hat{\xi} \times \hat{\zeta}}$$

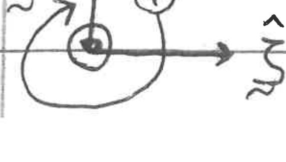
$$\boxed{\begin{aligned} \hat{b} &= \cos\theta_{Bn}\,\hat{n} + \sin\theta_{Bn}\,\hat{z} \\ \hat{\xi} &= \sin\theta_{Bn}\,\hat{n} - \cos\theta_{Bn}\,\hat{z} \\ \hat{\zeta} &= \hat{y} \end{aligned}}$$

ii) $$\boxed{\begin{aligned} \hat{n} &= \cos\theta_{Bn}\,\hat{b} + \sin\theta_{Bn}\,\hat{\xi} \\ \hat{z} &= \sin\theta_{Bn}\,\hat{b} - \cos\theta_{Bn}\,\hat{\xi} \\ \hat{y} &= \hat{\zeta} \end{aligned}}$$

d. Particle motion

$$\boxed{\underset{\sim}{V} = -v_{\parallel}\,\hat{b} + v_{\perp}\left[\cos(\Omega_i t + \phi_0)\,\hat{\xi} - \sin(\Omega_i t + \phi_0)\,\hat{\zeta}\right]}$$

initial gyrophase (ϕ_0)

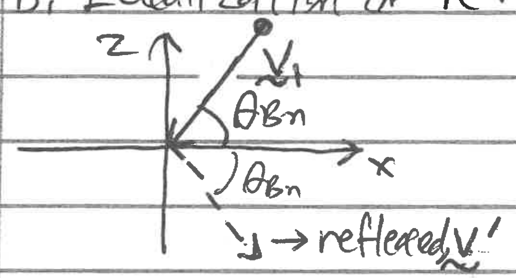


$$\Omega_i = \frac{q_i B_1}{m_i} \quad \text{cyclotron frequency}$$

5. Specular Reflection:

a. Assume particle with FAC velocity $(-V_\parallel, V_\perp)$ reaches the shock discontinuity at $x=0$ at $t=0$, within phase ϕ_0.

b. Idealization of reflection: Specular reflection



i) Reverse normal component of V_1

ii) $$\boxed{V' = V_1 - 2(V\cdot\hat{n})\,\hat{n}}$$

6. One can determine the new parallel & perpendicular components of particle velocity (in FAC) after reflection:

a. $$V_\parallel' = V_\parallel \cos 2\theta_{Bn} - V_\perp \cos(\Omega_i t + \phi_0)\sin 2\theta_{Bn}$$

b. $$V_\perp' = \left\{ \left(V_\parallel \sin 2\theta + V_\perp \cos(\Omega_i t + \phi_0)\cos 2\theta\right)^2 + V_\perp^2 \sin^2(\Omega_i t + \phi_0) \right\}^{\frac{1}{2}}$$

7. Integrating the position for particle velocity, we obtain

a. $$x(t) = V_\parallel' t\,\hat{b} + \frac{V_\perp'}{\Omega_i}\left[\sin(\Omega_i t + \phi_0)\,\hat{\zeta} + \cos(\Omega_i t + \phi_0)\,\hat{\xi}\right]$$

b. The normal component is given by

$$x_n(t) = x(t)\cdot\hat{n} = V_\parallel' t \cos\theta_{Bn} + \frac{V_\perp'}{\Omega}\sin(\Omega_i t + \phi_0)\sin\theta_{Bn}$$

8. The reflected particle turns back toward the shock if the normal velocity reaches zero at $t>0$.

a. $V_n = \underset{\sim}{V} \cdot \underset{\sim}{\hat{n}} = V_\parallel' \cos\theta_{Bn} + V_\perp' \cos(\Omega_i t + \phi_0) \sin\theta_{Bn} = 0$

b. Substituting in $V_\parallel'$ & $V_\perp'$ and manipulating, one obtains

$$\Omega_i t + \phi_0 = \cos^{-1}\left\{ \frac{1 - 2\cos^2\theta_{Bn}}{2\sin^2\theta_{Bn}} \right\}$$

c. $\boxed{\text{For } \phi_0 = 0\text{, this equation has a solution for } t>0 \text{ \underline{only} when } \theta_{Bn} > 45^\circ \text{ [positive argument for } \cos^{-1}(\,)]}$

9. Thus, for $\theta_{Bn} > 45^\circ$ particles return downstream

for $\theta_{Bn} < 45^\circ$ particles escape upstream.

$\Rightarrow$ Motivation of quasi-perpendicular/quasi-parallel boundary.

10. Caveats:

a. This boundary shifts for $\phi_0 \neq 0^\circ$.

b. This result only holds for cold ions (where particle parallel velocity $\boxed{V_\parallel = U_\parallel}$ upstream inflow).

c. Specular reflection is a significant idealization

i) Particles actually penetrate downstream

ii) Increased downstream B_2 helps ions reflect

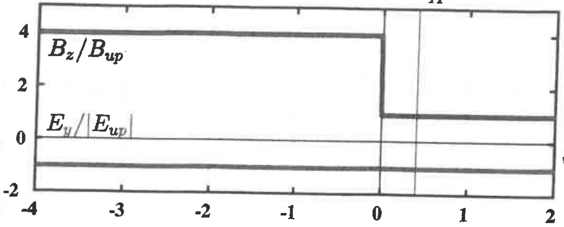
d. Ignores any effect of kinetic instabilities in shock layer

I. B. (Continued)

More Realistic Particle Dynamics

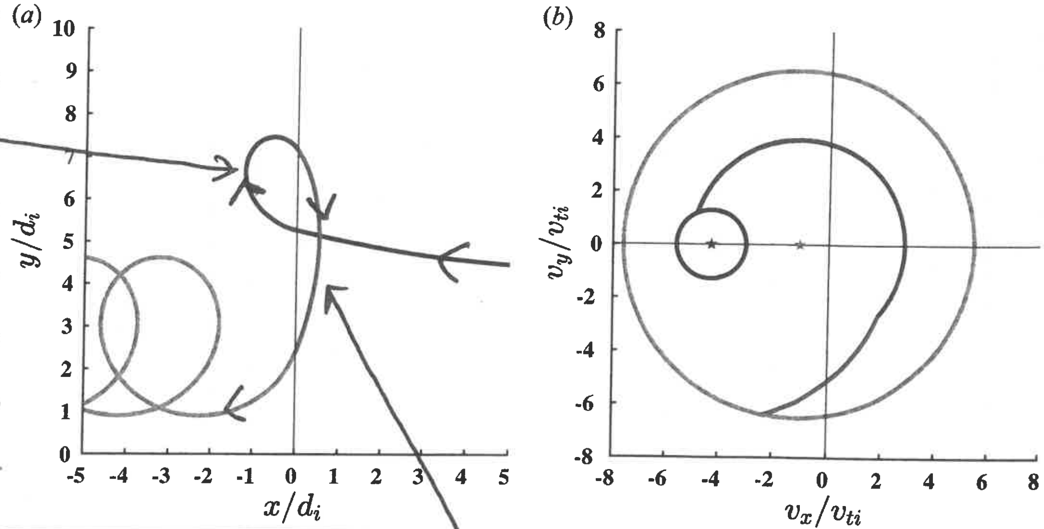
II.

Perpendicular Shock $\theta_{Bn}=90°$



← Discontinuous jump in $\underset{\sim}{B}_{t1}$

← Constant Motional $\underset{\sim}{E}_{t1}$



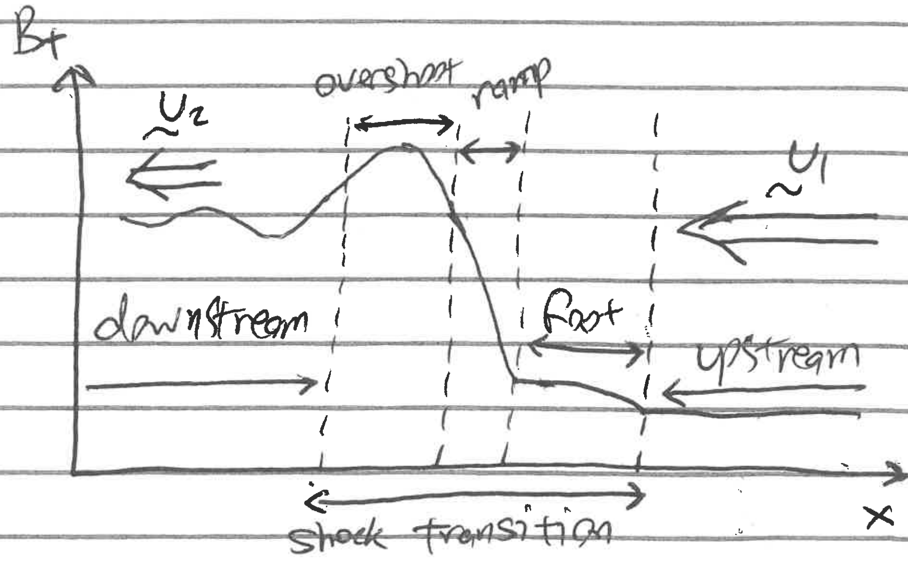
Ion penetrates downstream, but increased $\underset{\sim}{B}_{t2}$ causes gyromotion to return it upstream

Reflected Ion returns upstream briefly

C. Quasi-Perpendicular Shock Structure

1. Typical Features:

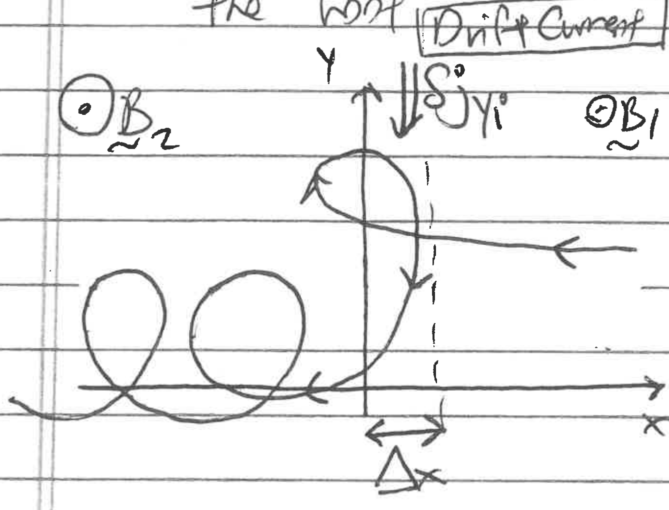
a. Upstream
b. Foot
c. Ramp
d. Overshoot
e. Downstream



I. C. (Continued)

2. Foot:

a. Reflected ions at the supercritical shock generate the foot



b. Reflected ions penetrate only about 1 Larmor radius upstream

$$\Delta x \simeq \rho_i \equiv \frac{v_\perp}{\Omega_i}$$

c. For $\theta_{Bn} = 90°$, $\Delta x \simeq 0.7 \rho_i$

d. The reflected ions (often called gyrating ions) have a net flow in the $-\hat{y}$ direction

~~i) ⇒ Generates a drift current δj_{y_i}~~

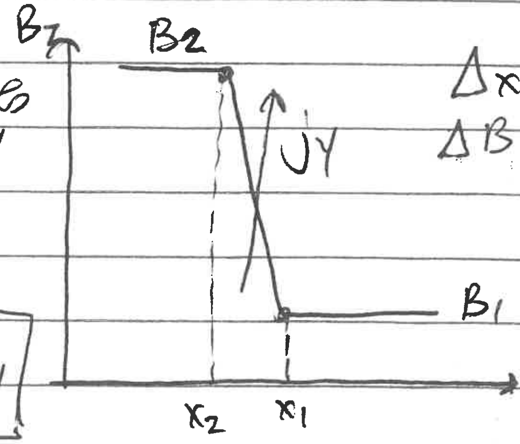
~~ii) Causes a perturbation to $\underset{\sim}{B}_t$ upstream of ramp where reflected ions drift.~~

3. Currents & Magnetic Field: Perpendicular Shock

a. Ampere-Maxwell Law $\quad \nabla \times \underset{\sim}{B} = \mu_0 \left(\underset{\sim}{j} + \cancel{\varepsilon_0 \frac{\partial \underset{\sim}{E}}{\partial t}} \right)$ 0

Stationary Shock Frame.

b. $\nabla \times \underset{\sim}{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \cancel{\partial_y}^{0} & \cancel{\partial_z}^{0} \\ \cancel{B_x}^{0} & \cancel{B_y}^{0} & B_z \end{vmatrix} = -\frac{\partial B_z}{\partial x}\hat{y} = \mu_0 j_y \hat{y}$



$\Delta x = x_1 - x_2$

$\Delta B = B_2 - B_1$

c. Current must exist to support magnetic field jump

$$j_y = -\frac{1}{\mu_0}\frac{\partial B_z}{\partial x} = -\frac{1}{\mu_0}\frac{B_2 - B_1}{x_2 - x_1}$$

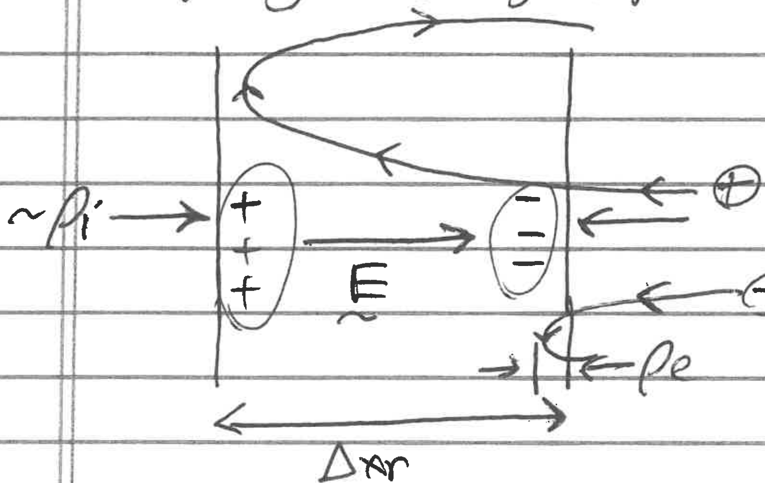
$$= -\frac{1}{\mu_0}\frac{\Delta B}{(-\Delta x)} = \boxed{\frac{1}{\mu_0}\frac{\Delta B}{\Delta x} = j_y}$$

I. C. 3. (Continued) Howes ⑧

d. This current must arise self-consistently to support jump in magnetic field (mostly due to electrons)

4. <u>Charge Separation due to Ion penetration</u>

a. In the ramp, ions penetrate further than electrons, leading to charge separation



b. Net positive at depth $\sim \rho_i$ into shock ramp

c. Net negative at depth $\sim \rho_e$ into shock ramp.

d. <u>Def:</u> <u>Cross-Shock Electric Field:</u> The charge separation creates a significant $\underset{\sim}{E} = E_x \hat{x} > 0$

e. Since $\rho_e \ll \Delta x_r$, electrons experience a $\underset{\sim}{E} \times \underset{\sim}{B}$ drift:

i) $\underset{\sim}{E} \times \underset{\sim}{B} = (E_x \hat{x}) \times (B_z \hat{z}) = -E_x B_z \hat{y}$

ii) For negatively charged electrons, this leads to a significant $j_{ye} > 0$.

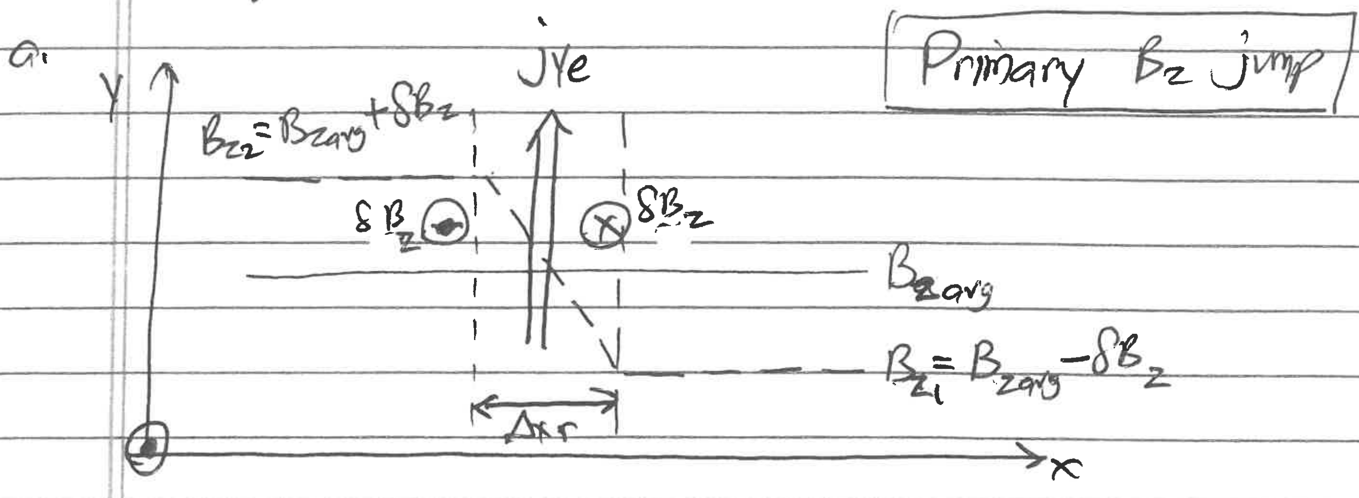
f. Ions have $\rho_i \gtrsim \Delta x_r$, so do not experience a similar $\underset{\sim}{E} \times \underset{\sim}{B}$ drift. But do have a small $\delta j_{yi} > 0$ at end of ramp.

g. Thus, j_y needed for B_z jump is largely due to electrons

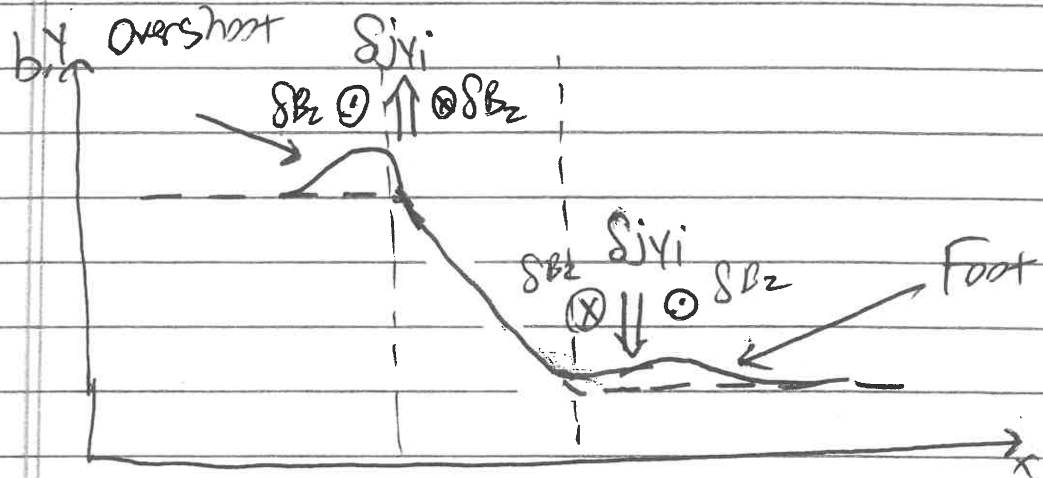
I. C. (Continued)

5. Magnetic Fields in Foot and Overshoot

a.



b.



6. Cross-Shock Electric Field

Ref: Goodrich & Scudder, JGR 89:6654 (1984)

a. The cross-shock electric field is generated by different dynamics of ions and electrons in shock ramp

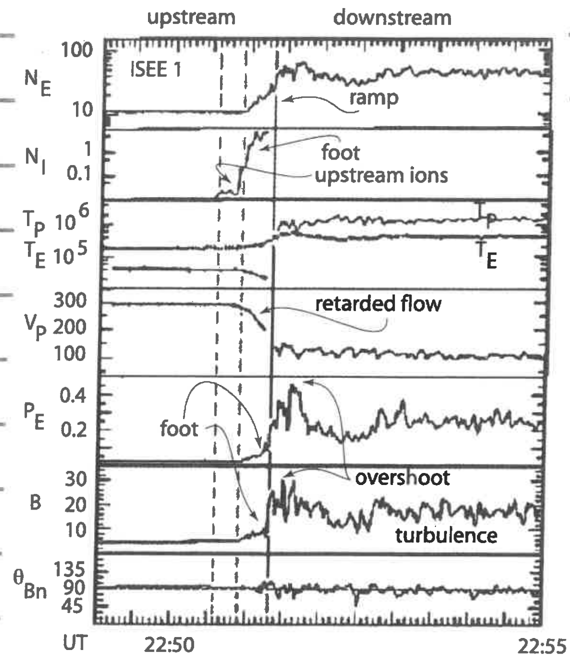
b. Can be estimated as potential arising from gradient in electron pressure:

$$\boxed{\Delta \Phi(x) \simeq \int_0^x \frac{1}{e n_e(x)} \frac{\partial p_e}{\partial x} dx}$$

c. Impedes ion inflow and accelerates electrons

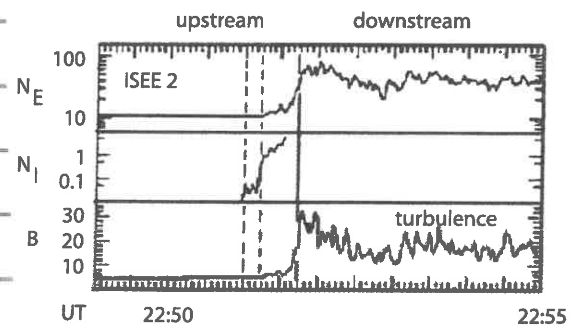
D. Quasi-perpendicular Shock Observations

1. ISEE: Ref: Sckopke et al. JGR 88:6121 (1983)

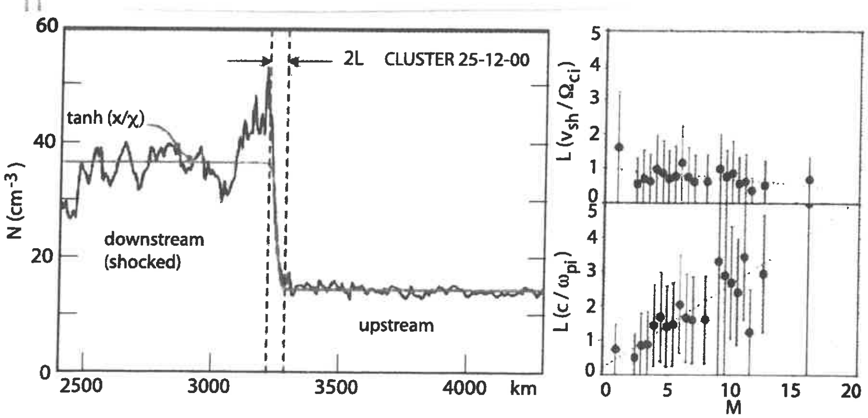


← Energetic (reflected) ions in foot

← Magnetic foot, ramp, and overshoot ⇒ downstream turbulence



2. Cluster: Ref: Bale et al. PRL 91:265004 (2003)



a. Ramp width seems to scale with gyroradius from shock speed & upstream $\underset{\sim}{B}$!

$$\boxed{\Delta x_r \sim \rho_{i2} \sim \frac{U_1}{\Omega_{i2}}}$$

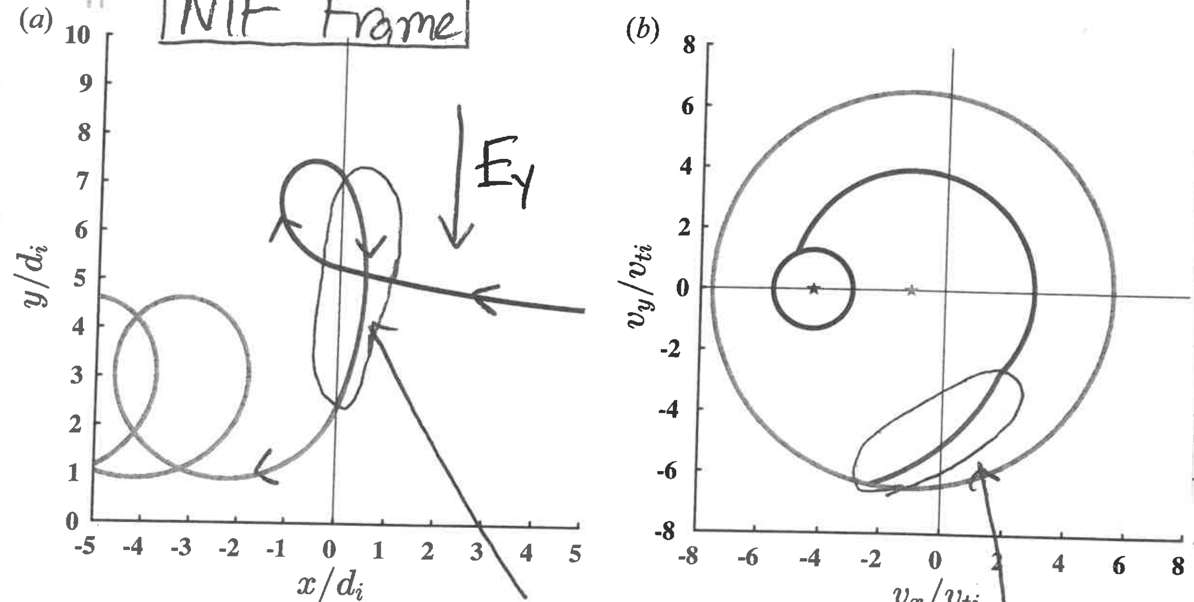
II. Particle Acceleration, Instabilities, & Reformation

A. Shock Drift Acceleration:

Perpendicular Shock:

1. Reflected ions undergo acceleration by the motional electric field, $\underset{\sim}{E} = E_y \hat{y} < 0$

$\boxed{\text{NIF Frame}}$



Ions gain energy

$$\boxed{\frac{dW_i}{dt} \sim q E_y v_y > 0}$$

In velocity space, ions move away from origin, gaining energy.

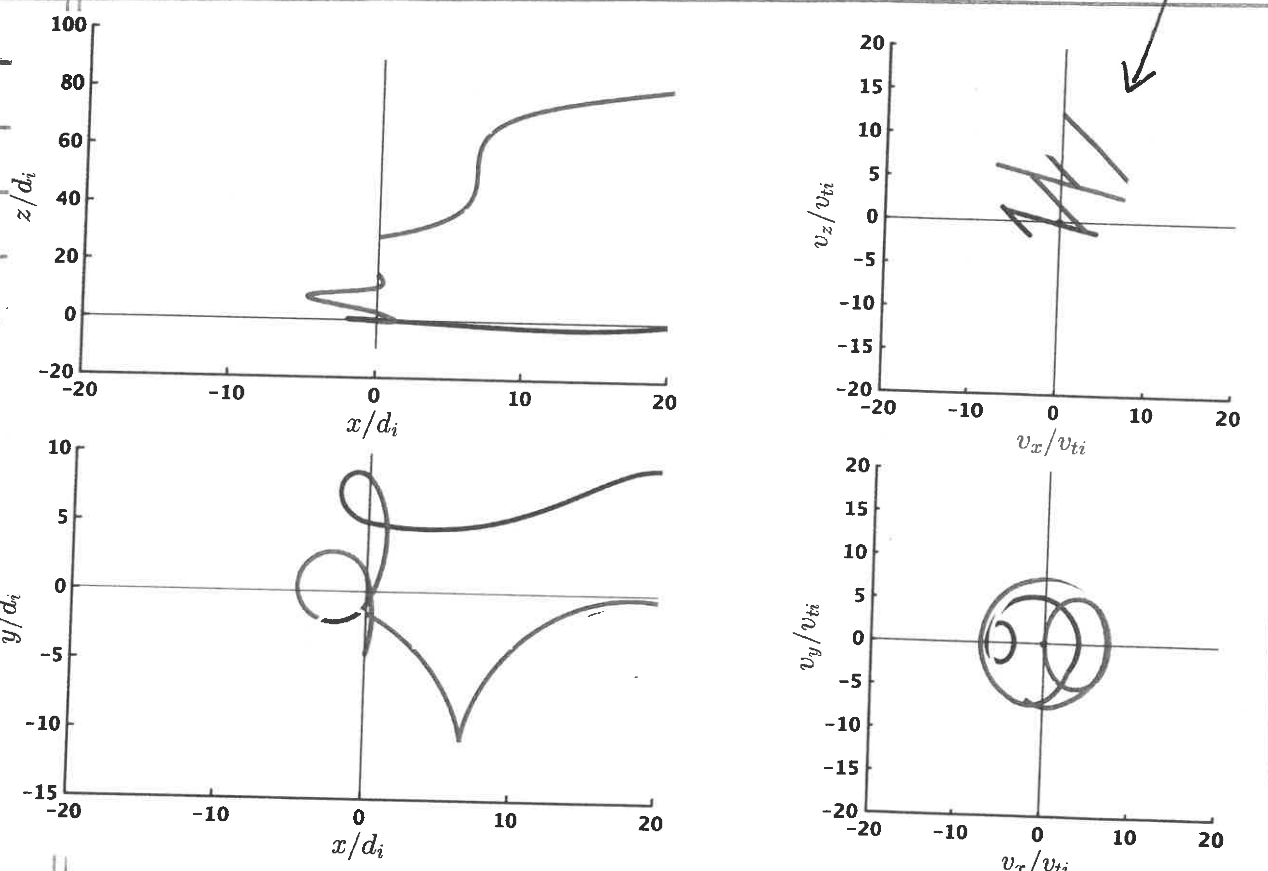
II A. (Continued)

2. Quasi-Perpendicular ($\theta_{Bn} = 45^\circ$) Shock

a. Reflected Back upstream

i) Three Reflections

ii) Shock-Drift Acceleration: Significant net energy gain in v_z



B. Kinetic Instabilities:

1. Reflected ion populations, as well as strong electron tangential drifts, yield particle distribution functions unstable to kinetic instabilities

2. Instabilities in the shock upstream (due to counterstreaming reflected particles), foot, and ramp impact shock dynamics & energization.
   a. Rippling of planar shock front
   b. Non-adiabatic heating of electrons
   c. Scattering reflected particles back to shock front
      $\Rightarrow$ Diffusive Shock Acceleration (Fermi Accel.)

C. Shock Reformation

1. At Mach numbers above the Second Critical Mach Number, M_{2c}, shocks become non-stationary

$$M_{2c} \simeq \left(\frac{m_i}{m_e}\right)^{\frac{1}{2}} \cos\theta_{Bn}$$

2. Shock propagates in steps
   a. Ramp reflects ions into foot
   b. Foot rises in amplitude
   c. Foot develops into new ramp