Lecture #24: The Field-Particle Correlation Technique

Refs: 1) Klein & Howes, ApJ Lett. 826: L30 (2016)
2) Howes, Klein, & Li, J. Plasma Phys. 83: 705830102 (2017)
3) Klein, Howes, & TenBarge, J. Plasma Phys 83: 535830401 (2017)

I. The Kinetic Physics of Particle Energization

A. Maxwell-Boltzmann Equations (cgs)

1. Boltzmann Equation

$$\frac{\partial f_s}{\partial t} + \underset{\sim}{v} \cdot \nabla f_s + \frac{q_s}{m_s}\left[\underset{\sim}{E} + \frac{\underset{\sim}{v} \times \underset{\sim}{B}}{c}\right] \cdot \frac{\partial f_s}{\partial \underset{\sim}{v}} = \left(\frac{\partial f_s}{\partial t}\right)_{coll}$$

2. Maxwell's Equations

$$\nabla \times \underset{\sim}{E} = -\frac{1}{c}\frac{\partial \underset{\sim}{B}}{\partial t} \qquad \nabla \cdot \underset{\sim}{E} = 4\pi \rho_q$$

$$\nabla \times \underset{\sim}{B} = \frac{4\pi}{c} \underset{\sim}{j} + \frac{1}{c}\frac{\partial \underset{\sim}{E}}{\partial t} \qquad \nabla \cdot \underset{\sim}{B} = 0$$

3. Source terms: Charge density $\rho_q = \sum_s q_s \int d^3\underset{\sim}{v}\, f_s$

Current density $\underset{\sim}{j} = \sum_s q_s \int d^3\underset{\sim}{v}\, \underset{\sim}{v} f_s$

4. Variables: a. Distribution function for each species, $f_s(\underset{\sim}{r}, \underset{\sim}{v}, t)$

b. Electromagnetic Fields: $\underset{\sim}{E}(\underset{\sim}{r}, t)$ $\underset{\sim}{B}(\underset{\sim}{r}, t)$

c. Six dimensional phase space (3D-3V) plus time.

I. (Continued)

B. Particle Energization

1. In many space & astrophysical plasmas, the diffuse and hot plasma conditions lead to weak collisionality

2. The timescale for particle energization is much shorter than the collisional timescale
   a. Thus, we may drop the collision term, recovering the Vlasov Equation

$$\frac{\partial f_s}{\partial t} + \underset{\sim}{v} \cdot \nabla f_s + \underbrace{\frac{q_s}{m_s}\left[\underset{\sim}{E} + \frac{\underset{\sim}{v} \times \underset{\sim}{B}}{c}\right] \cdot \frac{\partial f_s}{\partial \underset{\sim}{v}}}_{\text{Lorentz Force term is responsible for interactions between fields and particles.}} = 0$$

   b. Particle energization occurs via collisionless interactions between the electromagnetic fields and plasma particles.

⇒ $\boxed{\text{Field-Particle Correlations can be used to diagnose the mechanisms of particle energization in 3D-3V phase space}}$

B. Poynting's Theorem:

1. $\underset{\sim}{B} \cdot$ Faraday's law with $\underset{\sim}{B}$ and $\underset{\sim}{E} \cdot$ Ampere-Maxwell law with $\underset{\sim}{E}$

$$\underset{\sim}{B} \cdot \frac{\partial \underset{\sim}{B}}{\partial t} = -c\,\underset{\sim}{B} \cdot (\nabla \times \underset{\sim}{E})$$

$$\underset{\sim}{E} \cdot \frac{\partial \underset{\sim}{E}}{\partial t} = c\,\underset{\sim}{E} \cdot (\nabla \times \underset{\sim}{B}) - 4\pi \underset{\sim}{j} \cdot \underset{\sim}{E}$$

and Sum

2. $$\underbrace{\mathbf{E}\cdot\frac{\partial \mathbf{E}}{\partial t} + \mathbf{B}\cdot\frac{\partial \mathbf{B}}{\partial t}}_{=\frac{1}{2}\frac{\partial}{\partial t}(|\mathbf{E}|^2+|\mathbf{B}|^2)} = c\Big[\underbrace{\mathbf{E}\cdot(\nabla\times\mathbf{B}) - \mathbf{B}\cdot(\nabla\times\mathbf{E})}_{-\nabla\cdot(\mathbf{E}\times\mathbf{B})}\Big] - 4\pi\mathbf{j}\cdot\mathbf{E}$$

$$-\nabla\cdot(\mathbf{E}\times\mathbf{B}) = \mathbf{E}\cdot(\nabla\times\mathbf{B}) - \mathbf{B}\cdot\nabla\times\mathbf{E}$$

3. Divide by 4π

$$\boxed{\frac{\partial}{\partial t}\left[\frac{|\mathbf{E}|^2+|\mathbf{B}|^2}{8\pi}\right] + \frac{c}{4\pi}\nabla\cdot(\mathbf{E}\times\mathbf{B}) = -\mathbf{j}\cdot\mathbf{E}}$$

4. Integrate over a volume $\int d^3\mathbf{r}$ and use Gauss's Theorem on second term:

$$\frac{c}{4\pi}\int d^3\mathbf{r}\ \nabla\cdot(\mathbf{E}\times\mathbf{B}) = \frac{c}{4\pi}\oint d^2\mathbf{S}\cdot(\mathbf{E}\times\mathbf{B})$$

5. This yields Poynting's Theorem

$$\boxed{\underbrace{\frac{\partial}{\partial t}\int d^3\mathbf{r}\left(\frac{|\mathbf{E}|^2+|\mathbf{B}|^2}{8\pi}\right)}_{\text{Electric and Magnetic Field energy densities}} + \underbrace{\frac{c}{4\pi}\oint d^2\mathbf{S}\cdot(\mathbf{E}\times\mathbf{B})}_{\text{Poynting Flux through surface of volume } \frac{c}{4\pi}(\mathbf{E}\times\mathbf{B})} = \underbrace{-\int d^3\mathbf{r}\ \mathbf{j}\cdot\mathbf{E}}_{\text{Work done by } \mathbf{E} \text{ on plasma}}}$$

C. Conserved Vlasov-Maxwell Energy

1. Multiply the Vlasov Equation by $\frac{1}{2}m_s v^2$ and integrate $\underbrace{\int d^3\mathbf{r}\int d^3\mathbf{v}}_{=\int_{\mathbf{r}}\int_{\mathbf{v}}}$

$$\frac{\partial}{\partial t}\int_{\mathbf{r}}\int_{\mathbf{v}}\left(\frac{1}{2}m_s v^2 f_s\right) + \frac{1}{2}m_s\int_{\mathbf{v}} v^2\mathbf{v}\cdot\underbrace{\left[\int_{\mathbf{r}}\nabla f_s\right]}_{Ⓐ} + q_s\int_{\mathbf{v}}\frac{v^2}{2}\int_{\mathbf{r}}\Big[\underbrace{\mathbf{E}}_{Ⓑ} + \underbrace{\frac{\mathbf{v}}{c}\times\mathbf{B}}_{Ⓒ}\Big]\cdot\frac{\partial f_s}{\partial \mathbf{v}} = 0$$

2. Ⓐ $= \int d^3\underset{\sim}{r}\, \nabla f_s \underset{\uparrow}{=} \oint d\underset{\sim}{S}\, f_s$

Gauss' Theorem

a. Infinite Boundary Conditions: $\lim_{|\underset{\sim}{r}| \to \infty} f_s(\underset{\sim}{r}, \underset{\sim}{v}, t) = 0$

$\Rightarrow \oint d\underset{\sim}{S}\, f_s = 0$

b. Periodic Boundary Conditions: $f_s(x=-L_x, y, z, \underset{\sim}{v}, t) = f_s(x=+L_x, y, z, \underset{\sim}{v}, t)$

Same for $y = \pm L_y$ and $z = \pm L_z$

$$\int_{-L_x}^{L_x} dx \frac{\partial f_s}{\partial x} \hat{\underset{\sim}{x}} = \int_{-L_x}^{L_x} df_s\, \hat{\underset{\sim}{x}} = \left[f_s(x=L_x) - f_s(x=-L_x) \right] \hat{\underset{\sim}{x}} = 0!$$

c. Thus, advective (ballistic) term yields 0 for appropriate BCs.

3. Electric Field: Ⓑ $= \int d^3\underset{\sim}{r}\, q_s \underset{\sim}{E} \cdot \left[\int d^3\underset{\sim}{v}\, \frac{v^2}{2} \frac{\partial f_s}{\partial \underset{\sim}{v}} \right]$

a. $[\quad] = \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z \frac{(v_x^2 + v_y^2 + v_z^2)}{2} \left[\frac{\partial f_s}{\partial v_x} \hat{\underset{\sim}{x}} + \frac{\partial f_s}{\partial v_y} \hat{\underset{\sim}{y}} + \frac{\partial f_s}{\partial v_z} \hat{\underset{\sim}{z}} \right]$

b. $\int_{-\infty}^{\infty} dv_x \frac{v_x^2}{2} \frac{\partial f_s}{\partial v_x} \underset{\text{by parts}}{=} \cancel{\left[\frac{v_x^2}{2} f_s \right]_{-\infty}^{\infty}}^{\,0} - \int_{-\infty}^{\infty} dv_x\, v_x f_s$

$u = \frac{v_x^2}{2}$ $dv = \frac{\partial f_s}{\partial v_x} dv_x$

$du = v_x dv_x$ $v = f_s$

$\rightarrow \lim_{v_x \to \pm\infty} v_x^2 f_s = 0 \leftarrow$ always true!

c. Other terms with v_y^2 and v_z^2 over $\int dv_x$ are perfect differentials $\rightarrow 0$.

d. What remains? Ⓑ $= \int d^3\underset{\sim}{r}\, \underset{\sim}{E} \cdot \left[\int d^3\underset{\sim}{v}\, q_s \underset{\sim}{v} f_s \right] = \int d^3\underset{\sim}{r}\, \underset{\sim}{E} \cdot \underset{\sim}{j}_s$

Species Current density

4. Magnetic Field: (C) $= \int d^3\mathbf{r} \frac{q_s}{c} \int d^3\mathbf{v} \frac{v^2}{2} (\mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}}$

a. Carefully separating components and performing an integration by parts, the velocity integral becomes

$$= \int d^3\mathbf{v} \underbrace{\mathbf{v} \cdot (\mathbf{v} \times \mathbf{B})}_{=0} f_s = 0$$

b. $\boxed{\text{Magnetic Fields do no net work on particles!}}$

5. We are left with

$$\boxed{\frac{\partial}{\partial t} \int d^3\mathbf{r} \int d^3\mathbf{v} \left(\frac{1}{2} m_s v^2 f_s\right) - \int d^3\mathbf{r}\, \mathbf{j}_s \cdot \mathbf{E} = 0}$$

6. Summing this equation over species, where $\mathbf{j} = \sum_s \mathbf{j}_s$, and taking the boundary to infinity in space to eliminate Poynting flux term, we obtain

$$\frac{\partial}{\partial t} \sum_s \int d^3\mathbf{r} \int d^3\mathbf{v} \frac{1}{2} m_s v^2 f_s + \frac{\partial}{\partial t} \int d^3\mathbf{r} \left[\frac{|\mathbf{E}|^2 + |\mathbf{B}|^2}{8\pi}\right] = 0$$

7. Conserved Vlasov-Maxwell Energy:

(I) $$\boxed{\underbrace{\int d^3\mathbf{r} \left[\frac{|\mathbf{E}|^2 + |\mathbf{B}|^2}{8\pi}\right]}_{\text{EM Field Energy}} + \underbrace{\sum_s \int d^3\mathbf{r} \int d^3\mathbf{v} \frac{1}{2} m_s v^2 f_s}_{\text{Particle Kinetic energy}} = 0}$$

$$\boxed{W_s = \int d^3\mathbf{r} \int d^3\mathbf{v} \frac{1}{2} m_s v^2 f_s}$$

D. Constraints on Particle Energization

1. Equation (I) makes clear that any net gain in particle energy must come at the expense of field energy

$\Rightarrow$ $\boxed{\text{collisionless interactions between particles \& fields!}}$

2. We want to determine the change of energy of a particle species s:

Measure change in particle energy...

$$\frac{\partial W_s}{\partial t} = \int d^3\underset{\sim}{r} \int d^3\underset{\sim}{v}\, \frac{1}{2} m_s v^2 \frac{\partial f_s}{\partial t}$$

... using measurements of the change in the particle distribution function!

3. Substitute Vlasov Eq: $\displaystyle \frac{\partial f_s}{\partial t} = -\underset{\sim}{v}\cdot\nabla f_s - \frac{q_s}{m_s}\left[\underset{\sim}{E} + \frac{\underset{\sim}{v}\times\underset{\sim}{B}}{c}\right]\cdot\frac{\partial f_s}{\partial \underset{\sim}{v}}$

$$\frac{\partial W_s}{\partial t} = \int d^3\underset{\sim}{r}\int d^3\underset{\sim}{v}\, \frac{1}{2} m_s v^2 \left[\underbrace{\cancel{-\underset{\sim}{v}\cdot\nabla f_s}}_{\rightarrow 0} - \frac{q_s}{m_s}\underset{\sim}{E}\cdot\frac{\partial f_s}{\partial \underset{\sim}{v}} - \underbrace{\cancel{\frac{q_s}{m_s}\left(\frac{\underset{\sim}{v}\times\underset{\sim}{B}}{c}\right)\cdot\frac{\partial f_s}{\partial \underset{\sim}{v}}}}_{\rightarrow 0}\right]$$

Perfect Differential

Magnetic Field does no work

Electric Field does work on particles, changing their energy

4. Rate of Change of Particle Energy: $\boxed{\displaystyle \frac{\partial W_s}{\partial t} = \int d^3\underset{\sim}{r}\int d^3\underset{\sim}{v}\left[-q_s\frac{v^2}{2}\underset{\sim}{E}(\underset{\sim}{r},t)\cdot\frac{\partial f_s(\underset{\sim}{r},\underset{\sim}{v},t)}{\partial \underset{\sim}{v}}\right]}$

However, with spacecraft data, we cannot integrate over space $\int d^3\underset{\sim}{r}$!

II. The Field-Particle Correlation Technique

A. Energy Flow in 3D-3V Phase Space

1. Def: Phase-space energy density $\boxed{w_s(\mathbf{r},\mathbf{v},t) \equiv \frac{1}{2} m_s v^2 f_s(\mathbf{r},\mathbf{v},t)}$

2. Multiplying Vlasov equation by $\frac{1}{2} m_s v^2$, we obtain

Rate of Change of Phase-Space Energy Density

$$\boxed{\frac{\partial w_s(\mathbf{r},\mathbf{v},t)}{\partial t} = \underbrace{-\mathbf{v}\cdot\nabla w_s}_{\text{Advection of Particle energy}} \underbrace{- q_s \frac{v^2}{2} \mathbf{E}\cdot\frac{\partial f_s}{\partial \mathbf{v}}}_{\text{Work done by Electric Field}} \underbrace{- \frac{q_s}{c}\frac{v^2}{2}(\mathbf{v}\times\mathbf{B})\cdot\frac{\partial f_s}{\partial \mathbf{v}}}_{\text{Work done by Magnetic Field}}}$$

Advection of Particle energy $\Rightarrow$ ~~Pressure forces in~~ fluid theory

Work done by Electric Field $\Downarrow$ Responsible for net change in energy!

Work done by Magnetic Field $\Rightarrow$ Integrates to zero over velocity space

3. Energization

a. Advective (pressure) and magnetic terms can convert particle energy e.g., bulk flow kinetic energy to thermal kinetic energy

b. Only electric field can lead to a net change in particle energy.

4. So, we'll use the term $-q_s \frac{v^2}{2} \mathbf{E}\cdot\frac{\partial f_s}{\partial \mathbf{v}}$ to develop the field-particle correlation technique.

B. Defining the Field-Particle Correlation

1. We aim to correlate the Electric field & distribution function measurement to obtain a velocity-space signature of particle energization.

2. First, separate $\mathbf{E} \cdot \frac{\partial f_s}{\partial \mathbf{v}} = E_{\perp 1} \frac{\partial f_s}{\partial v_{\perp 1}} + E_{\perp 2} \frac{\partial f_s}{\partial v_{\perp 2}} + E_{\parallel} \frac{\partial f_s}{\partial v_{\parallel}}$

   in a magnetic field-aligned coordinate system (FAC)

3. Def: Field-Particle Correlations: At a position $\mathbf{r}_0$

   a. $$\boxed{C_{E_\parallel}(\mathbf{r}_0, \mathbf{v}, t) = C\left(-q_s \frac{v_\parallel^2}{2} \frac{\partial f_s(\mathbf{r}_0, \mathbf{v}, t)}{\partial v_\parallel}, E_\parallel(\mathbf{r}_0, t)\right)}$$

   b. $$\boxed{C_{E_\perp}(\mathbf{r}_0, \mathbf{v}, t) = C\left(-q_s \frac{v_{\perp 1}^2}{2} \frac{\partial f_s(\mathbf{r}_0, \mathbf{v}, t)}{\partial v_{\perp 1}}, E_{\perp 1}(\mathbf{r}_0, t)\right) + C\left(-q_s \frac{v_{\perp 2}^2}{2} \frac{\partial f_s(\mathbf{r}_0, \mathbf{v}, t)}{\partial v_{\perp 2}}, E_{\perp 2}(\mathbf{r}_0, t)\right)}$$

   where $$C(A, B) \equiv \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} dt\, A(t)\, B(t)$$

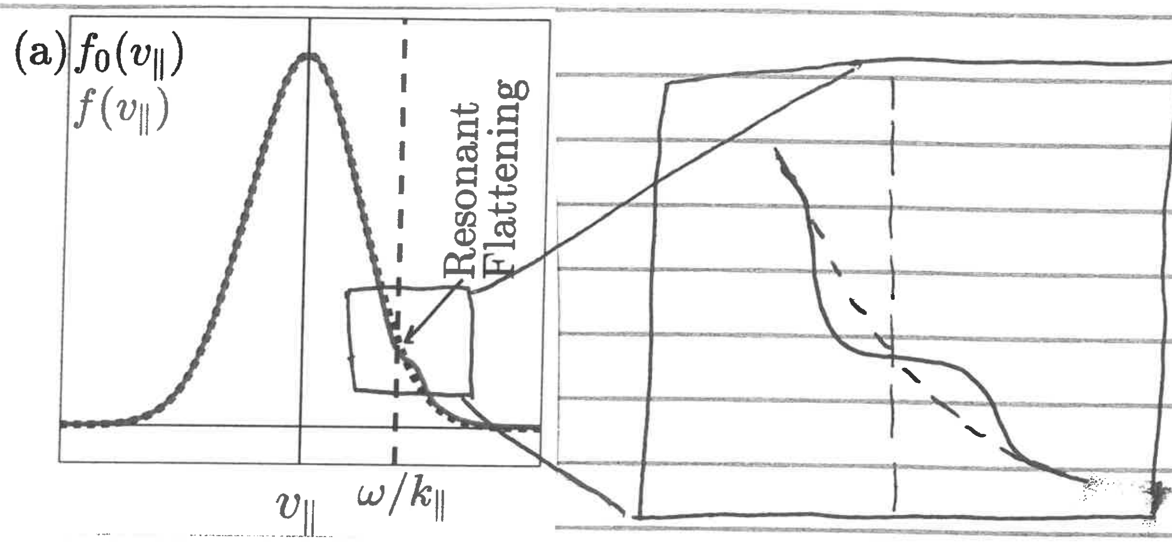
   is the time-average over a correlation interval τ.

4. The time average enables the (often small) secular change in particle energy to be extracted in the presence of (often large) oscillatory energy transfer associated with undamped waves.

III. (Continued)

C. Example: Landau Damping:

1. Consider Landau damping of a Single Kinetic Alfven Wave.

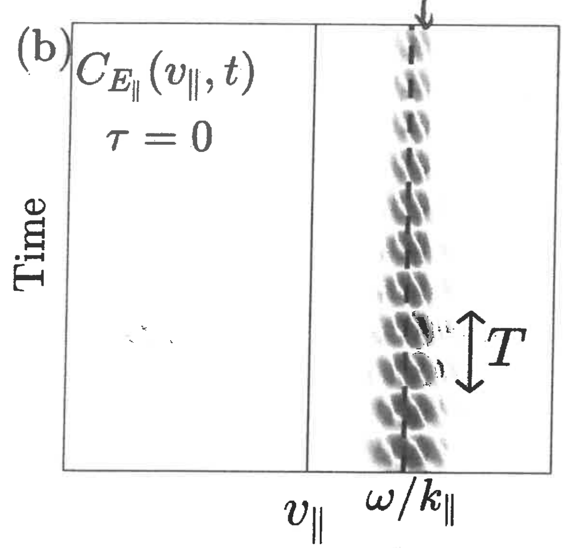


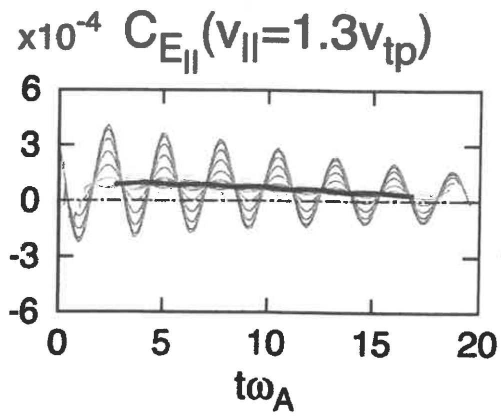
Instantaneous ($\tau=0$) correlation vs. time

At a fixed $v_\parallel$ value, $C_{E_\parallel}$ oscillates in time!

Time Stack plot.

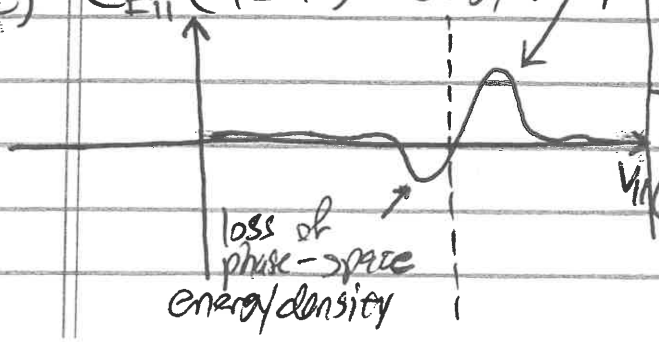
$C_{E_\parallel}(v_\parallel, t)$





c) $C_{E_\parallel}(\tau = T)$

bipolar signature

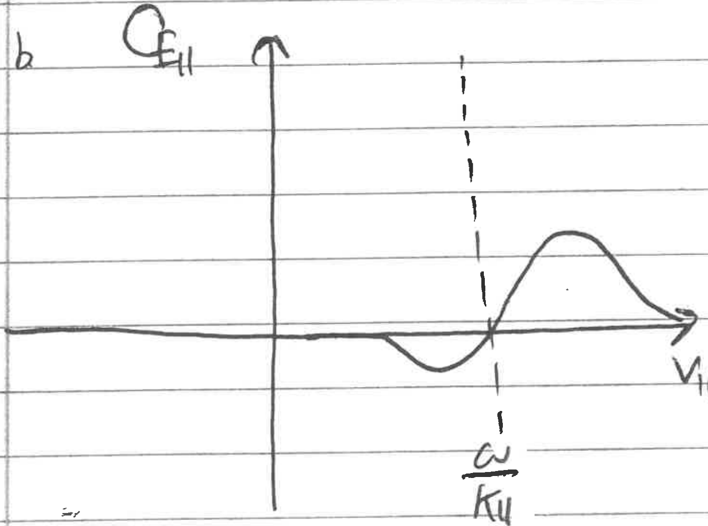


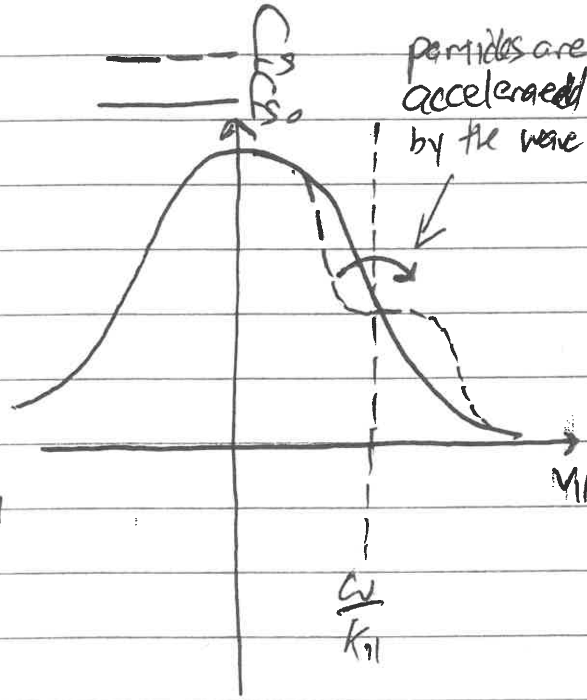
A sufficiently long correlation interval τ eliminates the large oscillation, exposing the non-zero (but small) secular energy transfer.

2. <u>Physical Interpretation of Bipolar Signature of Landau Damping:</u>

a. $W_s \equiv \underbrace{\tfrac{1}{2} m_s v^2}_{\text{coordinate}} \underbrace{f_s(x, \underset{\sim}{v}, t)}_{\text{only } f_s \text{ changes}}$

b.





c. Particles with $v_\parallel < \frac{\omega}{k_\parallel}$ are accelerated by the wave, ending up at a position with $v_\parallel > \frac{\omega}{k_\parallel}$

d. Thus f_s decreases at $v_\parallel < \frac{\omega}{k_\parallel}$

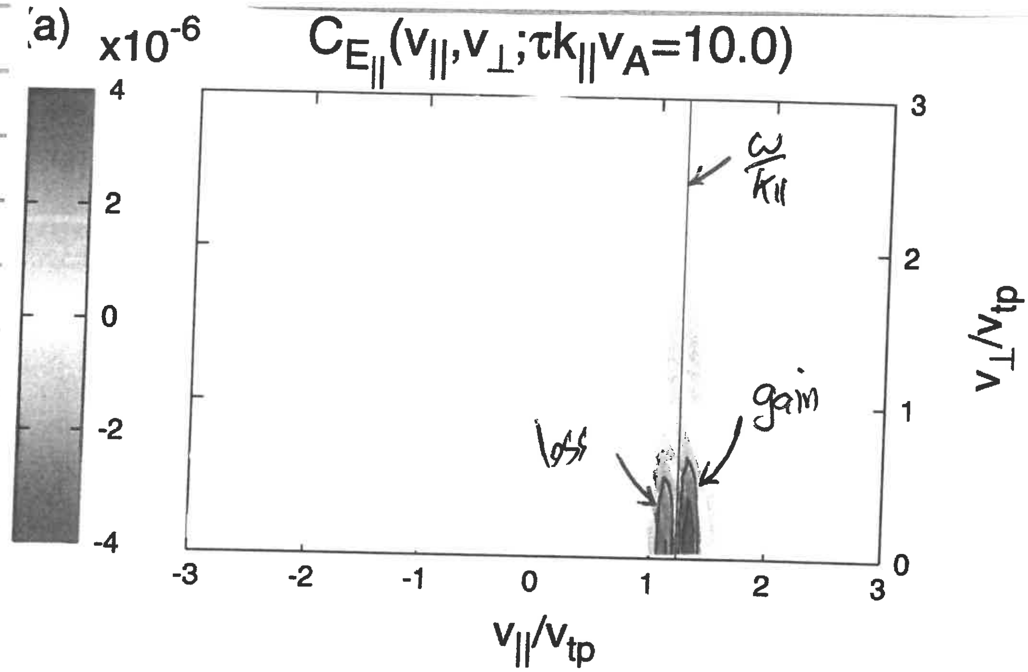
and increases at $v_\parallel > \frac{\omega}{k_\parallel}$

e. $\boxed{\text{Bipolar signature in } C_{E_\parallel}(v_\parallel) \text{ corresponds to quasilinear flattening of } f_s \text{ at } \frac{\omega}{k_\parallel}}$

3. NOTE: $\frac{\partial W_s}{\partial t} = \int d^3\underset{\sim}{v}\, C_{E_\parallel}(\underset{\sim}{v}, t) = \int d^3v \left[-q_s \frac{v_\parallel^2}{2} \frac{\partial f_s}{\partial v_\parallel} E_\parallel \right]$

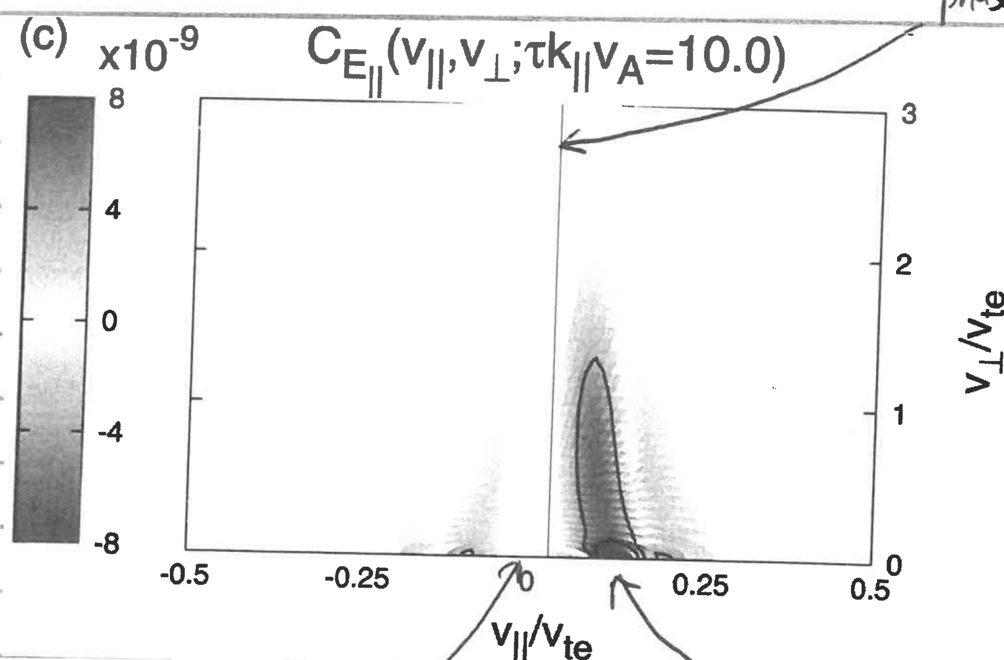
$= \left\{ \int d^3\underset{\sim}{v}\, q_s v_\parallel f_s \right\} E_\parallel = j_{\parallel s} E_\parallel \leftarrow$ Rate of work done by $E_\parallel$ on species s

4. Ex (a) Ion Landau Damping: KAW with $\beta_i = 1$, $\frac{T_i}{T_e} = 1$, $k_\perp \rho_i = 1.3$



b) Electron Landau Damping (Same KAW as above)

phase velocity $\frac{\omega}{k_\parallel} \ll v_{te}$



loss is cut off by $v_\parallel^2$ weighting

gain

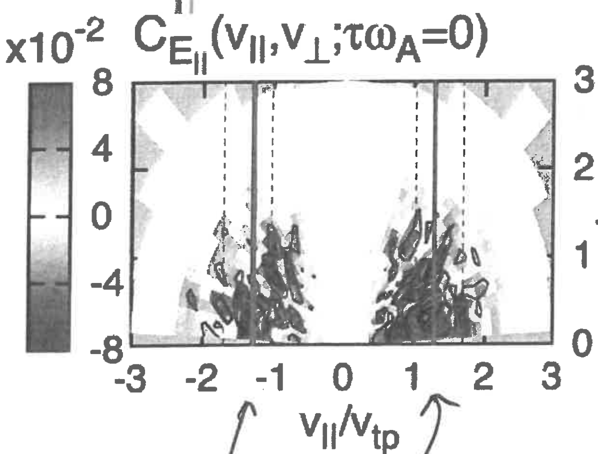
Ref: Howes, PoP 24:055907 (2017)

III. (Continued)

D. Diagnosing Particle Energy in Kinetic Plasma Turbulence

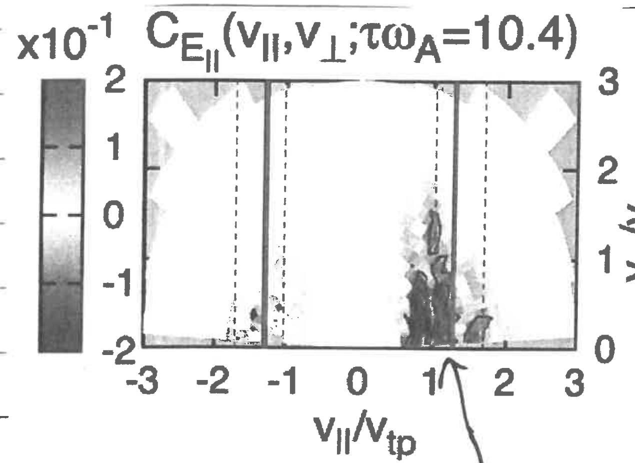
1. From a gyrokinetic simulation of plasma turbulence with $\beta_i = 1$, $T_i/T_e = 1$

Instantaneous ($\tau = 0$)



Total Mess

Sufficiently long τ:



Clear Bipolar Signature.

Ref: Klein, Howes, & Ten Barge (2017)

2. Demonstrates that ion Landau damping plays a role in damping strong plasma turbulence!

E. Kinetic Energization Mechanisms

1. FPC method can be used to generate velocity-space signatures of many different physical processes.
   a. Turbulent Plasma Heating
   b. Particle Acceleration at collisionless Shocks
   c. Magnetic Reconnection
   d. Kinetic Instabilities.

2. Can be used to create a Rosetta Stone for identifying mechanisms of particle energization in space & astrophysical plasmas.