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A dynamical model of plasma turbulence in the solar wind

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A dynamical approach, rather than the usual statistical approach, is taken to explore the physical mechanisms underlying the nonlinear transfer of energy, the damping of the turbulent fluctuations, and the development of coherent structures in kinetic plasma turbulence. It is argued that the linear and nonlinear dynamics of Alfvén waves are responsible, at a very fundamental level, for some of the key qualitative features of plasma turbulence that distinguish it from hydrodynamic turbulence, including the anisotropic cascade of energy and the development of current sheets at small scales. The first dynamical model of kinetic turbulence in the weakly collisional solar wind plasma that combines self-consistently the physics of Alfvén waves with the development of small-scale current sheets is presented and its physical implications are discussed. This model leads to a simplified perspective on the nature of turbulence in a weakly collisional plasma: the nonlinear interactions responsible for the turbulent cascade of energy and the formation of current sheets are essentially fluid in nature, while the collisionless damping of the turbulent fluctuations and the energy injection by kinetic instabilities are essentially kinetic in nature.

1. Introduction

Five centuries ago, Leonardo da Vinci first marvelled at the intricate flow patterns arising in a turbulent cascade of water. But the first quantitative understanding of the physics of hydrodynamic turbulence had to wait until the seminal work by Kolmogorov in the 1940s, and humankind's grasp of hydrodynamic turbulence remains incomplete. To advance the frontiers of our knowledge beyond the limits of our terrestrial environment, we strive to understand the impact of turbulence on the astrophysical plasmas that constitute most of the visible matter in the Universe.

THE ROYAL SOCIETY PUBLISHING

On the face of it, it would appear that unravelling the details of plasma turbulence, with electromagnetic forces in addition to the gas pressure force that occurs in hydrodynamics, would be a much more challenging task than the study of turbulence in water and air. But the study of plasma turbulence is facilitated by one fundamental difference between a hydrodynamic fluid and a plasma: in a magnetized plasma, magnetic tension provides a restoring force that is absent in hydrodynamics. The importance of this distinction cannot be overstated because magnetic tension supports the propagation of a fundamental wave mode, the Alfvén wave, that plays a governing role in plasma turbulence.

The linear Alfvénic response, due to magnetic tension, of the plasma to applied perturbations provides a critical theoretical foothold in the quest to understand the dynamics and energetics of plasma turbulence. Specifically, in the limit of weak nonlinearity,[1] it is possible to derive rigorous asymptotic analytical solutions of the nonlinear dynamics. It is precisely this nonlinear physics that underlies the turbulent cascade of energy from large to small scales, one of the most important impacts of turbulence on astrophysical environments. In practice, such rigorous solutions in the limit of weak nonlinearity can often be pushed to the limits of strong turbulence while remaining at least qualitatively correct, even though such solutions are formally well beyond their regime of applicability.

Lacking this foothold of linear physics, hydrodynamics ruthlessly forces the physicist to resort almost immediately to a statistical approach to describe the turbulent evolution, with the typical focus on the power spectra, the development of intermittency and the scaling of higher order statistics. In the weakly collisional solar wind, researchers at the forefront strive to illuminate the physics of the nonlinear energy transfer to small scales, the kinetic mechanisms of dissipation of the turbulent fluctuations, and the resulting plasma heating. To illuminate these mechanisms, I contend that we must step beyond the usual statistical treatments, adopting instead a dynamical approach, to determine definitively the dominant physical processes at play in the evolution of plasma turbulence. Only with a dynamical understanding of both the nonlinear wave–wave interactions responsible for the turbulent cascade and the collisionless wave–particle interactions responsible for the dissipation will it be possible to construct a predictive theory of plasma turbulence and its effect on energy transport and plasma heating.

A significant fraction of the heliospheric turbulence research community appears to believe, probably based on the analogy with hydrodynamics, that linear physics properties are not relevant to strong plasma turbulence. On the contrary, there are numerous counter-examples in which linear physics properties have been shown to be relevant to strong plasma turbulence [1–11] and in some cases significant insights into the nature of magnetized plasma turbulence have been achieved by the exploitation of intuition from the linear physics properties [12–19]. One particularly significant example is that the qualitative picture of the nonlinear energy transfer in the weak turbulence limit persists in the limit of strong turbulence [20]. In fact, there is a long history of linear wave properties being explicitly or implicitly used to analyse spacecraft measurements and numerical simulations of plasma turbulence [5,7,21–45].

In fact, as will be explained below, the linear and nonlinear dynamics of Alfvén waves are responsible, at a very fundamental level, for some of the key qualitative features of plasma turbulence that distinguish it from hydrodynamic turbulence, including the anisotropic cascade of energy and the development of current sheets at small scales. In the opinion of this author, to attempt a purely statistical investigation of plasma turbulence, analogous to the common approach in hydrodynamic turbulence, without exploiting the plasma physics of the turbulent fluctuations is bound to meet with moderate success at best.

The current frontier of research on plasma turbulence in the weakly collisional solar wind is framed by three fundamental questions:

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[1]Weak nonlinearity means that the magnitude of the nonlinear terms in the equations of evolution are small relative to the magnitude of the linear term that supports the propagation of Alfvén waves.

(i) *What is the physical mechanism underlying the nonlinear transfer of energy from large to small scales*? This question must be answered both for the magnetohydrodynamic (MHD) regime of the inertial range and the kinetic regime of the dissipation range.[2]

(ii) *What are the dominant physical mechanisms responsible for damping the turbulent electromagnetic fluctuations*? Note that which mechanisms dominate may depend on both the plasma parameters and the characteristics of the turbulence, such as the scale and amplitude of the energy injection. These mechanisms determine the partitioning of dissipated turbulent energy into heat, or other energization, of the protons, electrons, and minor ions.

(iii) *How do coherent structures arise from and/or affect both the nonlinear energy transfer and the dissipation mechanism*? The concentration of dissipation in coherent structures, specifically current sheets, is well established by numerical simulations of plasma turbulence and is inferred from spacecraft measurements in the solar wind.

The ultimate goal of the study of space and astrophysical plasma turbulence is to develop a sufficiently detailed understanding of the physics to enable the construction of predictive models of the nonlinear energy transfer, the damping of the turbulent fluctuations and the resulting heating of the plasma species, for example the heating of the solar corona [47]. In addition to establishing a more thorough knowledge of the physics of the heliosphere—our home in the Universe—it will provide a crucial foundation for an improved understanding of complex astrophysical phenomena in remote regions of the cosmos.

In the following section, I present the first dynamical model of kinetic turbulence in the weakly collisional solar wind plasma that combines self-consistently the physics of Alfvén waves with the development of small-scale current sheets. Based on this model, I will address each of the questions above, as well as numerous subsidiary issues, as we follow the turbulent cascade of energy from large to small scales in the solar wind.

2. The dynamics of the turbulent cascade

Incompressible MHD is one of the most simple, self-consistent descriptions of turbulent plasma dynamics. Although incompressible MHD lacks much of the rich physical behaviour possible in the weakly collisional plasma conditions relevant to the solar wind, I argue here that incompressible MHD systems appear to contain the minimum number of physical ingredients necessary to yield the key qualitative features of plasma turbulence that distinguish it from hydrodynamic turbulence, specifically the anisotropic cascade of energy and dissipation dominantly occurring within current sheet structures. These key features persist as more physically complete plasma descriptions are adopted, yet do not occur in hydrodynamic turbulence.

Early research on incompressible MHD turbulence in the 1960s [48,49] suggested that nonlinear interactions between counter-propagating Alfvén waves—or Alfvén wave collisions—support the turbulent cascade of energy from large to small scales. The incompressible MHD equations, expressed here in the symmetrized Elsasser form [50], are

$$\frac{\partial \mathbf{z}^\pm}{\partial t} \mp \mathbf{v}_A \cdot \nabla \mathbf{z}^\pm = -\mathbf{z}^\mp \cdot \nabla \mathbf{z}^\pm - \frac{\nabla P}{\rho_0} \qquad (2.1)$$

and $\nabla \cdot \mathbf{z}^\pm = 0$. Here $\mathbf{v}_A = \mathbf{B}_0/\sqrt{4\pi\rho_0}$ is the Alfvén velocity due to the local mean magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$, where the total magnetic field is $\mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B}$. Note that the direction of the local mean magnetic field plays a key role in the physics of incompressible MHD turbulence [51], and here the coordinate system is chosen so that the unit vector direction $\hat{\mathbf{z}}$ is aligned with the mean-field

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[2]I denote here the MHD regime as $k_\perp \rho_i \ll 1$ and the kinetic regime as $k_\perp \rho_i \gtrsim 1$, although compressible fluctuations in the weakly collisional solar wind require a kinetic treatment, even at the large scales of the inertial range [14,39,46]. Here $k_\perp$ refers to the component of the wavevector perpendicular to the local mean magnetic field $\mathbf{B}_0$ and $\rho_i = v_{ti}/\Omega_i$ is the thermal ion Larmor radius, where the ion thermal velocity is defined by $v_{ti}^2 = 2T_i/m_i$ (with temperature expressed in units of energy) and the ion cyclotron frequency is given by $\Omega_i = q_i B_0/(m_i c)$.

direction. P is total pressure (thermal plus magnetic), ρ_0 is mass density and $\mathbf{z}^{\pm} = \mathbf{u} \pm \delta\mathbf{B}/\sqrt{4\pi\rho_0}$ are the Elsasser fields which represent waves that propagate up or down the mean magnetic field. The $\mathbf{z}^{\mp} \cdot \nabla\mathbf{z}^{\pm}$ term governs the nonlinear interactions between counter-propagating Alfvén waves, denoted *Alfvén wave collisions*.

The strength of the nonlinearity in incompressible MHD turbulence may be characterized by the *nonlinearity parameter*, $\chi \equiv |\mathbf{z}^- \cdot \nabla\mathbf{z}^+|/|\mathbf{v}_A \cdot \nabla\mathbf{z}^+|$, the ratio of the magnitude of the nonlinear term to that of the linear term in (2.1). Weak incompressible MHD turbulence corresponds to the limit $\chi \ll 1$, while a state of strong incompressible MHD turbulence is characterized by $\chi \sim 1$ [52,53].

(a) How is energy transferred to small scales?

Following significant previous studies on weak incompressible MHD turbulence [52,54–56], the nonlinear energy transfer in Alfvén wave collisions has recently been solved analytically in the weakly nonlinear limit, $\chi \ll 1$ [17]. The key physics is illustrated in figure 1 for the interaction between two perpendicularly polarized, counter-propagating Alfvén waves with wavevectors $\mathbf{k}_1^+ = k_\perp \hat{\mathbf{x}} - k_\parallel \hat{\mathbf{z}}$ and $\mathbf{k}_1^- = k_\perp \hat{\mathbf{y}} + k_\parallel \hat{\mathbf{z}}$, denoted by the red circles in figure 1a. In this section, $k_\perp$ and $k_\parallel$ are positive constants, and the components of a particular wavevector are denoted k_x, k_y and k_z. The asymptotic solution is ordered by the small parameter $\epsilon \sim |\mathbf{z}^{\pm}|/v_A \ll 1$. The lowest order nonlinear interaction creates an inherently nonlinear, purely magnetic mode with wavevector $\mathbf{k}_2^{(0)} = k_\perp \hat{\mathbf{x}} + k_\perp \hat{\mathbf{y}}$ (green triangle). The nonlinear evolution is presented in figure 1b, where Elsasser field amplitudes $|\mathbf{z}^{\pm}|$ are represented by Elssaser potentials, $\zeta^{\pm}$ [39], given by $\mathbf{z}^{\pm} = \hat{\mathbf{z}} \times \nabla_\perp \zeta^{\pm}$. The Elsasser potential is normalized by $\zeta_{\mathrm{NL}} \equiv k_\parallel v_A/k_\perp^2$ and time is normalized by the primary Alfvén wave frequency $\omega_0 = k_\parallel v_A$. In figure 1b, the secondary mode (middle green line) has $k_z = 0$ and frequency $\omega = 2\omega_0$, but does not grow secularly in time. At next order, the primary modes then interact with this secondary mode to transfer energy secularly (with an amplitude that grows linearly in time) to two nonlinearly generated Alfvén waves $\mathbf{k}_3^+$ and $\mathbf{k}_3^-$ (blue squares), where $\mathbf{k}_1^+$ transfers energy to an Alfvén wave with $\mathbf{k}_3^+ = 2k_\perp \hat{\mathbf{x}} + k_\perp \hat{\mathbf{y}} - k_\parallel \hat{\mathbf{z}}$, and $\mathbf{k}_1^-$ transfers energy to $\mathbf{k}_3^- = k_\perp \hat{\mathbf{x}} + 2k_\perp \hat{\mathbf{y}} + k_\parallel \hat{\mathbf{z}}$. This process is the fundamental mechanism by which turbulence transfers energy from large to small scales [17].

This fundamental energy transfer mechanism in the weakly nonlinear limit, derived in the limit of incompressible MHD, has been confirmed numerically with gyrokinetic numerical simulations in the MHD regime [18] and verified experimentally in the laboratory [57–59], establishing Alfvén wave collisions as the fundamental building block of astrophysical plasma turbulence. This nonlinear mechanism is likely to be the physics leading to a non-zero third-moment of turbulent magnetic field measurements, a statistical measure often used to estimate the turbulent energy cascade rate [60].

The validation of the incompressible MHD analytical solution for the nonlinear evolution of Alfvén wave collisions with a kinetic numerical simulation has two important implications. First, the properties of the incompressible MHD solution persist even under the weakly collisional plasma conditions relevant to realistic space and astrophysical plasmas. Second, the dynamical behaviour of the turbulent cascade in the MHD regime, $k_\perp \rho_i \ll 1$, in particular the nonlinear energy transfer, is adequately described by the simplified framework of incompressible MHD. This second point highlights that, even in a kinetic plasma, the nonlinear energy transfer is essentially fluid in nature, and can be modelled as a nonlinear wave–wave interaction. On the other hand, the physical mechanisms responsible for damping of the turbulent fluctuations under weakly collisional plasma conditions, as will be argued below, must be essentially kinetic in nature, dominated by wave–particle interactions.

I also note that the physics of Alfvén wave collisions highlights the fact that plasma turbulence is inherently three dimensional [9,17,61]. The linear term in (2.1) governs the propagation of the Alfvén waves along the equilibrium magnetic field and is non-zero only when the parallel wavenumber $k_\parallel \neq 0$, requiring the inclusion of the field-parallel dimension. And the vector
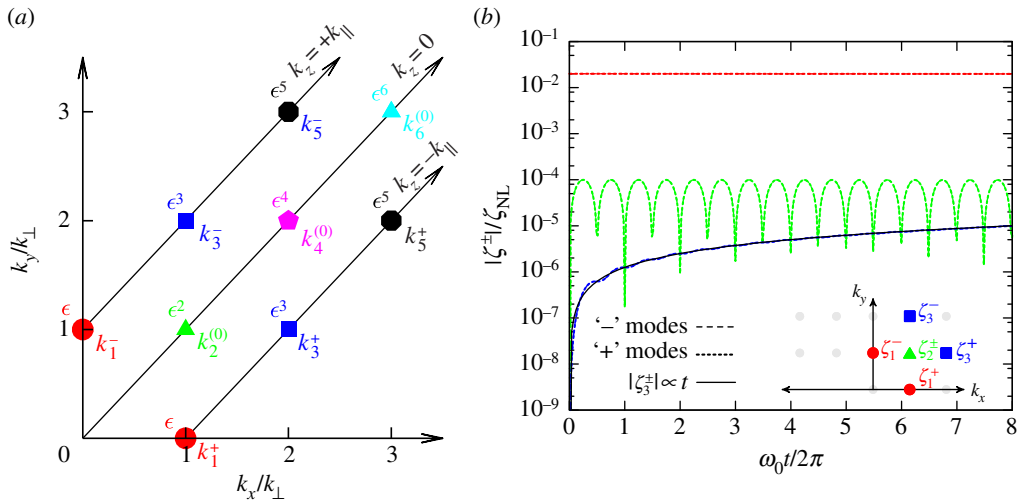
Figure 1. (*a*) Perpendicular Fourier modes nonlinearly generated by an Alfvén wave collision between counter-propagating parent Alfvén waves $\mathbf{k}_1^+$ and $\mathbf{k}_1^-$. (Adapted from Howes [20].) (*b*) In the weakly nonlinear limit, evolution of the normalized amplitude $|\zeta^\pm|/\zeta_{NL}$ of the key Fourier modes involved in the nonlinear energy transfer versus time $\omega_0 t/2\pi$ over eight periods of the primary Alfvén waves. Here $\mathbf{z}^\pm = \hat{\mathbf{z}} \times \nabla_\perp \zeta^\pm$, $\zeta_{NL} \equiv k_\parallel v_A/k_\perp^2$ and $\omega_0 = k_\parallel v_A$. (Adapted from Nielson *et al.* [18].) (Online version in colour.)

nature of the nonlinear term requires variation in both of the directions perpendicular to the magnetic field for the nonlinearity to be non-zero. The manifestly three-dimensional nature of plasma turbulence not only applies to incompressible MHD plasmas but also persists as a general characteristic of the turbulence for more complex plasmas, such as compressible MHD plasmas or kinetic plasmas [9].

(b) What causes the anisotropic cascade in turbulent plasmas?

The anisotropic nature of the nonlinear energy transfer in Alfvénic plasma turbulence has long been recognized in laboratory plasmas [62–64] and in the solar wind [22], as well as in the results of numerical simulations [65,66]. It is observed that energy is preferentially transferred to small scales perpendicular to the local magnetic field direction, leading to anisotropic, small-scale turbulent fluctuations that are elongated along the local magnetic field, characterized by the wavevector anisotropy $k_\parallel \ll k_\perp$.

Why is magnetized plasma turbulence anisotropic? This question can be answered rather simply, in the context of incompressible MHD, through physical intuition derived from the picture of nonlinear energy transfer presented above. Basically, the anisotropy arises as a direct consequence of two facts: (i) only counter-propagating Alfvén waves interact nonlinearly and (ii) the nonlinear energy transfer is maximized for interactions between perpendicularly polarized Alfvén waves.

Consider the nonlinear interaction between two plane Alfvén waves with arbitrary wavevectors $\mathbf{k}_1$ and $\mathbf{k}_2$. We adopt the convention that the wave frequency must be non-negative, $\omega \geq 0$, so that the propagation direction of the wave along the mean magnetic field is given by the sign of the parallel component of the wavevector. The nonlinear interaction between these two modes is non-zero only for counter-propagating Alfvén waves [17,48,49], so the parallel components $k_{\parallel 1}$ and $k_{\parallel 2}$ must be opposite in sign. A mode $\mathbf{k}_3$ receiving energy via the nonlinear interaction between these two modes must satisfy $\mathbf{k}_3 = \mathbf{k}_1 + \mathbf{k}_2$, so its parallel component must have $k_{\parallel 3} \leq k_{\parallel 1}$ and $k_{\parallel 3} \leq k_{\parallel 2}$. The nonlinear term in (2.1) has the form $\mathbf{z}^\mp \cdot \nabla \mathbf{z}^\pm$. The magnitude of this nonlinear term is maximized when the perpendicular components of the wavevectors of the interacting Alfvén waves are orthogonal, $\mathbf{k}_{\perp 1} \cdot \mathbf{k}_{\perp 2} = 0$, so we expect the nonlinear energy

transfer to be dominated by interactions in which the colliding Alfvén waves are perpendicularly polarized [17]. For this case of orthogonal perpendicular wavevector components, $\mathbf{k}_{\perp 1} \cdot \mathbf{k}_{\perp 2} = 0$, the resulting magnitude of the perpendicular component of $\mathbf{k}_3$ will satisfy $k_{\perp 3} \geq k_{\perp 1}$ and $k_{\perp 3} \geq k_{\perp 2}$. In summary, the parallel wavevector component of Alfvénic fluctuations decreases or remains constant due to the fact that only counter-propagating Alfvén waves interact nonlinearly, while the perpendicular wavevector component of the fluctuations generally increases as a result of the dominance of nonlinear interactions between perpendicularly polarized Alfvén waves. Thus, the resulting nonlinear energy transfer is expected to be anisotropic, with energy preferentially flowing towards smaller scales in the perpendicular direction rather than in the parallel direction.

The argument presented above is similar to a previous explanation of anisotropic energy transfer [52,54–56,65,66] which relies on the wavevector $\mathbf{k}_3 = \mathbf{k}_1 + \mathbf{k}_2$ and frequency $\omega_3 = \omega_1 + \omega_2$ matching conditions in the weak turbulence limit for resonant interactions between Alfvén waves with wavevectors $\mathbf{k}_1$ and $\mathbf{k}_2$. Retaining the convention of non-negative frequencies, the linear Alfvén wave frequency is given by $\omega = |k_\parallel| v_A$. Therefore, the parallel component of the wavevector matching condition and the frequency matching condition lead to the equations $k_{\parallel 3} = k_{\parallel 1} + k_{\parallel 2}$ and $|k_{\parallel 3}| = |k_{\parallel 1}| + |k_{\parallel 2}|$. Since only counter-propagating Alfvén waves interact nonlinearly, and thus $k_{\parallel 1}$ and $k_{\parallel 2}$ must be opposite in sign (or zero), these equations have a solution only if either $k_{\parallel 1} = 0$ or $k_{\parallel 2} = 0$. This argument establishes the necessity of modes with $k_\parallel = 0$ in resonant nonlinear interactions in the weak turbulence limit, but elucidates neither the origin nor the nature of such modes. In summary, the parallel wavenumber does not increase through nonlinear interactions in weak incompressible MHD turbulence, while the perpendicular wavenumber generally does increase, yielding an anisotropic transfer of energy to modes with $k_\parallel \ll k_\perp$.

The analytical solution for Alfvén wave collisions in the weakly nonlinear limit presented in §2a can be used to illustrate this previous explanation for the anisotropic energy transfer and to clarify the origin and role of the $k_\parallel = 0$ modes. Using the wavevector notation from §2a, where k_z represents the wavevector component parallel to the local mean magnetic field, the nonlinear interaction between counter-propagating plane Alfvén waves given by wavevectors $\mathbf{k}_1^+$ and $\mathbf{k}_1^-$ may lead to energy transfer to two other Fourier modes $\mathbf{k}_2^{(0)} = \mathbf{k}_1^- + \mathbf{k}_1^+$ and $\mathbf{k}_2^{(+2)} = \mathbf{k}_1^- - \mathbf{k}_1^+$. The detailed analytical solution [17] demonstrates that the resonant interactions that lead to the secular transfer of energy to smaller scales are mediated by a nonlinearly generated mode with $k_z = 0$, so this result singles out $\mathbf{k}_2^{(0)}$ as the key mode mediating the nonlinear energy transfer. For this particular problem, the primary counter-propagating Alfvén waves $\mathbf{k}_1^+$ and $\mathbf{k}_1^-$ each then interact nonlinearly with this self-consistently generated mode $\mathbf{k}_2^{(0)}$ to transfer energy resonantly to Alfvén waves with wavevectors $\mathbf{k}_3^+$ and $\mathbf{k}_3^-$. It is these two subsequent interactions that correspond directly to the previous explanation [52,54–56,65,66] of anisotropic energy transfer in the weak turbulence limit. In the presence of pre-existing energy in $k_z = 0$ modes, this energy transfer corresponds to a dominant three-wave interaction. In the absence of such pre-existing $k_z = 0$ modes, it is the self-consistently generated $\mathbf{k}_2^{(0)}$ mode that mediates the energy transfer. In this case, there is no energy transfer via three-wave interactions, but rather the combined interactions $\mathbf{k}_2^{(0)} = \mathbf{k}_1^- + \mathbf{k}_1^+$ and $\mathbf{k}_3^\pm = \mathbf{k}_1^\pm + \mathbf{k}_2^{(0)}$, together equivalent to a four-wave interaction, constitute the dominant mechanism of nonlinear energy transfer. Note that the mode $\mathbf{k}_2^{(0)}$ has no associated velocity fluctuation, $\mathbf{u} = 0$, but does include a non-zero perpendicular magnetic field fluctuation, $\delta \mathbf{B}_\perp \neq 0$ [17], so this mode has both $\mathbf{z}^+ \neq 0$ and $\mathbf{z}^- \neq 0$, and therefore interacts with Alfvén waves propagating in either direction along the magnetic field. Physically, the mode $\mathbf{k}_2^{(0)}$ can be interpreted as a shear in the magnetic field that the Alfvén waves travelling in either direction attempt to follow.

These arguments demonstrate that, in the weak turbulence limit, $\chi \ll 1$, no parallel cascade occurs, yielding a maximally anisotropic energy transfer only to smaller perpendicular scales. Does the physical picture of energy transfer mediated by self-consistently generated $k_z = 0$ modes persist in the strong turbulence limit, $\chi \sim 1$? Let us consider an Alfvén wave collision

problem equivalent to the weakly nonlinear case presented in §2a, but with the initial Alfvén wave amplitudes increased to yield a strongly nonlinear case with $\chi = 1$. Simulations of Alfvén wave collisions in this strong turbulence limit [20] demonstrate that the energy transfer in perpendicular Fourier space flows primarily to modes along the three diagonal lines represented in figure 1a. Because of the wavevector matching conditions for this particular problem, the parallel wavenumber[3] is constant along these diagonal lines, with values $k_z = +k_\parallel$, $k_z = 0$ and $k_z = -k_\parallel$, while the perpendicular wavenumber increases with the distance from the origin of the plot. Thus, the energy transfer is anisotropic in this strongly nonlinear case, consistent with previous findings of anisotropic energy transfer in strong plasma turbulence. Of course, in contrast to the weakly nonlinear case where the energy transfer is dominated by interactions involving just the modes $\mathbf{k}_1^\pm$, $\mathbf{k}_2^{(0)}$ and $\mathbf{k}_3^\pm$, the strongly nonlinear limit involves significant contributions from higher order terms in the asymptotic expansion. These terms lead to significant nonlinear energy transfer involving many other three-wave couplings, ultimately leading to a parallel transfer of energy to modes with $|k_z| > k_\parallel$, but at a rate slower than the cascade of energy to higher perpendicular wavenumber. The observed channelling of energy along the three diagonal lines in figure 1a suggests that $k_z = 0$ modes continue to play a key role in mediating the nonlinear energy transfer in strong turbulence. It is important to note that, for an Alfvén wave collision with $\chi \sim 1$, the self-consistently generated $k_z = 0$ modes rise to amplitudes with the same order of magnitude as the primary Alfvén waves, so pre-existing energy in $k_z = 0$ modes is unnecessary to yield a strong cascade of turbulent energy.

In summary, the anisotropic cascade of energy in incompressible MHD turbulence is a consequence of the two facts that only counter-propagating Alfvén waves interact nonlinearly and that the nonlinear term is greatest for interactions between perpendicularly polarized Alfvén waves. Note that this explanation of the anisotropy in terms of the linear and nonlinear properties of the turbulent fluctuations differs substantially from a statistical argument in this theme issue [67] relying on the coupling of third-order to second-order correlations in the turbulence. It is an open question why the property of anisotropic energy transfer appears to persist under less restrictive plasma conditions than incompressible MHD (in particular for the dispersive modes at scales below the ion Larmor radius) which allow for co-propagating modes to interact nonlinearly. An analytical solution of the nonlinear energy transfer in the dispersive regime of kinetic Alfvén waves may help to answer this open question.

(c) What is low-frequency turbulence in a weakly collisional plasma?

As a relevant aside, it is worthwhile considering the nature of low-frequency[4] turbulence in a weakly collisional plasma. Turbulence arises as a result of nonlinearities in the equations of evolution for the plasma. The Boltzmann equation describes the evolution of the six-dimensional distribution function $f_s(\mathbf{r}, \mathbf{v}, t)$ for a plasma species s,

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \nabla f_s + \frac{q_s}{m_s}\left[\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c}\right] \cdot \frac{\partial f_s}{\partial \mathbf{v}} = \left(\frac{\partial f_s}{\partial t}\right)_{\text{coll}}. \tag{2.2}$$

A Boltzmann equation for each plasma species together with Maxwell's equations form the closed set of Maxwell–Boltzmann equations governing the dynamics of a kinetic plasma. Since Maxwell's equations are linear, the only nonlinearities appearing in the Maxwell–Boltzmann equations occur in the electromagnetic Lorentz force term and the collisional term in (2.2). Under the weakly collisional conditions relevant to the solar wind, the collisional term is subdominant, and thus the physics of solar wind turbulence is controlled by the electromagnetic Lorentz force on the particles.

The charge density and current density give rise to the electromagnetic fields through Maxwell's equations, and, therefore, the fields are entirely determined by the two lowest-order

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[3]Recall that, in this notation, $k_\parallel$ is a positive constant describing the initial counter-propagating Alfvén waves.

[4]Low frequency here refers to frequencies much smaller than the electron plasma frequency, $\omega \ll \omega_{\text{pe}}$.

moments of the ion and electron distribution functions. Thus, the forces in weakly collisional plasma turbulence are essentially fluid in nature, since they depend on only the low-order moments. For the non-relativistic conditions of solar wind turbulence, the plasma fluctuations are quasi-neutral, with typical charge density fluctuations, $\sum_s q_s \delta n_s$, normalized by the total ion charge density, $q_i n_{0i}$, having an order of magnitude $\sum_s q_s \delta n_s / q_i n_{0i} \sim \mathcal{O}(v_A^2/c^2) \ll 1$ [68]. Therefore, with negligible charge density fluctuations, the electromagnetic fields mediating the low-frequency turbulence in the solar wind are determined entirely by the current density, given by the first moments of the ion and electron distribution functions.

The energy in a volume of plasma is given by

$$W = \int d^3\mathbf{r} \left(\frac{|\mathbf{E}|^2 + |\mathbf{B}|^2}{8\pi} + \sum_s \int d^3\mathbf{v} \frac{1}{2} m v^2 f_s \right), \tag{2.3}$$

the sum of the electromagnetic field energy and kinetic energy of the plasma particles [69]. Note that the kinetic energy density of a plasma species is given by $\varepsilon_s = \int d^3\mathbf{v} \frac{1}{2} m v^2 f_s$, and we may define a 'kinetic' temperature T_{ks} (expressed in units of energy) for a monatomic plasma species s in three spatial dimensions by

$$T_{ks} \equiv \frac{2}{3n_s} \left(\varepsilon_s - \frac{1}{2} n_s m_s |\mathbf{U}_s|^2 \right), \tag{2.4}$$

where $\mathbf{U}_s$ is the bulk fluid velocity of species s, given by the first moment of the distribution function. Therefore, the energy in a kinetic plasma may be expressed by

$$W = \int d^3\mathbf{r} \left[\frac{|\mathbf{E}|^2 + |\mathbf{B}|^2}{8\pi} + \sum_s \left(\frac{1}{2} n_s m_s |\mathbf{U}_s|^2 + \frac{3}{2} n_s T_{ks} \right) \right]. \tag{2.5}$$

Note that, if the distribution function for species s is Maxwellian (corresponding to a state of local thermodynamic equilibrium), the kinetic temperature is equal to the thermodynamic temperature of that species, $T_{ks} = T_s$. In this case, the last term in (2.5) corresponds entirely to the thermal energy. Energy cannot be extracted from a thermal distribution without lowering the entropy, so this energy is thermodynamically inaccessible. But, if the distribution function deviates from a Maxwellian, as will generally be the case for a turbulent weakly collisional plasma, then the last term contains not only thermal energy but also non-thermal free energy associated with the deviations from a Maxwellian distribution. These deviations generally have the form of small-scale structure in velocity space. In summary, the first term in (2.5) represents the electromagnetic fluctuation energy, the second term represents the kinetic energy of the bulk fluid velocity, and the third term includes both non-thermal free energy in the distribution function as well as thermal energy.

The energy given in (2.5) is conserved by the Maxwell–Boltzmann equations independent of collisionality. It is worthwhile considering the various physical mechanisms that can lead to the transfer of energy among the different terms in (2.5). The low-frequency linear wave response of the kinetic plasma leads to the continual exchange of energy between the electromagnetic energy and bulk kinetic energy—the magnetic tension in Alfvén waves, for example, leads to the periodic transfer of energy between the magnetic energy and the kinetic energy of the bulk plasma motion, an exchange that is purely fluid in nature. Collisionless wave–particle interactions, such as Landau damping, lead to the transfer of energy from the electromagnetic fields into the non-thermal free energy in the distribution function. Conversely, unstable distribution functions can lead to the transfer of energy from the non-thermal free energy in the velocity distributions to electromagnetic fluctuations via collisionless wave–particle interactions, such as kinetic temperature anisotropy instabilities [70]. Both of these physical mechanisms of energy transfer by wave–particle interactions are essentially kinetic in nature. In addition to conserving energy, all of the processes mentioned above also conserve entropy since the collisionless Boltzmann equation (or the Vlasov equation) conserves entropy. Collisions are the final physical mechanism that converts non-thermal free energy in the distribution function to thermal energy,

a process that increases the entropy of the plasma, thereby realizing irreversible thermodynamic heating of the plasma. Collisions, of course, do not transfer energy between terms in (2.5), but merely convert non-thermal to thermal energy within the last term in the equation.

Based on these considerations, here I suggest a practical definition for what constitutes the turbulence in a kinetic plasma. The turbulence is represented by the first moments of the distribution functions, specifically the electromagnetic fluctuations (determined by the current density) and the bulk fluid velocities of the plasma species, corresponding to the first two terms in (2.5). The nonlinear interactions underlying the turbulent cascade of energy from large to small scales are mediated by the electromagnetic fields, and since these fields depend only on the first moments of the ion and electron distribution functions, the nonlinear dynamics of the turbulence is therefore fluid in nature. Particle motions contributing to the third term, containing both non-thermal and thermal energy in the distribution function, are not part of the turbulence by this definition. Collisionless wave–particle interactions transfer energy from the turbulent electromagnetic and bulk velocity fluctuations (the first two terms in (2.5)) into non-thermal free energy in the plasma species distribution functions (the third term). This process damps the turbulence, increasing the 'kinetic' temperature, but does not lead to thermodynamic heating of the plasma (an increase of the thermal temperature). The physical mechanisms that damp the turbulent fluctuations are therefore kinetic in nature. Collisions then act to smooth out the non-thermal fluctuations in the velocity distribution, irreversibly converting the non-thermal free energy in the distribution function to thermal energy. Thus, the dissipation of turbulence in a weakly collisional plasma is necessarily a two-step process, whereby first entropy-conserving collisionless wave–particle interactions damp the turbulent electromagnetic and bulk velocity fluctuations, and then entropy-increasing collisions thermalize the energy removed from the turbulent fluctuations. Other forms of particle energization, such as the development of beams or high-energy tails in the velocity distribution, may also occur via collisionless wave–particle interactions, and correspond to the transfer of energy from the electromagnetic fields to non-thermal free energy in the distribution functions, a transfer that is also essentially kinetic in nature.

(d) Why are parametric instabilities subdominant?

The inherently anisotropic nature of plasma turbulence, with turbulent fluctuations that satisfy $k_\parallel \ll k_\perp$, also leads to the prediction that the nonlinearity associated with Alfvén wave collisions dominates over other potential nonlinear mechanisms driven by gradients parallel to the magnetic field, in particular parametric instabilities, such as the decay [71–83], modulational [77,81,84–88] and beat [81,87] instabilities. For example, even for oblique Alfvén wave modes, numerical studies using both MHD [89,90] and kinetic ion/fluid electron hybrid [91] simulations find parametric instability growth rates that are proportional to $k_\parallel = k \cos \theta$, where θ is the angle between the wavevector and the magnetic field. The root of this property is that the nonlinearity associated with parametric instabilities is proportional to $k_\parallel \delta v$, whereas the nonlinearity associated with Alfvén wave collisions is proportional to $k_\perp \delta v$. Therefore, the relative magnitude of the effect of parametric instabilities to that of Alfvén wave collisions is $k_\parallel/k_\perp \ll 1$, so parametric instabilities are expected to be subdominant in anisotropic plasma turbulence [17]. A more thorough investigation is warranted to confirm this prediction.

(e) Why is critical balance natural for turbulent plasmas?

The nonlinearity parameter, $\chi \equiv |\mathbf{z}^- \cdot \nabla \mathbf{z}^+|/|\mathbf{v}_A \cdot \nabla \mathbf{z}^+|$, measures the strength of incompressible MHD turbulence using the ratio of the magnitude of the nonlinear term to that of the linear term in (2.1). In the weak turbulence limit, $\chi \ll 1$, the dynamics may be rigorously calculated analytically using perturbation theory [17,55,56,92], with the results of such a calculation described above in §2a. Strong incompressible MHD turbulence is conjectured to maintain a state of critical balance [53] in which the nonlinear and linear terms are of equal magnitude, or

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$\chi \sim 1$. Although the concept of critical balance is not universally accepted, a significant number of studies employing various techniques to estimate the variation along the *local* mean magnetic field direction support critical balance [1,51,93–98], while contradictory studies have uniformly used a global mean field [99–101], an approach that has been shown to give misleading results [51]. Since the linear terms are equal in magnitude to the nonlinear terms in critically balanced strong turbulence, the linear plasma physics continues to have an important impact on the dynamics in magnetized plasma turbulence, unlike hydrodynamic turbulence. An important consequence of this fact is that the turbulent fluctuations may exhibit some linear mode properties, properties which may be exploited to illuminate the nature of plasma turbulence [14,102]. The relevance of linear physics to plasma turbulence is responsible for the numerical finding that the picture of energy transfer to smaller scales presented in §2*a*, derived rigorously in the weakly nonlinear limit [17], remains qualitatively correct, even in the limit of strong, critically balanced turbulence, well beyond the formal regime of applicability of the solution [20].

It is worthwhile explaining physically why critical balance—equivalently described by a state in which the characteristic linear time scale $\tau \sim l_\parallel / v_A$ and nonlinear time scale $\tau_{nl} \sim l_\perp / \delta v_\perp$ are in balance—naturally arises through the dynamics of a turbulent plasma. In these variables, $\chi = (l_\parallel \delta v_\perp)/(l_\perp v_A)$. Consider an MHD plasma, initially at rest, that is shaken transverse to the magnetic field at one position with a sinusoidal variation of velocity of amplitude $\delta v_\perp$ over a distance $l_\perp$. The time scale associated with this perturbation of the plasma (and of the frozen-in magnetic field) is $\tau \sim l_\perp / \delta v_\perp$. Magnetic tension will lead to the propagation of Alfvén waves up and down the magnetic field, driven by the applied perturbation. One may ask, for the period of forcing τ, what is the parallel wavelength of the resulting waves? The parallel wavelength will be given by $l_\parallel = v_A \tau = v_A (l_\perp / \delta v_\perp)$, so this leads to the relation $l_\parallel / v_A = l_\perp / \delta v_\perp$ or $\chi = 1$. If the transverse perturbation of the plasma is due to the nonlinear terms in (2.1) such that $\tau_{nl} \sim l_\perp / \delta v_\perp$, then this leads to the relation between the linear and nonlinear time scales, $\tau \sim \tau_{nl}$. Alternatively, critical balance can be expressed as a balance between frequencies $\omega \sim \omega_{nl}$ or $k_\parallel v_A \sim k_\perp \delta v_\perp$.

One may wonder, why is the limit $\chi \gg 1$ not considered? In this case, the nonlinear term is much larger than the linear term, which occurs when $l_\parallel \to \infty$, or $k_\parallel \to 0$, the two-dimensional limit where fluctuations perpendicular to the magnetic field are correlated for long distances along the magnetic field. But, transverse fluctuations in a turbulent plasma will only remain correlated along a magnetic field if they can exchange information along the field. Since information propagates along the magnetic field at the Alfvén speed, for a transverse oscillation with a period $\tau \sim l_\perp / \delta v_\perp$, the distance along the magnetic field over which the transverse motions can remain correlated must have $l_\parallel \le v_A \tau = v_A (l_\perp / \delta v_\perp)$. Therefore, one obtains the constraint $l_\parallel / v_A \le l_\perp / \delta v_\perp$, or in terms of the nonlinearity parameter, $\chi \le 1$. Therefore, fluctuations with $\chi \gg 1$ will decorrelate and evolve towards a state with $\chi \le 1$. Thus, the turbulence cannot be any more two dimensional than allowed by the critical balance because fluctuations in any two planes perpendicular to the mean field can only remain correlated if an Alfvén wave can propagate between them in less than their perpendicular decorrelation time [1,39].

In closing, it is worthwhile pointing out the common misconception that plasma turbulence in the solar wind must be weak because the turbulent magnetic field fluctuations δB are small compared with the mean magnetic field magnitude B_0, or $\delta B / B_0 \ll 1$. In fact, the strength of the turbulence, estimated by the nonlinearity parameter $\chi \sim k_\perp \delta B_\perp / (k_\parallel B_0)$ as the ratio of the nonlinear to the linear term in (2.1), depends not only on the normalized amplitude of the fluctuations, $\delta B / B_0$ but also on the anisotropy of the fluctuations, $k_\perp / k_\parallel$. Direct multi-spacecraft measurements of turbulent magnetic field fluctuations at the small-scale end of the inertial range find $k_\perp / k_\parallel \gtrsim 10$ [7,103,104], so even the small amplitude magnetic field fluctuations measured at that scale, typically of magnitude $\delta B / B_0 \sim 0.1$, lead to strong nonlinear interactions.

(f) What is the nature of the compressible turbulent fluctuations?

In addition to the incompressible dynamics of Alfvén waves that dominate turbulence in the inertial range of magnetized plasmas, plasma turbulence also contains a small fraction (typically

10% or less) of energy in compressible fluctuations [105]. Recent high-frequency measurements of the density fluctuations are presented elsewhere in this theme issue [106]. Exploiting the properties of compressible fluctuations in the inertial range using linear kinetic theory, the nature of the compressible fluctuations may be identified by analysing the correlation between the density and parallel magnetic field fluctuations, $C(\delta n, \delta B_\parallel)$. A recent comparison of $C(\delta n, \delta B_\parallel)$ between 10 years of WIND spacecraft data at 1 AU and theoretical predictions using the synthetic spacecraft data method [14] has shown that there is negligible energy in the kinetic counterpart of the fast magnetosonic mode, and that the observed spectrum of compressible energy is composed entirely of kinetic slow mode[5] fluctuations [13]. This lack of energy in kinetic fast waves remains unexplained, but may be due to shock dissipation of fast wave energy in the inner heliosphere or to little generation of fast wave energy in the coronal plasma that is launched out into the solar wind. In the limit of significant anisotropy $k_\parallel \ll k_\perp$, it has been shown theoretically [17,39,61] and numerically [1] that the turbulent slow mode fluctuations (or pseudo-Alfvén waves in the case of incompressible MHD) are cascaded passively by the Alfvénic fluctuations, but that the slow waves do not impact the cascade of Alfvén waves. Therefore, the turbulent fluctuations in the solar wind inertial range consist of an active cascade of incompressible, counter-propagating Alfvén waves and a passive cascade of compressible slow wave fluctuations.

## (g) How does temperature anisotropy arise from or affect turbulence?

Recent numerical [107] and theoretical [108] results suggest that the compressible fluctuations in turbulence may control the observed spread of proton temperature anisotropy $T_{\perp i}/T_{\parallel i}$ in the solar wind plasma. The observations of $T_{\perp i}/T_{\parallel i}$ in the solar wind are distributed across the broad range of values between the kinetic instability boundaries due to the mirror instability at $T_{\perp i}/T_{\parallel i} > 1$ and the oblique firehose instability at $T_{\perp i}/T_{\parallel i} < 1$ [109–115]. Although a wide range of physical mechanisms affect $T_{\perp i}/T_{\parallel i}$—including collisions, ion parallel heat flux, spherical expansion, radial compression and expansion, turbulent heating, turbulent compression and kinetic instabilities— theoretical considerations show that kinetic slow mode fluctuations may be responsible for the observed spread of $T_{\perp i}/T_{\parallel i}$ values [108], and recent kinetic numerical simulations are consistent with this explanation [107].

In addition to the proton temperature anisotropy being affected by the turbulence, when $T_{\perp i}/T_{\parallel i}$ exceeds certain threshold values, kinetic temperature anisotropy instabilities can generate fluctuations[6] at characteristic ion kinetic length scales, thereby injecting energy into the turbulent plasma, as reviewed elsewhere in this theme issue [70]. Here I suggest that the fluctuations driven by such kinetic instabilities are distinct from the main body of turbulent fluctuations associated with the cascade of energy from large to small scales. The distinction is that the instability-driven fluctuations inhabit a completely different region of wavevector space from the fluctuations arising from the turbulent cascade (shown in figure 2), although to distinguish them from single-point spacecraft measurements requires careful analysis.

There exist four ion temperature anisotropy instabilities that may serve to inject energy into the turbulent plasma: the parallel (or whistler) firehose instability [116,117], the Alfvén (or oblique) firehose instability [118], the mirror instability [119–121] and the proton cyclotron instability [117]. For each of these instabilities, unstable wave growth peaks at $k_\parallel d_i \sim 1$, where $d_i = c/\omega_{pi} = v_A/\Omega_i$ is the ion inertial length. For the parallel firehose and proton cyclotron instabilities, these waves have $k_\perp d_i \ll 1$, and for the Alfvén firehose and mirror instabilities, they have $k_\perp d_i \sim 1$. Therefore, these instabilities inhabit a combined region with $k_\parallel d_i \sim 1$ and $k_\perp d_i \lesssim 1$, with a typical wavevector anisotropy of $k_\parallel/k_\perp \gtrsim 1$. By contrast, at the small scales around the ion inertial length, turbulent

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[5]Future work to confirm this finding should address the issue of whether modes driven by the kinetic mirror instability are also consistent with the WIND observations, but the ubiquity of the compressible fluctuations measured suggests that they are not restricted to plasma intervals with the temperature anisotropy $T_{\perp p}/T_{\parallel p} > 1$ necessary for the mirror instability to generate fluctuations.

[6]In addition to temperature anisotropy instabilities, other kinetic instabilities, such as those driven by beams or the relative drift between different plasma species, can also give rise to electromagnetic fluctuations.
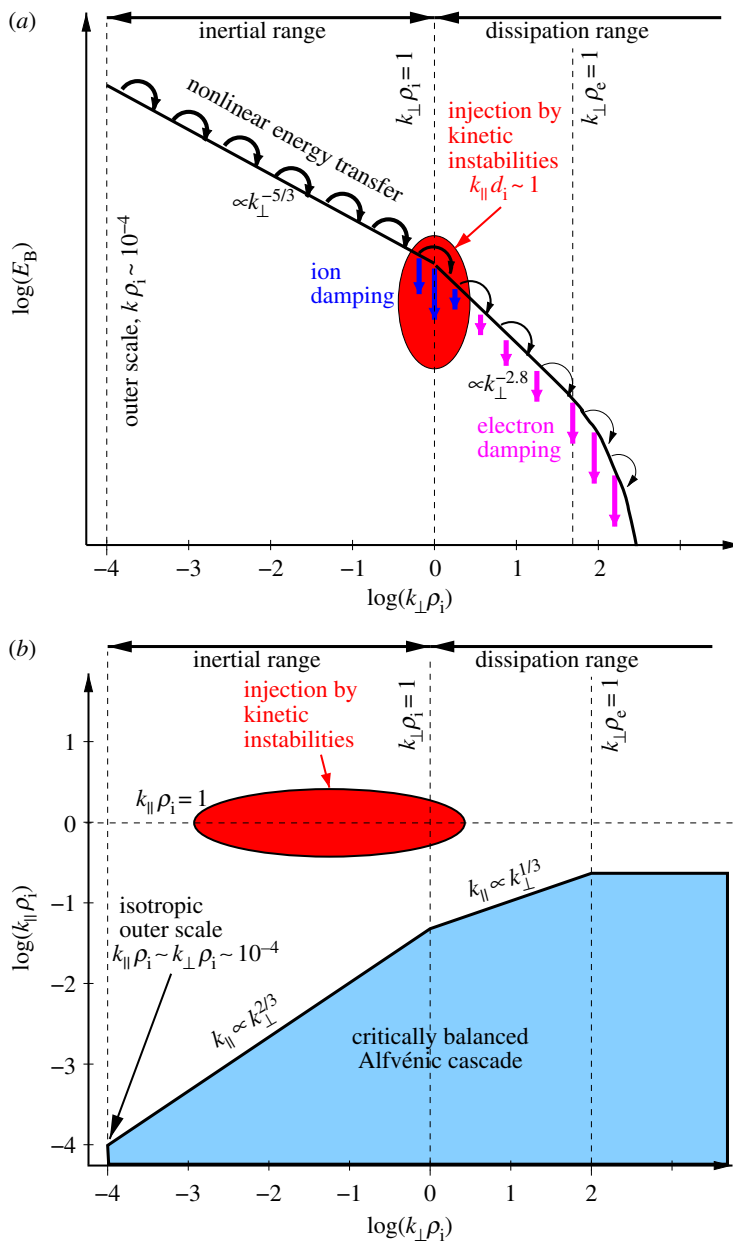
Figure 2. (a) Diagram of the magnetic energy spectrum in plasma turbulence, including the injection of energy by kinetic instabilities. (b) Anisotropic distribution of power in $(k_\perp, k_\parallel)$ wavevector space due to both the cascade of energy from large scales (shaded) and the injection of energy by kinetic instabilities (ellipse). (Online version in colour.)

fluctuations in a magnetic plasma have a typical wavevector anisotropy $k_\parallel/k_\perp \ll 1$ [5,39,53,122]. Therefore, the instability-driven fluctuations do not occupy the same region of wavevector space as the fluctuations of the turbulent cascade from large scales.

Note that kinetic instabilities of the electrons can also inject fluctuation energy into the solar wind at yet smaller scales, and recent work has indeed identified the signature of localized whistler wavepackets at scales $kd_i \gg 1$ which appear to be driven by the whistler heat flux instability [123]. These whistler modes have quasi-parallel wavevectors with $k_\parallel \gg k_\perp$, and,

therefore, also inhabit a region of wavevector space distinct from the anisotropic cascade of energy from large scales.

Discriminating fluctuations with $k_\parallel/k_\perp \gtrsim 1$ from those with $k_\parallel/k_\perp \ll 1$, however, is not generally possible using single-point spacecraft measurements. Such measurements only provide the projection of the wavevector along the solar wind velocity, so there is insufficient information to separate the parallel from the perpendicular component of the wavevector. Only when the magnetic field direction is parallel to the solar wind flow, a relatively rare occurrence in the near-Earth solar wind, can one uniquely determine the value of $k_\parallel$. Measurements of the fluctuating magnetic helicity σ_m as a function of the angle θ between the solar wind velocity $\mathbf{v}_{sw}$ and $\mathbf{B_0}$ indeed have discovered two distinct signatures in regions perpendicular and parallel to $\mathbf{B_0}$ at frequencies $f \sim 1\,\text{Hz}$ [124,125], corresponding to $kd_i \sim 1$. It has been suggested that the perpendicular signature arises from kinetic Alfvén waves with $k_\parallel \ll k_\perp$, while the parallel signature arises from either ion cyclotron waves propagating away from the Sun or whistler waves propagating towards the Sun with $k_\parallel \gg k_\perp$ [124–126]. An analysis comparing predictions from the synthetic spacecraft data method [14] with observations confirms that such a two-component model—consisting primarily of kinetic Alfvén waves with $k_\parallel \ll k_\perp$ with a small amount (5% in energy) of either ion cyclotron or whistler waves with $k_\parallel \gg k_\perp$—indeed reproduces the observations, supporting the hypothesis that kinetic-instability-driven waves persist in the solar wind alongside the Alfvénic turbulent fluctuations that mediate the nonlinear transfer of energy from large to small scales [19].

Are these instability-driven parallel waves turbulent themselves (do they transfer energy nonlinearly to small scales? [70]), or do they merely persist in the turbulent environment caused by the anisotropic Alfvénic cascade of energy from large to small scales? That the parallel waves have small amplitude and are propagating unidirectionally suggests they do not interact nonlinearly among themselves [19]. In addition, since the Alfvénic wave frequencies are linearly proportional to the parallel wavenumber, $\omega \propto k_\parallel$, the instability-driven waves with $k_\parallel d_i \sim 1$ have much higher frequencies than the anisotropic fluctuations of the turbulent cascade that have $k_\parallel d_i \ll 1$. Here I suggest that this frequency mismatch impedes significant nonlinear coupling of the parallel waves to the turbulent fluctuations, so the parallel waves are not turbulent but merely persist alongside the anisotropic Alfvénic turbulence. Therefore, the kinetic instabilities and their resulting fluctuations, distinguished observationally by the parallel magnetic helicity signature, operate separately from the turbulent cascade of energy from large to small scales.

(h) How are coherent structures generated in plasma turbulence?

Numerical simulations of plasma turbulence demonstrate the ubiquitous development of coherent structures, specifically current sheets, at small scales [1,127–131], and dissipation is largely concentrated in these current sheets [132–136]. There has been a flurry of activity recently seeking evidence for such localized heating through statistical analyses of solar wind observations [135,137–143]. But one fundamental question remains unanswered: what mechanism governs the generation of current sheets in plasma turbulence? New research suggests that current sheets arise naturally as a consequence of Alfvén wave collisions in the strong turbulence limit [20].

Consider first the physics of the Alfvén wave collision problem described in §2a in the weak turbulence limit, $\chi \ll 1$. For initial Alfvén waves that have equal amplitudes given by the constants $z_+ = z_-$, the nonlinearity parameter is $\chi = k_\perp z_\pm/(k_\parallel v_A)$. An asymptotic expansion of the nonlinear evolution in the weakly nonlinearly limit can be performed, ordered by the small expansion parameter $\epsilon = z_\pm/v_A \ll 1$ [17]. In this limit, the physics of the nonlinear energy transfer is dominated by just five modes, as depicted in figure 1a: the primary Alfvén waves $\mathbf{k}_1^\pm$ at order ϵ (red circles), the secondary mode $\mathbf{k}_2^{(0)}$ at order ϵ^2 (green triangles) and the tertiary Alfvén waves $\mathbf{k}_3^\pm$ at order ϵ^3 (blue squares). The secular transfer of energy to small scales is accomplished by just three nonlinear interactions: the interaction $\mathbf{k}_2^{(0)} = \mathbf{k}_1^- + \mathbf{k}_1^+$ at order ϵ^2 and

the two interactions $\mathbf{k}_3^{\pm} = \mathbf{k}_1^{\pm} + \mathbf{k}_2^{(0)}$ at order ϵ^3. These nonlinear interactions, governed by the form of the nonlinear terms in (2.1), determine the phase and amplitude relationships among these five relevant Fourier modes. For example, for the particular problem described in §2a, the nonlinearly generated Alfvén waves $\mathbf{k}_3^{\pm}$ are phase-shifted from the primary Alfvén waves $\mathbf{k}_1^{\pm}$ by 90° [17,58]. In fact, recent analytical and numerical work argues for the importance of intermittency, or phase synchronization, in weak incompressible MHD turbulence [144].

As the initial Alfvén wave amplitudes $z_{\pm}$ increase to the strongly nonlinear regime, $\chi \to 1$, the asymptotic expansion of the equations of evolution ceases to be well ordered, and higher order terms—terms that can safely be neglected in the weakly nonlinear limit—begin to contribute significantly. The nonlinear interactions associated with these higher order terms mediate significant energy transfer to additional Fourier modes in the perpendicular plane, as depicted in figure 1. Yet, even in the strong turbulence limit, the phase and amplitude relationships among all of these modes remain completely specified by the nonlinear terms in the equations of evolution.

Nonlinear gyrokinetic simulations using the Astrophysical Gyrokinetics Code, AstroGK [145], have demonstrated the development of a current sheet as a natural consequence of the nonlinear evolution of an Alfvén wave collision in the strong turbulence limit, $\chi \sim 1$ [20]. Through these simulations, the qualitative understanding of the nonlinear energy transfer gleaned from the analytical solution in the weakly nonlinear limit, $\chi \ll 1$ [17], is found to persist as one approaches and reaches the strongly nonlinear limit, $\chi \to 1$. In particular, the nonlinearly generated $k_z = 0$ modes continue to play a key role in the energy transfer. It is found that the energy transfer to higher perpendicular wavenumbers flows predominantly along the three diagonal lines in figure 1a, largely confined to these modes with $k_z = +k_{\parallel}$, $k_z = 0$ and $k_z = -k_{\parallel}$ [20]. Generally, it appears that the higher order $k_z = 0$ modes are also purely magnetic, inherently nonlinear modes, similar to $\mathbf{k}_2^{(0)}$. And, the energy in the higher order modes with $k_z = \pm k_{\parallel}$ appears to primarily consist of counter-propagating Alfvén waves, where the perpendicular wavenumber of these Alfvén waves increases, given by the distance of the mode from the origin of the plot. Further analytical work is necessary to confirm the nature of these higher order, nonlinearly generated modes. The modes along these three diagonals represent a strikingly small fraction of the total possible perpendicular Fourier modes, typically a few per cent. It is constructive interference among just this small number of modes that results in the appearance of a coherent structure, in this case a current sheet.

All of the modes shown in figure 1a can, in principle, be computed analytically by extending the asymptotic calculation to higher order, but such a calculation involves a rapidly growing number of terms. However, the confinement of the turbulent energy to a few modes at each order ϵ^n can provide a guide to limit the effort of detailed analytical calculations to just the nonlinear modes directly involved in the dominant flow of energy to small scales along the diagonal lines in figure 1. Another complication is that, as the nonlinearity parameter increases to the strong turbulence limit, $\chi \to 1$, the convergence of the asymptotic series becomes poor, and the calculation must be taken to increasingly higher powers of ϵ. Nevertheless, the onset of current sheet development in a moderately nonlinear numerical calculation with $\chi = \frac{1}{2}$ is well reproduced by an analytical calculation employing only the five modes up to and including order ϵ^3 [20]. This result suggests that the physical mechanism of nonlinear energy transfer in Alfvén wave collisions, outlined in §2a, is indeed ultimately responsible for the development of current sheets in plasma turbulence, and that such coherent structure development can be computed analytically.

Note that any coherent structure can be assembled through constructive interference among spatial Fourier modes with particular amplitudes and phases. For example, a square wave can be expressed as the sum of sinusoidal modes $f(x) = \sin x + \sin(3x)/3 + \sin(5x)/5 + \cdots$. In the Alfvén wave collision problem, it is constructive interference among wave modes in the perpendicular plane—specifically the Fourier modes depicted in figure 1, where the amplitudes and phases of these modes are determined by the form of the nonlinear terms and can be computed analytically—that results in the development of a coherent structure. The observation of coherent structures in plasma turbulence arising in the form of current sheets is often invoked as evidence

that their presence is inconsistent with the strongly interacting Alfvén wave interpretation of plasma turbulence. The results reviewed here suggest not only that Alfvén waves are consistent with, but also that they are directly responsible for, the development of current sheets. The development of a current sheet is a natural consequence of the nonlinear interaction between two counter-propagating Alfvén waves in the strong turbulence limit, a result that is not *a priori* obvious in any way. This key finding resolves the apparent contradiction between those investigations that view plasma turbulence as a sea of nonlinearly interacting Alfvén waves and those that focus on the development of coherent structures and their role as sites of enhanced dissipation. Ultimately, this investigation of the nonlinear dynamics of Alfvén waves enables a first-principles prediction of current sheet development in plasma turbulence through the linear superposition of the nonlinearly generated modes, and illuminates the nature of these current sheets as constructive interference between counter-propagating Alfvén waves.

Along with the picture of nonlinear energy transfer mediated by counter-propagating Alfvén waves, this discovery of Alfvén-wave-driven current sheet development completes the foundation necessary to construct the first dynamical model of plasma turbulence that self-consistently combines the physics of Alfvén waves and small-scale current sheets, as presented below. This dynamical understanding of the origin and nature of current sheets provides important constraints on how such coherent structures may be dissipated, to be discussed in the next section.

Before ending our discussion of current sheets, it is important to point out that there are other mechanisms, such as particular flow and magnetic field geometries, that can produce current sheets. The Orzsag–Tang vortex is one such example [146]. Do such alternative mechanisms play any role in the development of current sheets in solar wind turbulence, or are strong Alfvén wave collisions sufficient to account for all of the current sheets observed in plasma turbulence? To answer this open question, the properties of current sheets generated by Alfvén wave collisions must be carefully compared with those arising in numerical simulations and inferred from solar wind observations.

3. The damping of turbulent fluctuations and plasma heating

At the forefront of plasma turbulence research stands the effort to identify and characterize the physical mechanisms responsible for the dissipation of the turbulent fluctuations and the conversion of their energy to heat, or other energization, of the protons, electrons and minor ions. As discussed above, for a weakly collisional plasma, the dissipation of turbulence necessarily includes two stages: first, the entropy-conserving, collisionless damping of the turbulent fluctuations; then, the ultimate conversion of the damped non-thermal free energy through entropy-increasing collisions into plasma heat.

There are presently three leading candidates for the damping of the turbulent fluctuations: (i) coherent wave–particle interactions, in particular, Landau damping[7] [5,28–32,39,134]; (ii) stochastic wave–particle interactions [40,147–154]; and (iii) dissipation associated with the current sheets that are ubiquitously observed in plasma turbulence [132,133,137,139,140,143,155–159]. Observational constraints on the physical mechanisms responsible for the dissipation of the turbulence and resulting plasma heating are reviewed elsewhere in this theme issue [160]. Note that different mechanisms may dominate from one environment to another (corona, inner heliosphere, outer heliosphere) as a result of variations in the plasma parameters (β, T_i/T_e, etc.) or changing characteristics of the turbulence (scale and amplitude of the energy injection, size of inertial range, etc.). I present a dynamical model for the cascade of anisotropic Alfvénic turbulence down to and below ion kinetic length scales, its collisionless damping by coherent wave–particle interactions and the ultimate conversion of the turbulent energy into plasma heat in the following section. In addition, I discuss how stochastic heating and dissipation in current sheets relate to this model.

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[7]I include in my use of the term 'Landau damping' all damping associated with the Landau resonance, thereby also including transit-time damping, the magnetic analogue of Landau damping.

(a) A model of anisotropic Alfvénic turbulence

The observed one-dimensional magnetic energy frequency spectrum in the solar wind demonstrates a power law with spectral index $-5/3$ at low frequencies [161], a break at around $f \sim 0.4\,\mathrm{Hz}$, and a steeper spectrum at higher frequencies with a spectral index of approximately -2.8 [7,38,41,162,163]. The low-frequency range is denoted the *inertial range* of solar wind turbulence, the break is believed to be associated with an ion kinetic length scale, and the high-frequency range is commonly denoted the *dissipation range*, although whether dissipation is actively occurring at all scales in this range remains an open question. In addition, multi-spacecraft observations show that the turbulent fluctuations at the small scales near the break are highly anisotropic, with $k_\parallel \ll k_\perp$ [7,103,104].

The following model of critically balanced, strong Alfvénic turbulence has been proposed to explain these observed features and to provide a foundation upon which predictive models of solar wind turbulence, its dissipation and the resulting plasma heating may be constructed [5,17, 20,39,164,165]. In this model, the inertial range occurs at scales $k_\perp \rho_i \ll 1$ and consists of counter-propagating Alfvén waves that transfer energy to successively smaller scales via strong Alfvén wave collisions. At the scale $k_\perp \rho_i \sim 1$, the linear wave physics transitions from that of non-dispersive MHD Alfvén waves to dispersive kinetic Alfvén waves, leading to the steepening of the energy spectrum. At smaller scales, I suggest here that the nonlinear energy transfer continues to be mediated by counter-propagating wave collisions, only between kinetic Alfvén waves rather than MHD Alfvén waves.[8] Although the nonlinear evolution of kinetic Alfvén wave collisions remains to be solved rigorously, I expect that, although the resulting energy transfer in the kinetic regime may differ quantitatively from that of MHD Alfvén wave collisions, the qualitative picture of nonlinear energy transfer by counter-propagating wave collisions persists.

Although the collisionless damping of the Alfvén waves is weak in the inertial range at $k_\perp \rho_i \ll 1$, ion Landau damping peaks at $k_\perp \rho_i \sim 1$ and is expected to transfer energy from the turbulent electromagnetic fluctuations into non-thermal free energy in velocity space through collisionless wave–particle interactions. The resulting small-scale structure in the ion velocity distribution, through a nonlinear phase mixing process, may undergo a dual-cascade to smaller scales in both configuration space and velocity space, termed the ion entropy cascade [39,167–170]. The entropy cascade enables efficient collisional thermalization of the non-thermal free energy associated with fluctuations in the ion distribution function even under conditions of arbitrarily weak collisionality. In addition to ion Landau damping, electron Landau damping becomes increasingly strong as the perpendicular wavenumber increases within the dissipation range, $k_\perp \rho_i \gtrsim 1$. Ultimately, the electron Landau damping becomes sufficiently strong to dominate over the nonlinear energy transfer, terminating the cascade with an exponential roll-off of the magnetic energy spectrum around the scale of the electron Larmor radius, $k_\perp \rho_e \sim 1$ [98,171].

Based on such a dynamical understanding of Alfvénic turbulence and its collisionless damping, it is possible to construct predictive models for the turbulent cascade of energy and its dissipation [5,43,165,172,173]. Turbulent cascade models have been used to predict: the density fluctuations in the solar corona and solar wind [11,174]; the ratio of ion to electron heating as a function of β_i and T_i/T_e [42]; and the proton-to-total turbulent heating ratio in the solar wind [12]. Also, such cascade models have proven invaluable in the interpretation of kinetic numerical simulations of weakly collisional plasma turbulence [4,98,165].

(b) Relation to stochastic heating

Although it may not be immediately obvious, the model of stochastic heating is entirely compatible with the dynamical model of anisotropic Alfvénic turbulence described above. Recent work has resulted in a very well-developed model of stochastic ion heating in heliospheric plasma turbulence [40,152–154]. A key component of this model is a turbulent spectrum of

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[8]The growing body of observational evidence in support of kinetic Alfvén waves as the dominant contributor to the dissipation range of solar wind turbulence has recently been reviewed [166].

Alfvén and kinetic Alfvén waves, such as the Alfvénic cascade described above. If a particular ion has a Larmor radius[9] $\rho = v_\perp / \Omega_i$, when the amplitude of the turbulent Alfvén and kinetic Alfvén fluctuations at $k_\perp \rho \sim 1$ exceeds a certain threshold, the ion's orbit becomes chaotic. The resulting stochastic interaction of the ion with the time-varying electrostatic potential associated with the turbulent fluctuations leads to a random walk of the ion's energy. For a distribution of ions that monotonically decreases with increasing energy, this leads to net damping of the turbulent electromagnetic fluctuations and an increase in the perpendicular temperature of the ion distribution. This stochastic ion heating mechanism is particularly effective under the low plasma beta conditions relevant to the solar corona, with estimates that half or more of the turbulent cascade power may be diverted into proton heating at the scale of the proton thermal Larmor radius, $k_\perp \rho_i \sim 1$ [40]. Thus, incoherent wave–particle interactions may serve as an effective channel to dissipate plasma turbulence.

In principle, stochastic heating could be incorporated into the cascade models mentioned in §3*a*, but to attempt such an implementation raises several unanswered questions: (i) what fraction of the large-scale turbulent cascade energy is stochastically lost to the ions? (ii) how is this energy loss distributed over different scales of the turbulent Alfvénic fluctuations? and (iii) does this mechanism impact the nature of the fluctuations at smaller scales that receive the remaining turbulent energy not lost stochastically to the ions? Indeed, a recent observational study [175] that found an amplitude-dependent drop in the magnetic energy spectrum at the ion kinetic scale may be evidence of turbulent cascade energy lost stochastically to ions.

(c) Dissipation associated with current sheets

Although the property that dissipation in plasma turbulence occurs dominantly in small-scale current sheets is well established [1,127–136], the kinetic physical mechanism by which dissipation occurs in current sheets has not been elucidated. Indeed, stochastic ion heating [156, 176] and Landau damping [134,177,178] have both been suggested as the physical process underlying the kinetic damping of current sheets. In addition, parallel electric fields [179–184] and Fermi acceleration [185], both arising through the process of magnetic reconnection, have also been proposed. It should be emphasized, however, that even though magnetic reconnection often springs to mind when current sheets are found, it has not yet been firmly established that magnetic reconnection plays a significant role in the dissipation of energy in three-dimensional plasma turbulence in the solar wind. The topics of the connection between magnetic reconnection and turbulence and the role played by current sheets in the dissipation of turbulence are central to several reviews in this theme issue [160,186,187].

One important question is whether the current sheets inferred from solar wind observations are all dynamically generated by the turbulence itself [188,189], or whether some fraction of the current sheets merely represent advected flux tube boundaries [190–193]. I have highlighted in §2*h* recent research that explains, from first principles, the development of current sheets in plasma turbulence as a natural consequence of the nonlinear energy transfer caused by strong Alfvén wave collisions [20]. If this mechanism operates in the solar wind, then at least some of the observed current sheets consist of a constructively interfering sum of counter-propagating Alfvén waves. This finding provides new insight into the underlying nature of dynamically generated current sheets in plasma turbulence, and the damping of the associated turbulent electromagnetic fluctuations may indeed be dominated by collisionless wave–particle interactions via the Landau resonance, as previously suggested by a recent analysis of gyrokinetic numerical simulations [134]. If these conjectures are correct, then *the fact that the heating is concentrated in current sheets is merely a consequence of the turbulent nonlinear dynamics but does not determine the mechanism of dissipation*. This idea does not conflict with results, reported elsewhere in this theme issue [187], that suggest intermittency develops identically in both ideal and resistive MHD simulations.

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[9]Note that this Larmor radius ρ is *not* the thermal ion Larmor radius $\rho_i = v_{ti}/\Omega_i$, but rather is the gyroradius for an ion with a particular perpendicular velocity $v_\perp$.

A statement that is likely to prove very controversial is that intermittency, although a well-established characteristic of plasma turbulence, may not be critically important for the prediction of the plasma heating due to the dissipation of turbulent fluctuations, particularly if coherent (or possibly even stochastic) wave–particle interactions dominate the dissipation. This directly contradicts the conclusion of Matthaeus *et al.* [187] that 'recent studies of kinetic dissipation of the turbulent cascade suggest that coherent structures and associated non-uniform dissipation play a very important and possibly dominant role in the termination of the cascade and the effectively irreversible conversion of fluid macroscopic energy into microscopic random motions'. In short, the viewpoint proposed here is that current sheets are not a cause of turbulent dissipation, but are merely a consequence of the nonlinear dynamics underlying the turbulent energy cascade. If this contentious statement is correct, it represents good news for the endeavour to develop predictive models of plasma heating arising from the dissipation of weakly collisional plasma turbulence. Simple cascade models would remain a valid means of predicting the differential heating of the plasma species since the collisionless damping would depend only on the energy content in different wavevectors, while the phases that lead to intermittency would not come into play.

4. Dynamical model of plasma turbulence

Here I present a brief summary of the first dynamical model of plasma turbulence that combines the physics of Alfvén waves with the self-consistent development of current sheets, illustrated by the diagrams of the magnetic energy spectrum and wavevector anisotropy in figure 2.

Isotropic fluctuations at the outer scale of the turbulent inertial range in the near-Earth solar wind, typically at $k\rho_i \sim 10^{-4}$, consist primarily of incompressible Alfvén waves with a small admixture of kinetic slow wave fluctuations (not depicted). Nonlinear energy transfer from large to small scales through the inertial range is governed by the physics of Alfvén wave collisions. The theory of the critically balanced Alfvénic turbulence [53] predicts a magnetic energy spectrum $E_B \propto k_\perp^{-5/3}$, where the turbulent fluctuations become increasingly more anisotropic as the cascade progresses to smaller scales, following a scale-dependent anisotropy $k_\parallel \propto k_\perp^{2/3}$ in the inertial range. Note that a refined version of critically balanced turbulence that accounts for the dynamic alignment of the turbulent fluctuations [122] predicts slightly different scalings, $E_B \propto k_\perp^{-3/2}$ and $k_\parallel \propto k_\perp^{1/2}$, and appears to be more consistent with the high-resolution numerical simulations of incompressible MHD turbulence [194].

At the perpendicular scale of the ion Larmor radius, $k_\perp \rho_i \sim 1$, the turbulent cascade transitions from the inertial range ($k_\perp \rho_i \ll 1$), in which the nonlinear energy transfer is mediated by Alfvén wave collisions, to the dissipation range ($k_\perp \rho_i \gtrsim 1$), in which the nonlinear energy transfer is mediated by kinetic Alfvén wave collisions. The dispersive nature of kinetic Alfvén waves leads to a steepening of the magnetic energy spectrum $E_B \propto k_\perp^{-2.8}$, and an extension of the concept of critical balance to this regime [5] predicts an anisotropy scaling as $k_\parallel \propto k_\perp^{1/3}$.

The nonlinear energy transfer is governed by Alfvén and kinetic Alfvén wave collisions, which transfer energy to Alfvénic waves at smaller scales. The particular phases and amplitudes of the nonlinearly generated waves, determined by the mathematical form of the nonlinear terms in the dynamical equations, leads to the development of coherent structures through constructive interference [20]. In plasma turbulence, these coherent structures take the form of current sheets, and dissipation is found to be concentrated within these current sheets. But, since the current sheets themselves are primarily made up of counter-propagating Alfvén and kinetic Alfvén waves, these turbulent electromagnetic fluctuations will damp by collisionless wave–particle interactions via the Landau resonance. Ion Landau damping peaks at scales $k_\perp \rho_i \sim 1$, and electron Landau damping begins to be significant at $k_\perp \rho_i \gtrsim 1$ and becomes increasingly strong as the wavenumber increases. At the electron scales $k_\perp \rho_e \sim 1$, electron Landau damping overwhelms the nonlinear energy transfer and the cascade is terminated, producing an exponential roll-off in the magnetic energy spectrum. The energy removed from the turbulent fluctuations by

collisionless wave–particle interactions will ultimately be converted to plasma heat by collisions. This process is facilitated by an entropy cascade (not shown) that mediates the transfer of non-thermal structure to sufficiently small scales in velocity space that weak collisions can thermalize the energy [39].

In addition to the physics of the Alfvénic turbulent cascade of energy from large scales that is ultimately terminated at electron scales, kinetic instabilities can inject energy into fluctuations at scales[10] $k_{\parallel} d_i \sim 1$. An important, but generally under-appreciated, property of these instability-driven fluctuations is that they occupy a distinct region of wavevector space from the anisotropic cascade of energy from large scales. However, since single-point spacecraft measurements cannot uniquely separate $k_{\parallel}$ from $k_{\perp}$, these instability-driven fluctuations will appear at $k d_i \sim k \rho_i \sim 1$ in the magnetic energy spectrum, making them quite difficult to distinguish.

5. Conclusion

An improved understanding of turbulence in the weakly collisional solar wind will impact our ability not only to predict physical processes within the heliosphere but also to illuminate complex astrophysical phenomena in remote regions of the Universe. The linear response due to magnetic tension in a magnetized plasma, a physical property absent in turbulent hydrodynamical flows, provides an important foothold to understand the dynamics of plasma turbulence. To unravel the physics of the nonlinear energy transfer to small scales, the kinetic mechanisms of dissipation of the turbulent fluctuations and the resulting plasma heating in kinetic plasma turbulence, I contend that we must step beyond the usual statistical treatments of turbulence, adopting instead a dynamical approach. It is the linear and nonlinear dynamics of Alfvén waves that are responsible, at a very fundamental level, for some of the key qualitative features of plasma turbulence that distinguish it from hydrodynamic turbulence, including the anisotropic cascade of energy and the development of current sheets at small scales. The ultimate goal is to create a predictive theory of plasma turbulence. Only a thorough understanding of the turbulent plasma dynamics will enable the development of a simplified theoretical framework upon which predictive models of plasma turbulence and its effect on energy transport and plasma heating can be constructed.

The arguments presented here support a simplified perspective on the nature of turbulence in a weakly collisional plasma. The nonlinear interactions responsible for the turbulent cascade of energy and the formation of current sheets are essentially fluid in nature, and can be understood in terms of nonlinear wave–wave interactions. On the other hand, the collisionless damping of the turbulent fluctuations and the energy injection by kinetic instabilities are essentially kinetic in nature, and can be understood in terms of linear collisionless wave–particle interactions.

From this perspective, it is easy to understand why reduced models, such as incompressible MHD and reduced MHD, are valuable tools in the study of plasma turbulence. The simplicity of such reduced models is that they satisfy a number of exact constraints that facilitate the development of an intuitive understanding of the turbulent dynamics. Although these constraints do not strictly hold under more general plasma conditions, often the behaviour observed in the simplified system persists, at least to lowest order, in the more general system. For example, we employ here our understanding of the nonlinear energy transfer between counter-propagating Alfvén waves in incompressible MHD to argue that the anisotropic cascade of energy ubiquitously observed in magnetized plasma turbulence is due to the facts that only counter-propagating Alfvén waves interact nonlinearly and that the nonlinear term is greatest for interactions between perpendicularly polarized Alfvén waves. Although the constraint that only counter-propagating waves interact ceases to hold strictly when compressibility or kinetic effects (dispersion due to finite Larmor radius effects) are introduced, the anisotropy of the turbulent cascade persists.

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[10]Note that $d_i = \rho_i / \sqrt{\beta_i}$, so for the typical value $\beta_i \sim 1$ in the near-Earth solar wind, $d_i \sim \rho_i$.

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The separation of essentially fluid versus essentially kinetic properties of kinetic plasma turbulence enables further simplifications. Although non-Maxwellian particle velocity distributions are widely found in the solar wind and in kinetic numerical simulations, the departures from a Maxwellian equilibrium (bi-Maxwellian, kappa or multi-component with core/halo structure) do not affect the electromagnetic dynamics of the turbulence very much [195]. Since the nonlinear energy transfer and development of current sheets in plasma turbulence are due to wave–wave interactions, they depend only on the first moments of the distribution functions, and are therefore not very sensitive to the particular form of the distribution functions. The physical mechanisms of dissipation and the injection of energy by kinetic instabilities, on the other hand, which depend on growth or damping rates due to collisionless wave–particle interactions, may depend sensitively on the form of the distribution functions. For example, a kappa distribution, having a much larger population of particles at high energy with respect to a Maxwellian, may experience significantly enhanced rates of collisionless damping of waves that are resonant with these high-energy particles. This viewpoint proposed here strongly contradicts the recent suggestion from a kinetic study of plasma turbulence [196] that 'it seems increasingly clear that significant kinetic effects including heating have strong association with coherent structures and with the turbulent cascade that produces intermittency'.

Looking towards the future, to identify definitively the kinetic mechanism responsible for the dissipation of solar wind turbulence, it is likely that it will be necessary to step beyond the analysis of the turbulent electromagnetic fields alone, delving much more deeply into the dynamic effect of turbulent fluctuations and their dissipation on the particle velocity distributions. These velocity distributions are already measured directly by instruments on existing spacecraft missions. And, on upcoming missions, such as *MMS* and *Solar Probe Plus*, with a combination of higher cadence and better resolution than previous missions, the distribution function data represent an important untapped potential source for discovery science. A new frontier will be the exploration of the dynamics of the perturbations to the distribution functions associated with the turbulent fluctuations and their kinetic dissipation, where kinetic plasma turbulence simulations will be essential to exploit fully the potential of *MMS* and *Solar Probe Plus*.

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