College Physics I: 1511 Mechanics & Thermodynamics

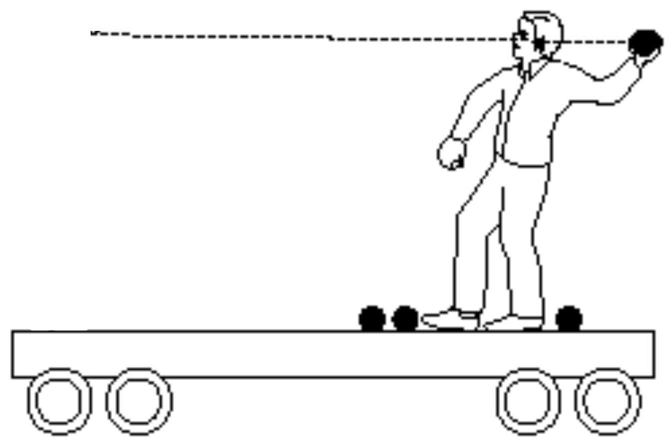
Professor Jasper Halekas
Van Allen Lecture Room 1
MWF 8:30-9:20 Lecture

Conservation of Momentum

- Conservation of momentum for two bodies:
  - $m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f} = m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i}$
  - True as long as no net external force acts on the bodies
  - True even if total mechanical energy is not conserved!

Concept Check

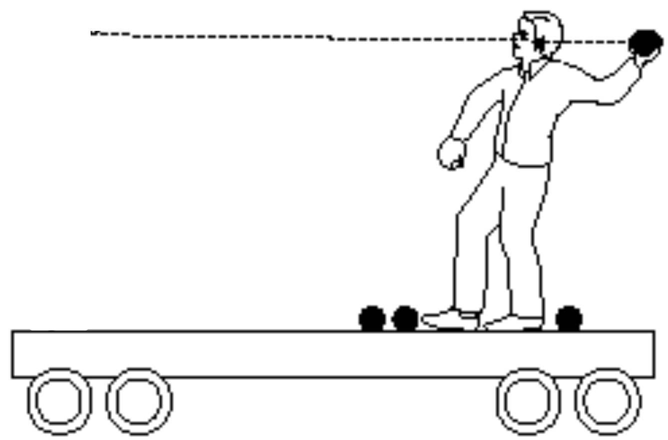
Suppose you are on a cart, initially at rest on a track with very little friction. You throw balls to the left. Is the cart put in motion?



A: Yes, it moves to the right.
B: Yes, it moves to the left.
C: No, it remains in place.

Concept Check

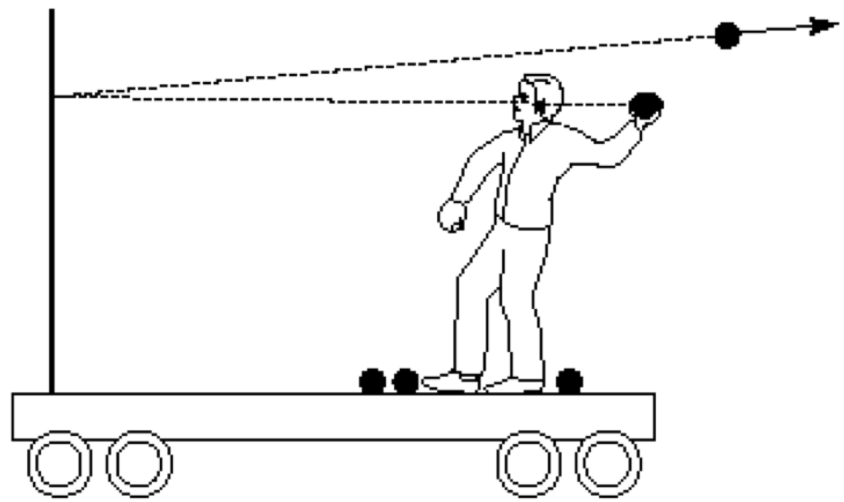
Suppose you are on a cart, initially at rest on a track with very little friction. You throw balls to the left. Is the cart put in motion?



A: Yes, it moves to the right.
B: Yes, it moves to the left.
C: No, it remains in place.

Concept Check

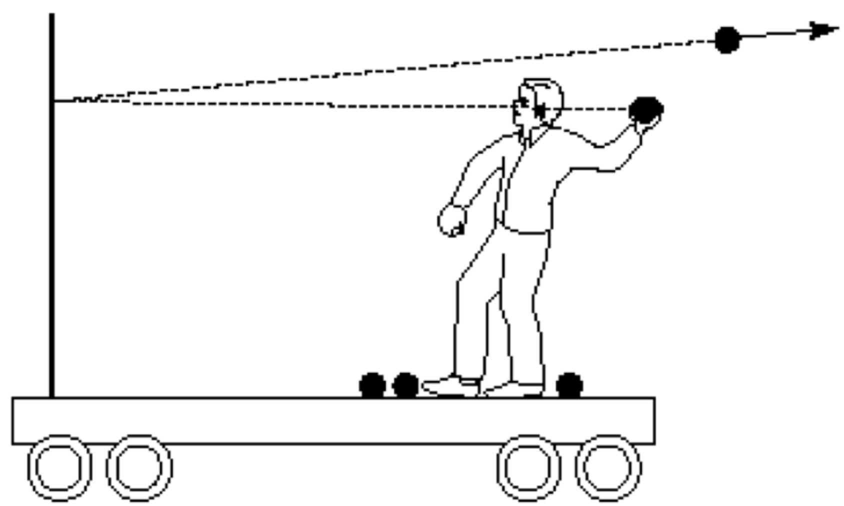
Suppose you are on a cart, initially at rest on a track with very little friction. You throw balls at a partition that is rigidly mounted on the cart. If the balls bounce straight back as shown in the figure, is the cart put in motion?



A: Yes, it moves to the right.
B: Yes, it moves to the left.
C: No, it remains in place.

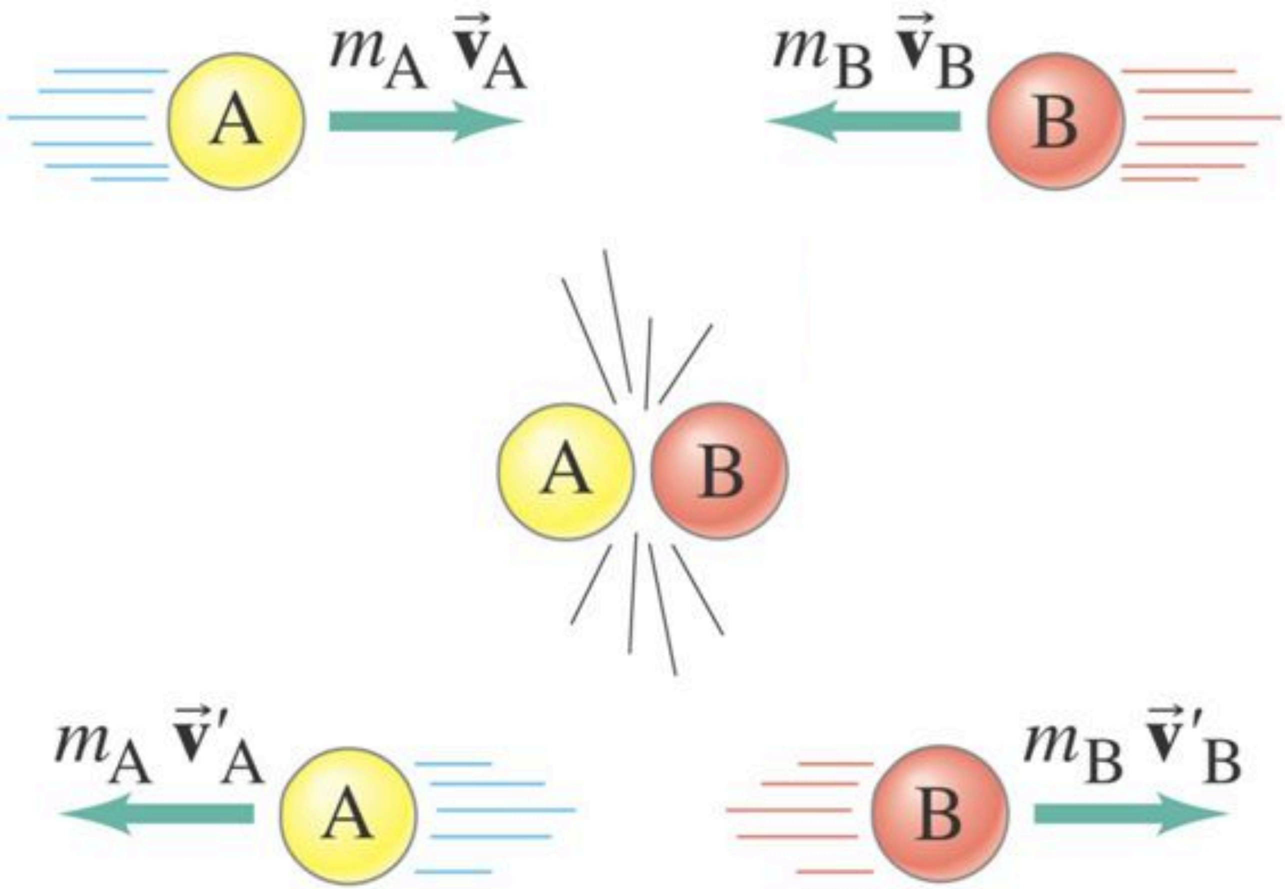
Concept Check

Suppose you are on a cart, initially at rest on a track with very little friction. You throw balls at a partition that is rigidly mounted on the cart. If the balls bounce straight back as shown in the figure, is the cart put in motion?



A: Yes, it moves to the right.
B: Yes, it moves to the left.
C: No, it remains in place.

Collisions

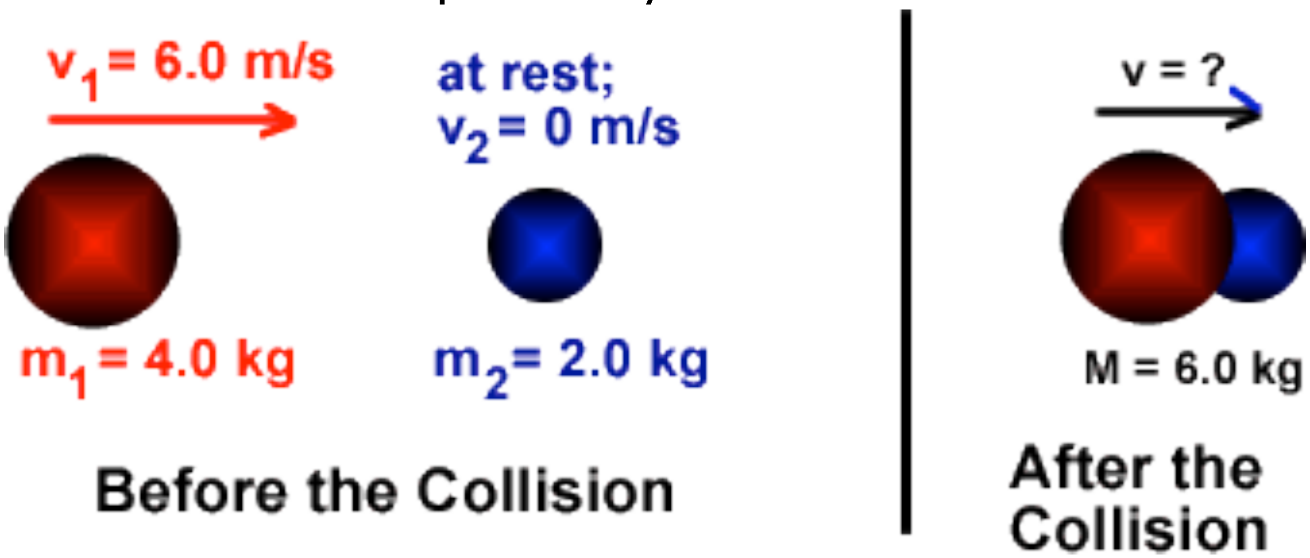


Definition: Types of Collisions

- Elastic collisions
  - Kinetic energy is conserved
    - Corresponds to objects that bounce perfectly
- Inelastic collisions
  - Kinetic energy is not conserved
    - Corresponds to objects that don't bounce perfectly
- Totally Inelastic Collisions
  - Corresponds to objects that stick together after colliding

Concept Check

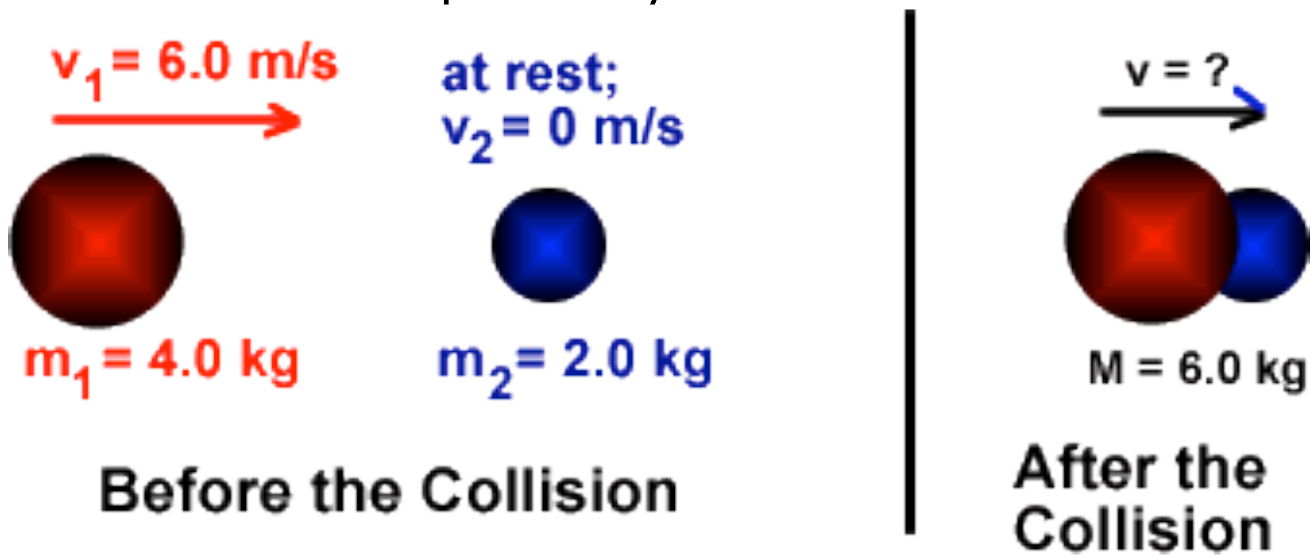
Assume a perfectly inelastic collision



A. $V_f = 2$ m/s
B. $V_f = 3$ m/s
C. $V_f = 4$ m/s
D. $V_f = 6$ m/s

Concept Check

Assume a perfectly inelastic collision



A. $V_f = 2$ m/s
B. $V_f = 3$ m/s
C. $V_f = 4$ m/s
D. $V_f = 6$ m/s

$$p_0 = m_1 v_{10} + m_2 v_{20}$$
$$= 4 \cdot 6 + 2 \cdot 0$$
$$= 24 \text{ kg m/s}$$

$$p_f = (m_1 + m_2) v_f$$
$$= 6 \cdot v_f$$

$$6 v_f = 24$$

$$\boxed{v_f = 4 \text{ m/s}}$$

What about energy?

$$KE_0 = \tfrac{1}{2} m_1 v_{10}^2 + \tfrac{1}{2} m_2 v_{20}^2$$
$$= \tfrac{1}{2}\, 4 \cdot 6^2 + \tfrac{1}{2}\, 2 \cdot 0$$
$$= 72 \text{ J}$$

$$KE_f = \tfrac{1}{2} (m_1 + m_2) v_f^2$$
$$= \tfrac{1}{2} \cdot 6 \cdot 4^2$$
$$= 48 \text{ J}$$

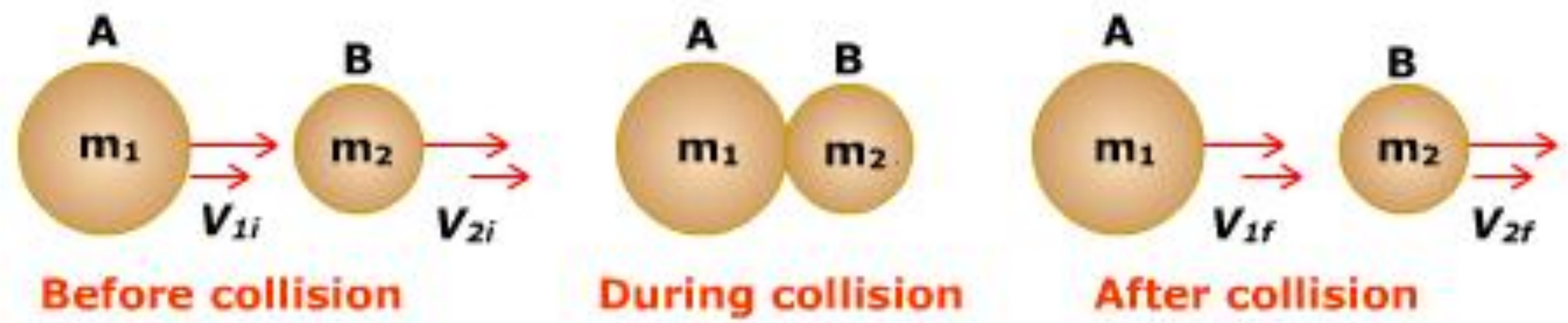
$$\Delta KE = 48 - 72 = -24 \text{ J}$$

We lost energy!
To heat or deformation

Final Velocities in Totally Inelastic Collisions

- By conservation of momentum:
- $(m_1+m_2)\mathbf{v}_f = m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i}$
- $\mathbf{v}_f = (m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i})/(m_1+m_2)$
- This is the “center of mass velocity”

1-d Elastic Collisions



Final Velocities in 1-d Elastic Collisions

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

Elastic Collisions

$$\vec{P}_1 + \vec{P}_2 = \text{const.}$$

$$KE_1 + KE_2 = \text{const.}$$

Stick to 1-d for now

$$m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1f} + m_2 V_{2f}$$

$$\tfrac{1}{2} m_1 V_{1i}^2 + \tfrac{1}{2} m_2 V_{2i}^2 = \tfrac{1}{2} m_1 V_{1f}^2 + \tfrac{1}{2} m_2 V_{2f}^2$$

Rewrite:

$$\tfrac{1}{2} m_1 \left(V_{1i}^2 - V_{1f}^2\right) = \tfrac{1}{2} m_2 \left(V_{2f}^2 - V_{2i}^2\right)$$

$$m_1 \left(V_{1i} - V_{1f}\right) = m_2 \left(V_{2f} - V_{2i}\right)$$

Divide $\quad V_{1i} + V_{1f} = V_{2f} + V_{2i}$

$$\Rightarrow V_{1f} = V_{2f} + V_{2i} - V_{1i}$$

Plug into Momentum

$$m_1 V_{1i} + m_2 V_{2i} = m_1 \left(V_{2f} + V_{2i} - V_{1i}\right) + m_2 V_{2f}$$

$$m_1 V_{1i} + m_2 V_{2i} - m_1 V_{2i} + m_1 V_{1i} = m_1 V_{2f} + m_2 V_{2f}$$

$$\Rightarrow \boxed{V_{2f} = \frac{2 m_1 V_{1i} + (m_2 - m_1) V_{2i}}{m_1 + m_2}}$$

Similarly:

$$V_{1f} = \frac{2m_2 V_{2i} + (m_1 - m_2) V_{1i}}{m_1 + m_2}$$

For equal masses:

$$m_1 - m_2 = 0$$

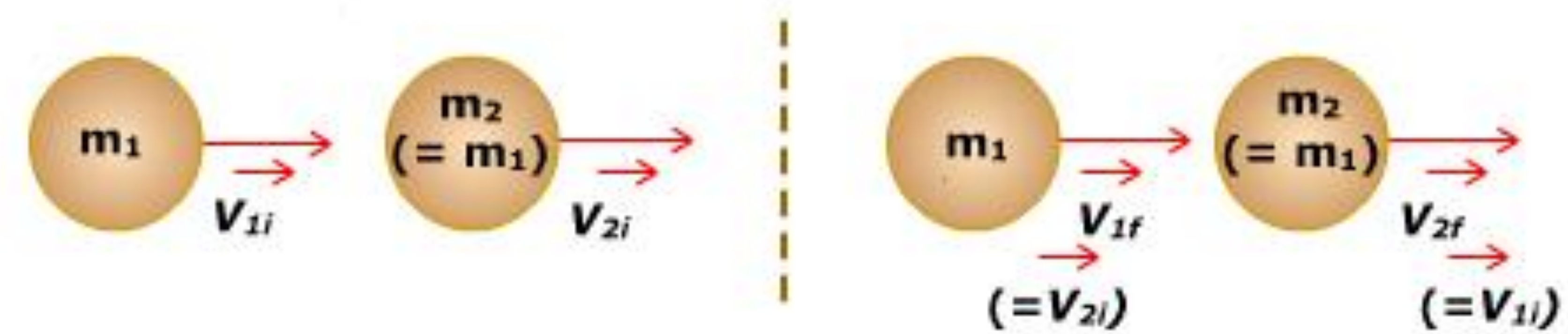
so $$V_{1f} = \frac{2m_2 V_{2i}}{m_1 + m_2} = V_{2i}$$

$$V_{2f} = \frac{2m_1 V_{1i}}{m_1 + m_2} = V_{1i}$$

Velocities Trade!

Final Velocities For Equal Mass

- For any 1-d collision between two objects with equal mass colliding elastically, the two objects simply trade velocities



Newton's Spheres

Why science teachers are not asked to monitor recess.

Concept Check

- You are given two carts, A and B. They look identical, and you've been told they are made of the same material.
- You place A at rest on an air track and give B a constant velocity to the right so that it collides elastically with A.
- After the collision, both carts move to the right, the velocity of B being smaller than what it was before the collision. What do you conclude?
- A: Cart A is hollow.
- B: The two carts are identical.
- C: Cart B is hollow.

Concept Check

- You are given two carts, A and B. They look identical, and you've been told they are made of the same material.
- You place A at rest on an air track and give B a constant velocity to the right so that it collides elastically with A.
- After the collision, both carts move to the right, the velocity of B being smaller than what it was before the collision. What do you conclude?

- A: Cart A is hollow.
- B: The two carts are identical.
- C: Cart B is hollow.

Final Velocities in 1-d Elastic Collisions

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i}$$

Unequal – small mass moving

$m_2 >> m_1$

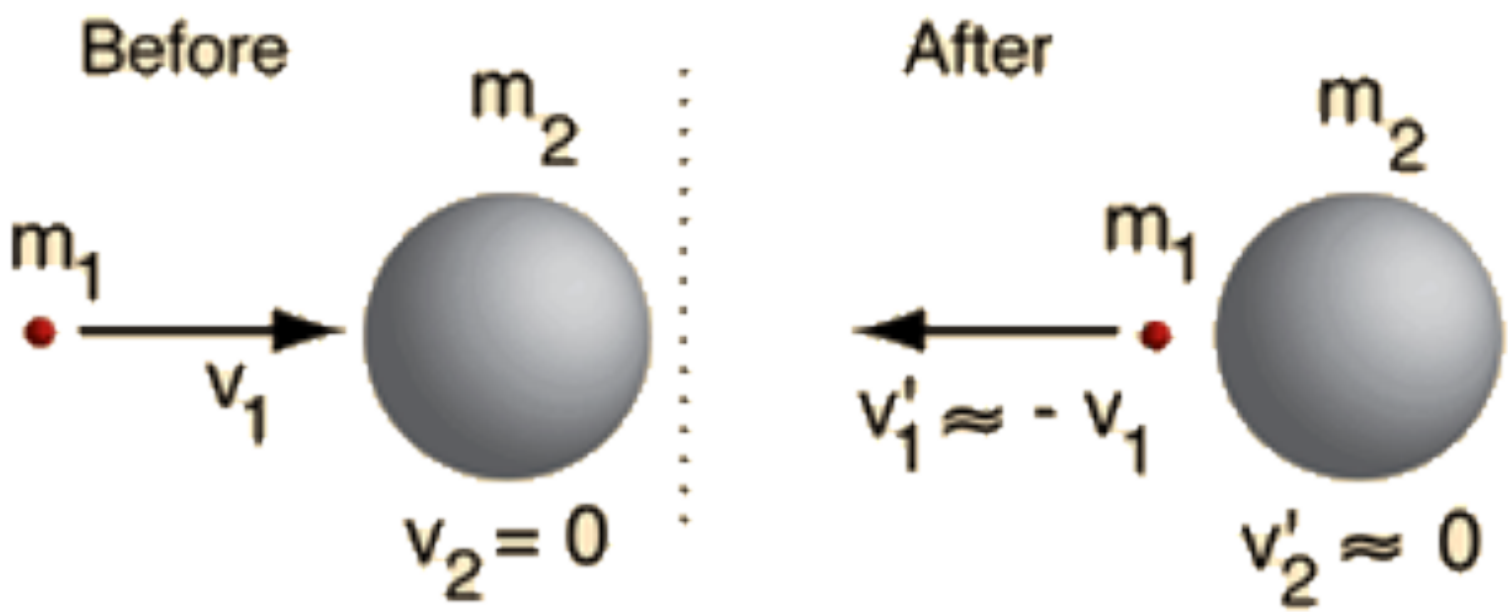
$v_{2i} = 0$

$v_{1f} \sim -\frac{m_2}{m_2} v_{1i} = -v_{1i}$

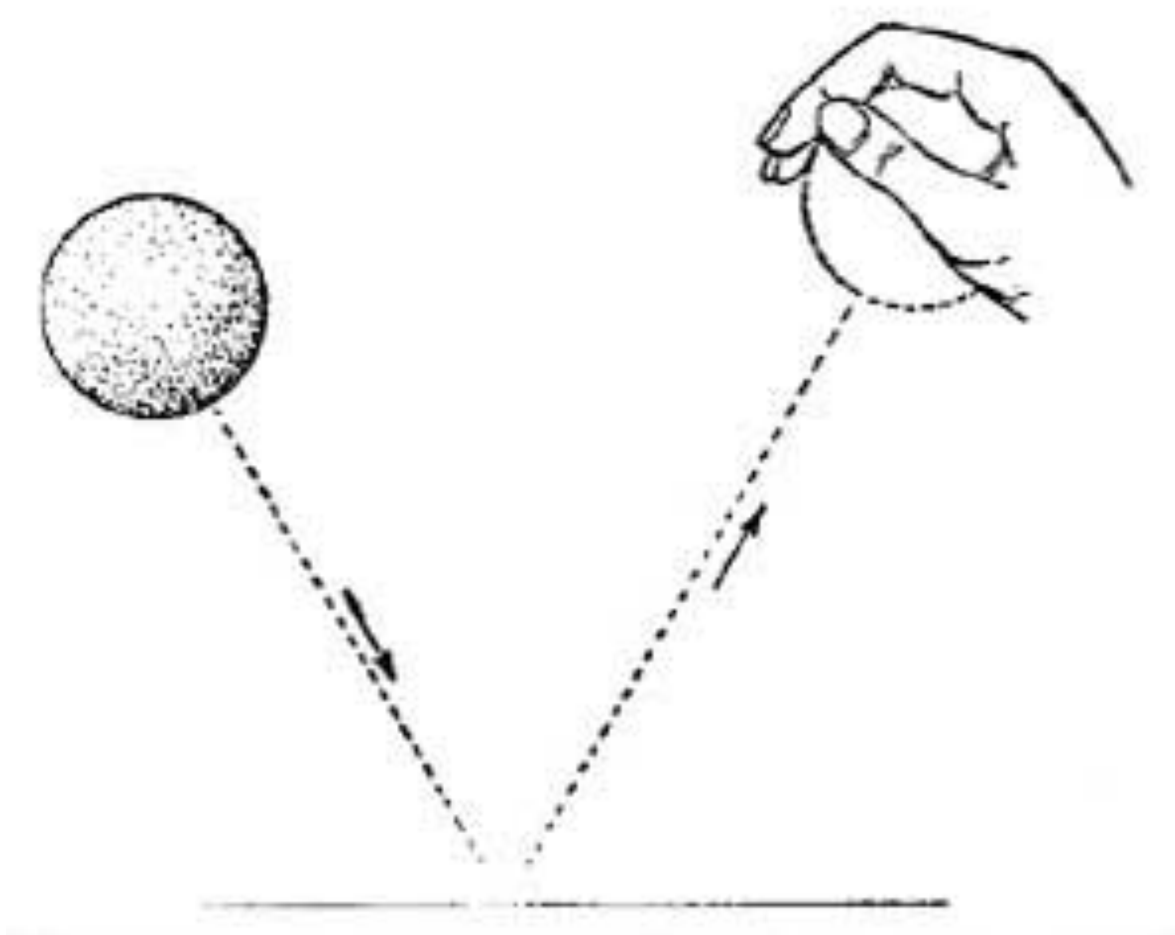
$v_{2f} \sim 0$

– Small bounces
– Big sits still

Final Velocities For Highly Unequal Mass: Small Mass Moving



Unequal Mass Example



Final Velocities in 1-d Elastic Collisions

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

Unequal – Big moving

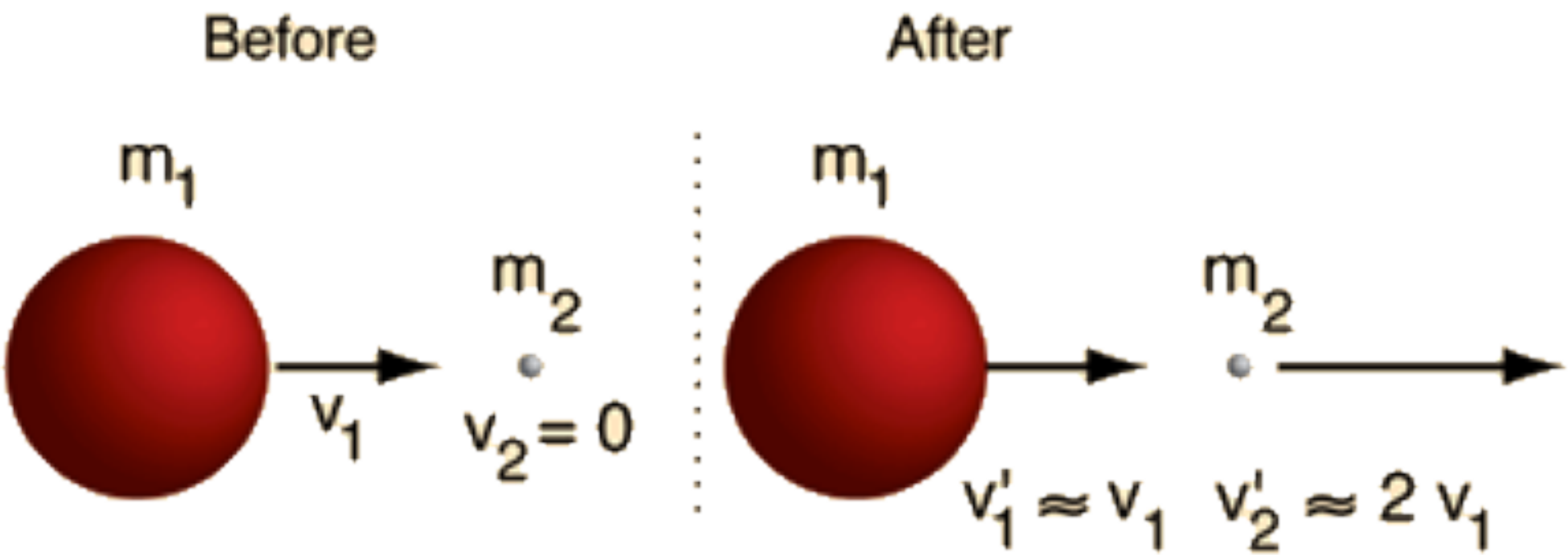
$m_2 >> m_1$
$v_{1i} = 0$

$v_{1f} \sim 2 m_2 / m_2 \, V_{2i} = 2 V_{2i}$

$v_{2f} \sim m_2 / m_2 \, V_{2i} = V_{2i}$

– Big keeps going
– small bounced ahead

Final Velocities For Highly Unequal Mass: Large Mass Moving



Unequal Mass Example