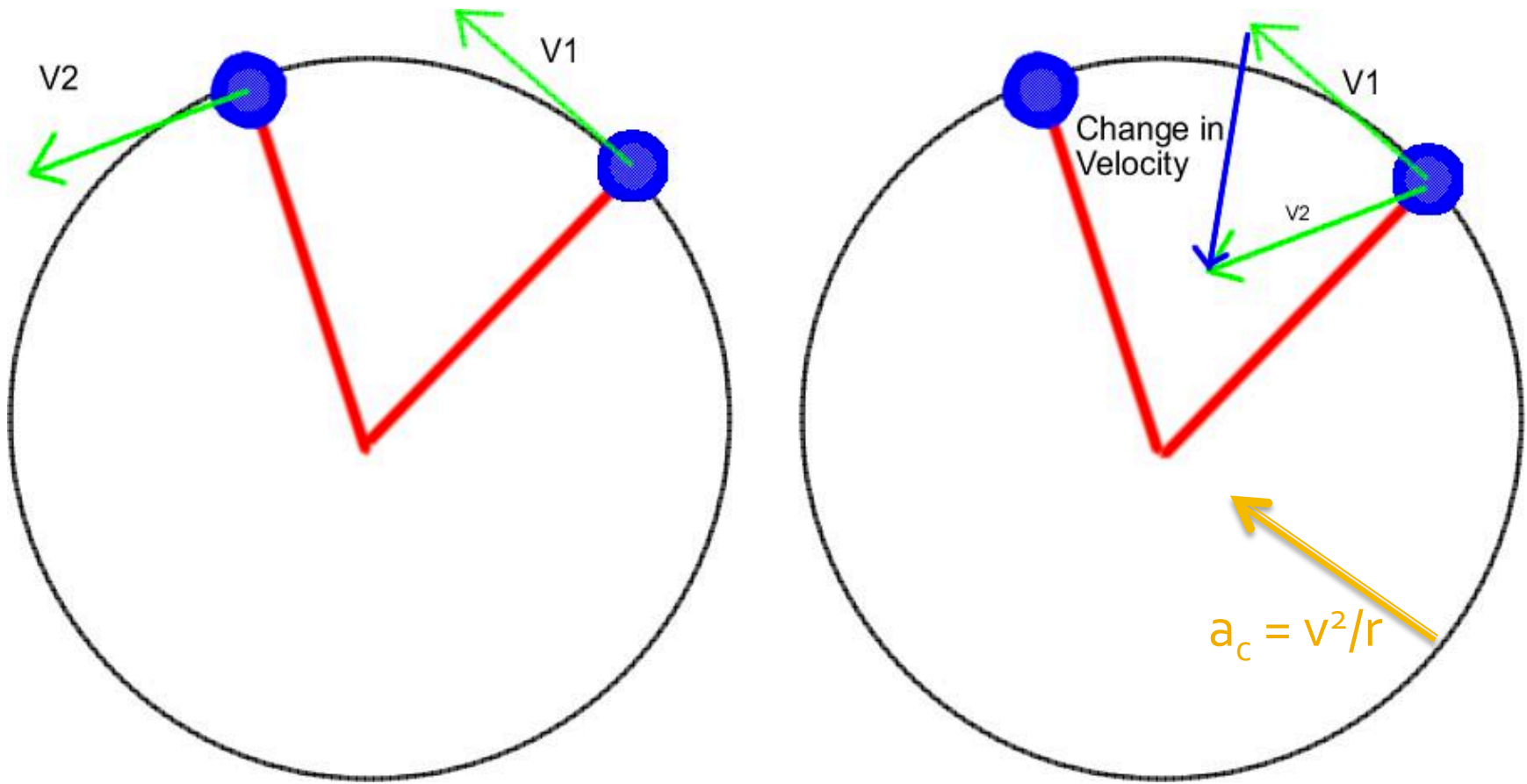


# College Physics I: 1511

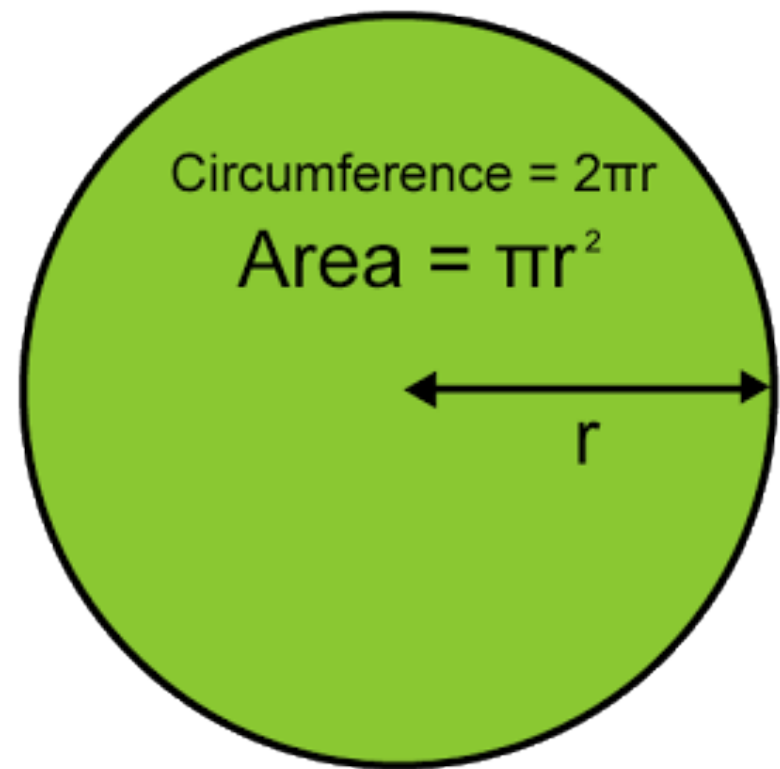
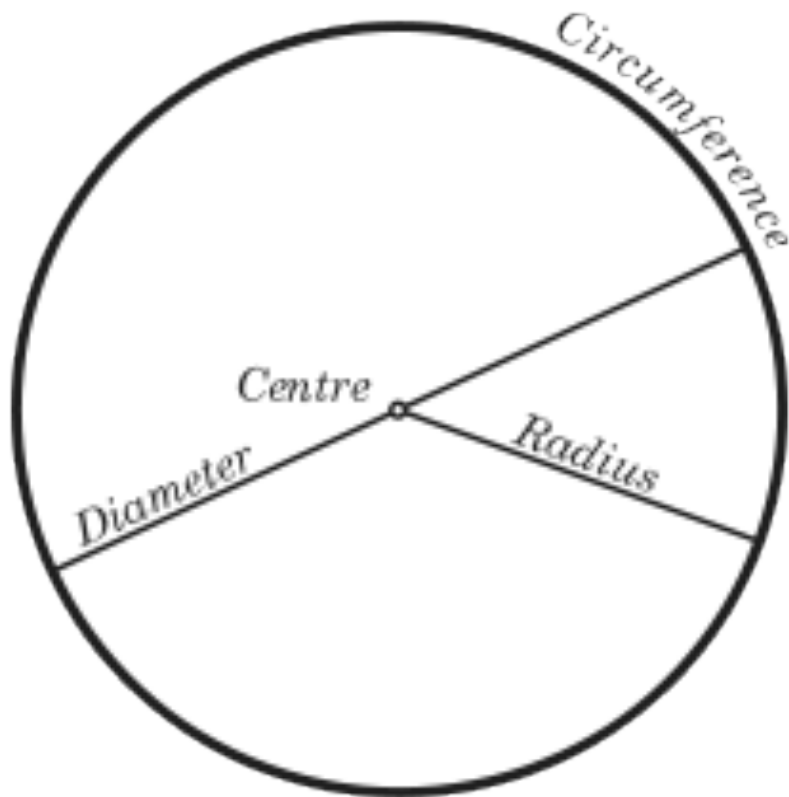
## Mechanics & Thermodynamics

Professor Jasper Halekas  
Van Allen Lecture Room 1  
MWF 8:30-9:20 Lecture

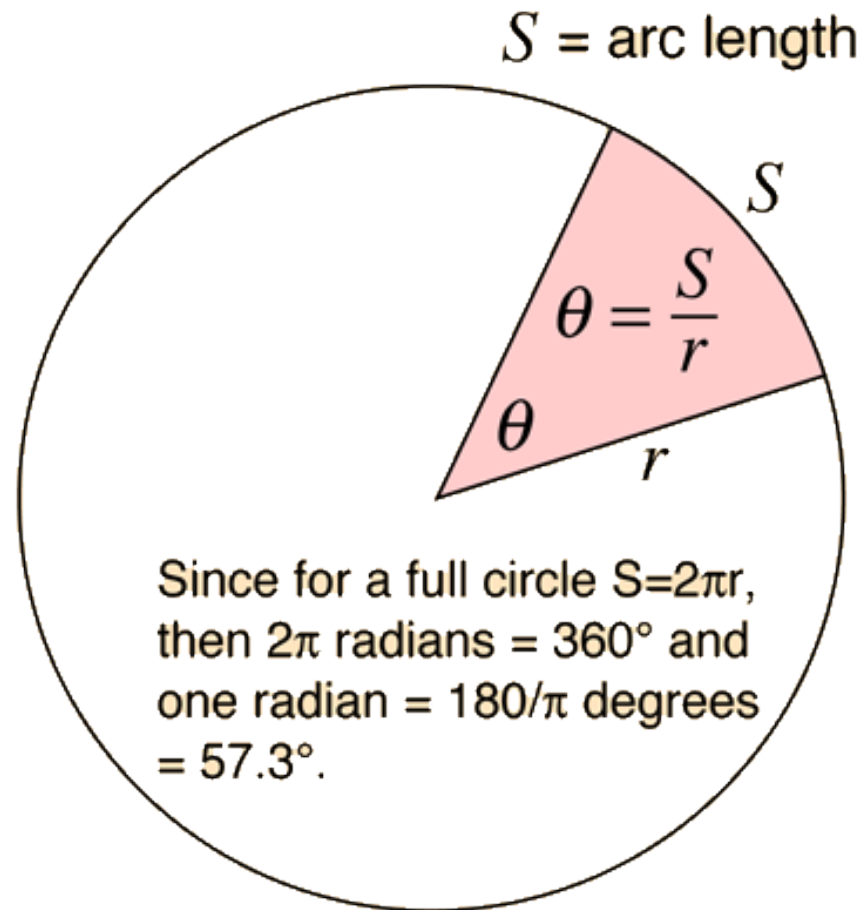
# Review: Uniform Circular Motion



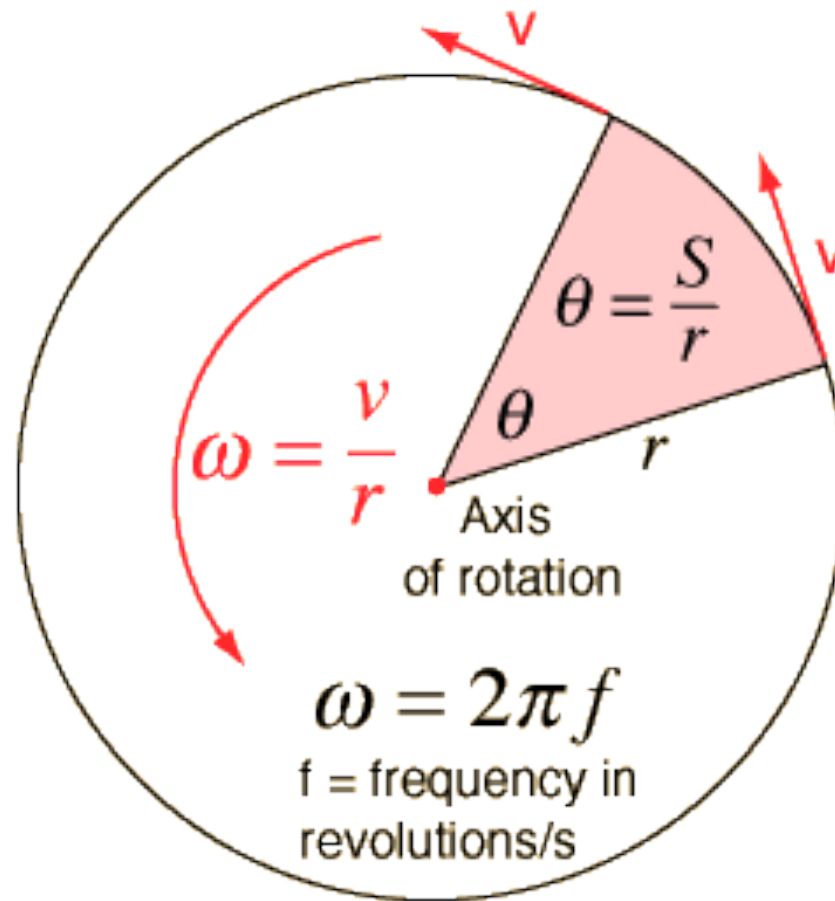
# Review: Circles



# Rotational Quantities I: Angle



# Rotational Quantities II: Angular Velocity



Tangential velocity

$$\langle v_T \rangle = \frac{s}{\Delta t}$$

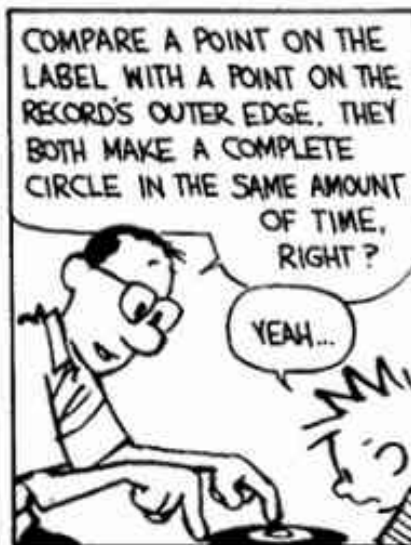
but  $s = r \Delta \theta$

$$\text{so } \langle v_T \rangle = \frac{r \Delta \theta}{\Delta t}$$

define  $\langle \omega \rangle = \frac{\Delta \theta}{\Delta t}$

$$\langle v_T \rangle = r \langle \omega \rangle$$

# Tangential Vs. Angular Velocity



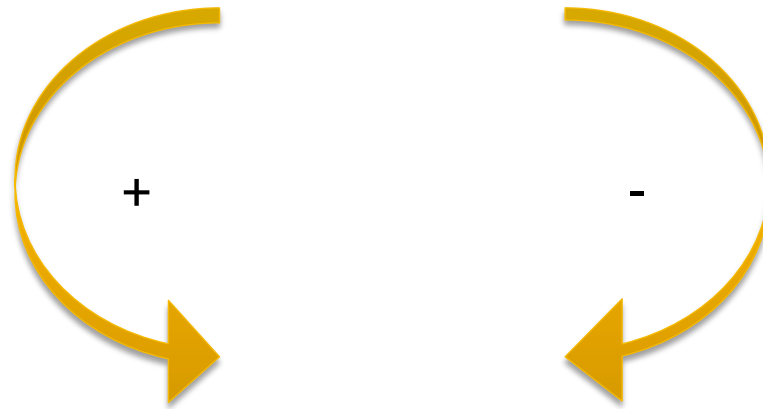
# Tangential Velocity Vs. Angular Velocity

- Tangential velocity is the velocity of a body in the direction of the tangent to a circle
  - Tangential velocity (SI Unit: m/s) depends on the radius of the motion
- Angular velocity is the rate at which the angle of a body with respect to a set of coordinate axes changes
  - Angular velocity (SI Unit: rad/s) does not depend on the radius of the motion



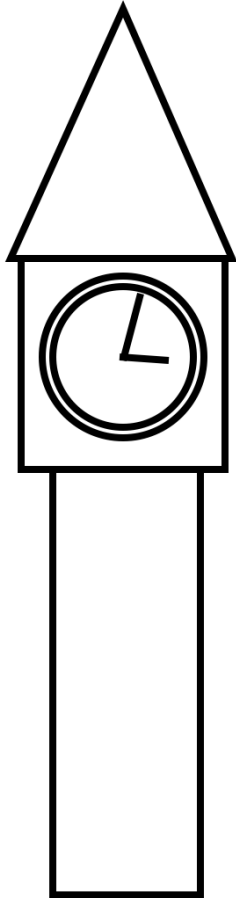
# Important Convention

- Angles and angular velocities are defined as positive if they are counter-clockwise, negative if they are clockwise



# Concept Check

BIG BEN and a little alarm clock both keep perfect time.  
Which minute hand has the bigger angular velocity  $\omega$ ?

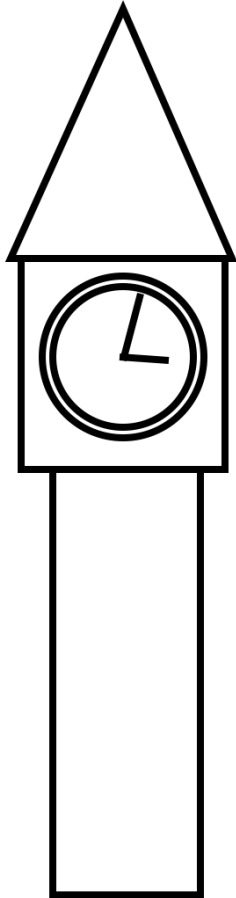


- A) Big Ben
- B) little alarm clock
- C) Both have the same  $\omega$



# Concept Check

BIG BEN and a little alarm clock both keep perfect time.  
Which minute hand has the bigger angular velocity  $\omega$ ?



A) Big Ben

B) little alarm clock

C) Both have the same  $\omega$



Big Ben:

$$\omega_b = \frac{2\pi \text{ rad}}{60 \text{ s}}$$

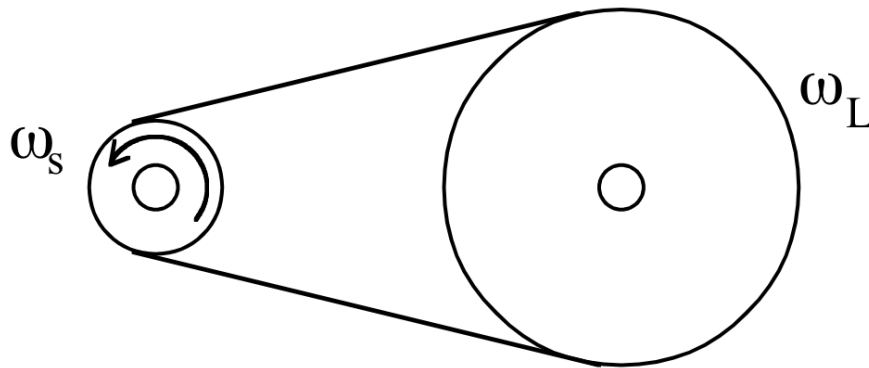
Little Clock:

$$\omega_c = \frac{2\pi \text{ rad}}{60 \text{ s}}$$

$$\omega_b = \omega_c$$

# Concept Check

A small wheel and a large wheel are connected by a belt. The small wheel is turned at a constant angular velocity  $\omega_s$ . How does the magnitude of the angular velocity of the large wheel  $\omega_L$  compare to that of the small wheel?



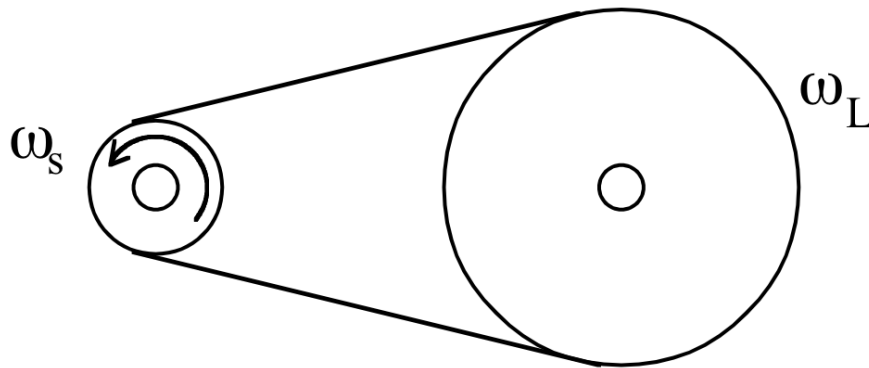
A:  $\omega_s = \omega_L$

B:  $\omega_s > \omega_L$

C:  $\omega_s < \omega_L$

# Concept Check

A small wheel and a large wheel are connected by a belt. The small wheel is turned at a constant angular velocity  $\omega_s$ . How does the magnitude of the angular velocity of the large wheel  $\omega_L$  compare to that of the small wheel?



A:  $\omega_s = \omega_L$

B:  $\omega_s > \omega_L$

C:  $\omega_s < \omega_L$

Bicycle:

Big wheel has angular  
velocity  $\omega_L$

$$\Delta\theta = \omega_L \cdot t$$

$$s_L = r_L \Delta\theta_L = r_L \omega_L \cdot t$$

since belt connects  
two wheels

$$s_s = s_L$$

$$s_s = r_s \Delta\theta_s = r_s \omega_s \cdot t$$

$$\text{so: } r_L \omega_L \cdot t = r_s \omega_s \cdot t$$

$$\Rightarrow r_L \omega_L = r_s \omega_s$$

$$\Rightarrow \omega_s = \frac{r_L}{r_s} \omega_L$$

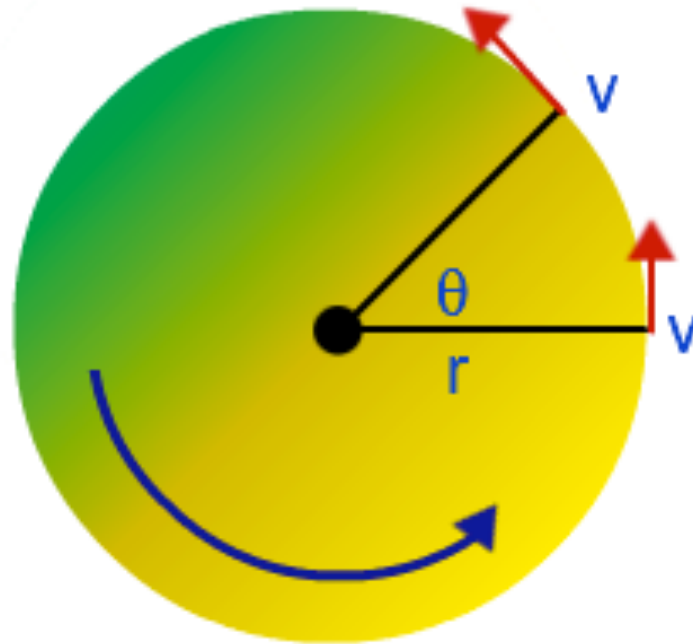
$$\boxed{> \omega_L}$$

# Bicycle Gears





# Rotational Quantities III: Angular Acceleration



Average angular acceleration =  $\frac{\text{Change in angular velocity}}{\text{Elapsed time}}$

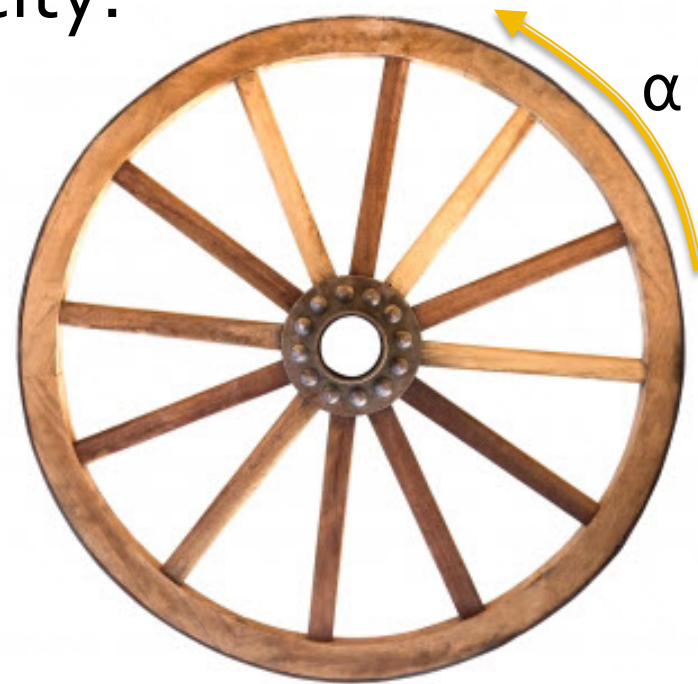
$$\bar{\alpha} = \frac{\omega - \omega_o}{t - t_o} = \frac{\Delta\omega}{\Delta t}$$

**SI Unit of Angular acceleration:** radian per second per second (rad/s<sup>2</sup>)

# Concept Check

- A wheel starts from rest and undergoes an angular acceleration of  $4 \text{ rad/s}^2$ . After  $5 \text{ s}$ , what is its angular velocity?

- A.  $10 \text{ rad/s}$
- B.  $20 \text{ rad/s}$
- C.  $40 \text{ rad/s}$
- D.  $5 \text{ rad/s}$
- E.  $0 \text{ rad/s}$



# Concept Check

- A wheel starts from rest and undergoes an angular acceleration of  $4 \text{ rad/s}^2$ . After  $5 \text{ s}$ , what is its angular velocity?

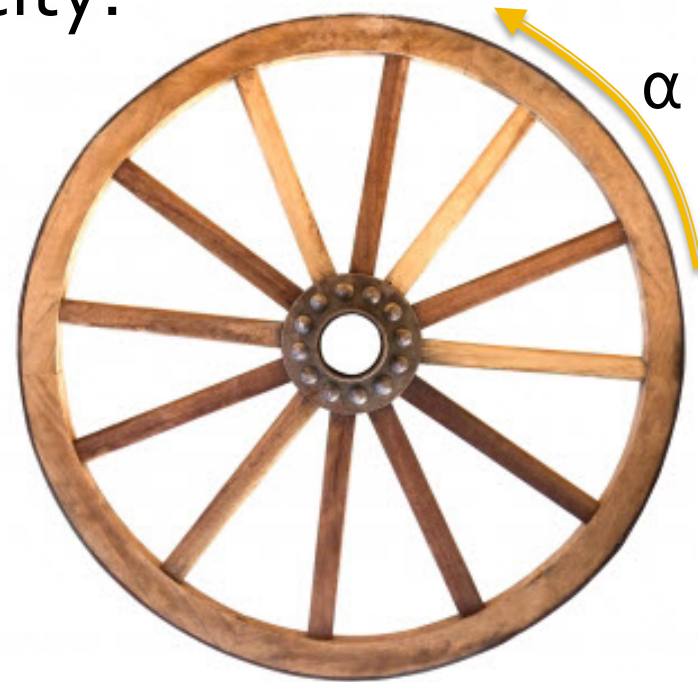
A.  $10 \text{ rad/s}$

B.  $20 \text{ rad/s}$

C.  $40 \text{ rad/s}$

D.  $5 \text{ rad/s}$

E.  $0 \text{ rad/s}$



$$\alpha = \frac{\Delta \omega}{\Delta t}$$

$$= (\omega - \omega_0) / \Delta t$$

$$\Rightarrow \omega - \omega_0 = \alpha \Delta t$$

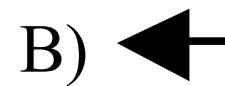
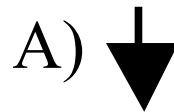
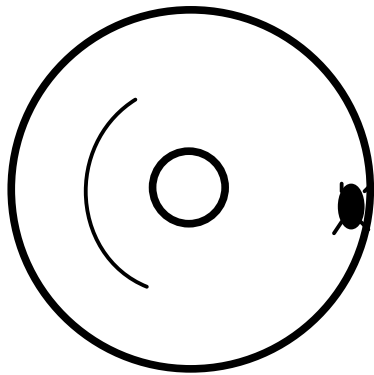
$$\text{or } \omega = \omega_0 + \alpha \Delta t$$

$$= 0 + 4 \cdot 5$$

$$= \boxed{20 \text{ rad/s}}$$

# Concept Check

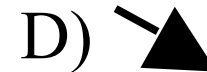
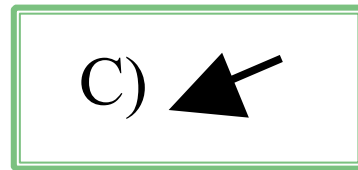
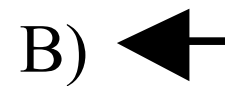
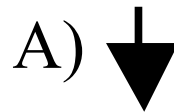
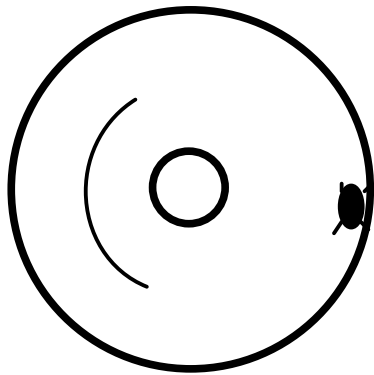
A ladybug is clinging to the rim of a spinning wheel which is spinning CCW very fast and is slowing down. At the moment shown, what is the approximate direction of the ladybug's total acceleration?



E) None of these

# Concept Check

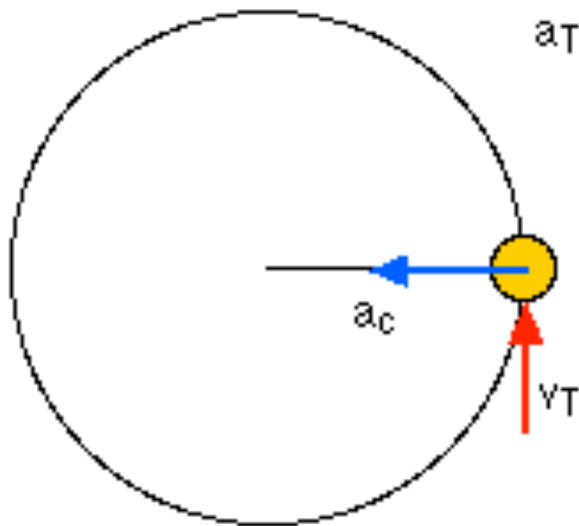
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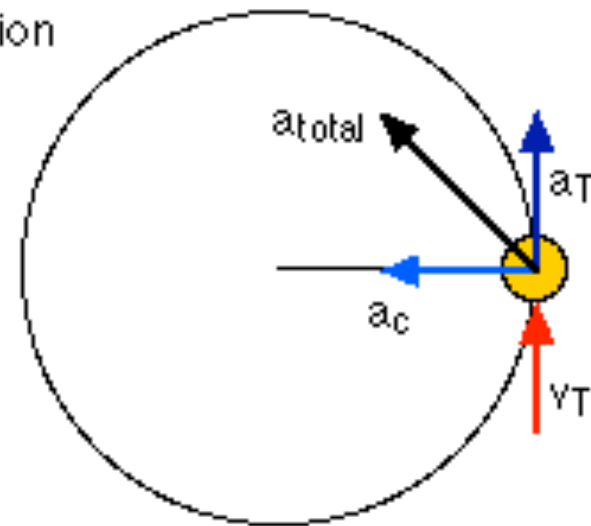
E) None of these

# Centripetal and Tangential Acceleration

$a_c$  = centripetal acceleration  
 $a_T$  = tangential acceleration  
 $v_T$  = tangential velocity

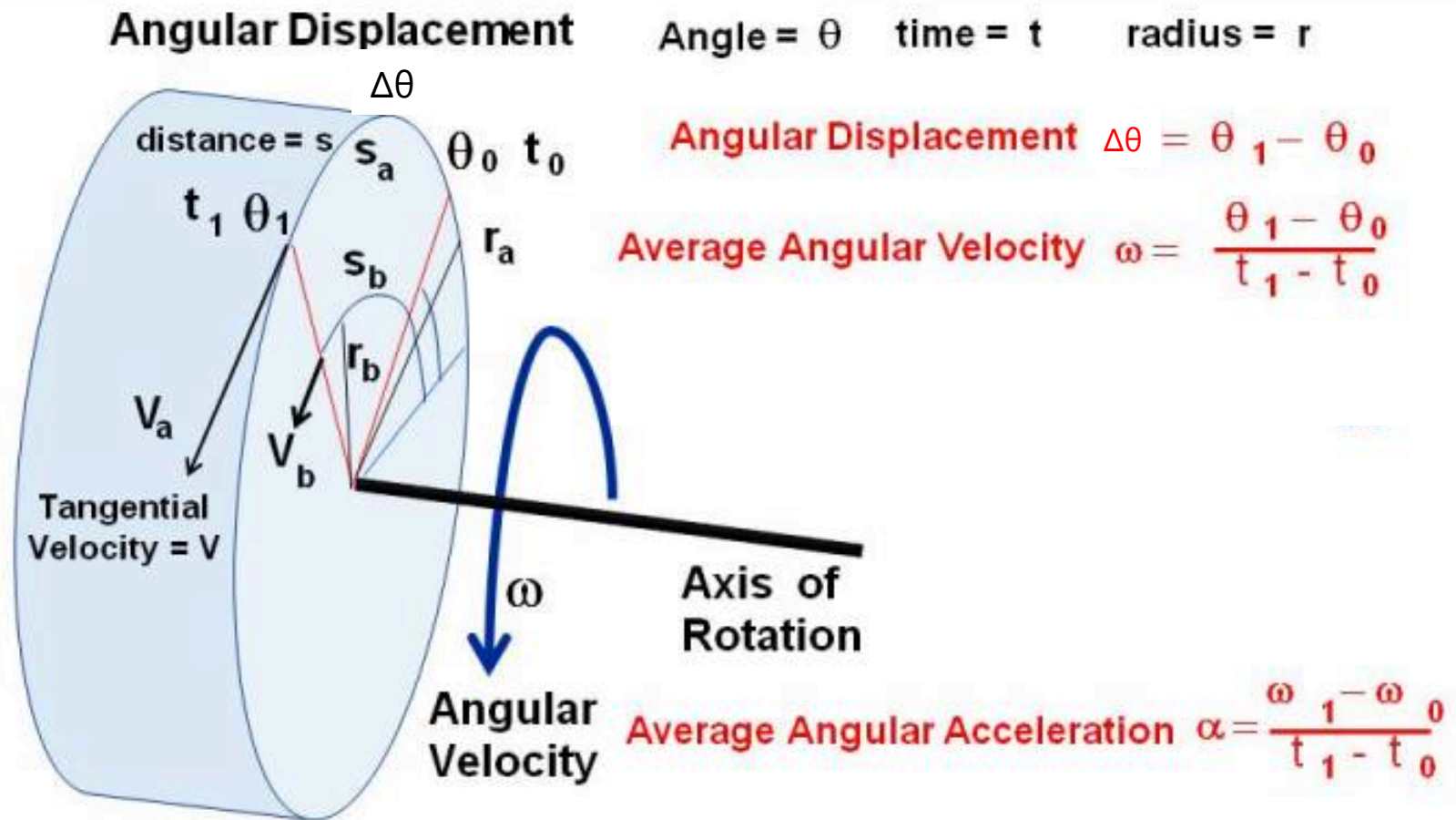


Uniform circular motion :  
constant speed,  
constant angular velocity



Non-uniform circular motion:  
changing speed,  
changing angular velocity  
(this picture represents increasing speed)

# Angular Kinematic Variables





# Angular Vs. Tangential Variables

## Linear and Rotational Quantities

Linear	Type	Rotational	Relation ( $\theta$ in radians)
$s = x$	displacement	$\theta$	$s = x = R\theta$
$v$	velocity	$\omega$	$v = R\omega$
$a_{\text{tan}}$	acceleration	$\alpha$	$a_{\text{tan}} = R\alpha$