College Physics I: 1511 Mechanics & Thermodynamics

Professor Jasper Halekas
Van Allen Lecture Room 1
MWF 8:30-9:20 Lecture

Public Service Announcement

- Iowa Voter Registration Deadline is October 29 (two weeks away).
- Make sure you are registered
- Make sure your voice is heard!

Equilibrium

$\Sigma F = ma$ $\Sigma \tau = I\alpha$

- Acceleration = 0
  - Net External Force = 0
    - All external forces balanced

- Angular acceleration = 0
  - Net External Torque = 0
    - All external torques balanced

Concept Check

- A meter stick is balanced at its midpoint. I put a 1 kg mass at the end of the stick (x = 50 cm). If I also have a 5 kg mass, where should I place it so that the meter stick is balanced?

A. x = -50 cm
B. x = -10 cm
C. x = 10 cm
D. x = -20 cm
E. x = -25 cm

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$$\sum \tau = \sum F r \sin\theta_{Fr}$$

$$= \sum F l$$

$$\tau_1 = F_1 x_1 \quad CW$$

$$= -m_1 g x_1$$

$$= -g \cdot 0.5$$

$$\tau_2 = F_2 x_2 \quad CCW$$

$$= -m_2 g \, x_2$$

$$= -5 \cdot g \, x_2$$

$$\sum \tau = \tau_1 + \tau_2$$

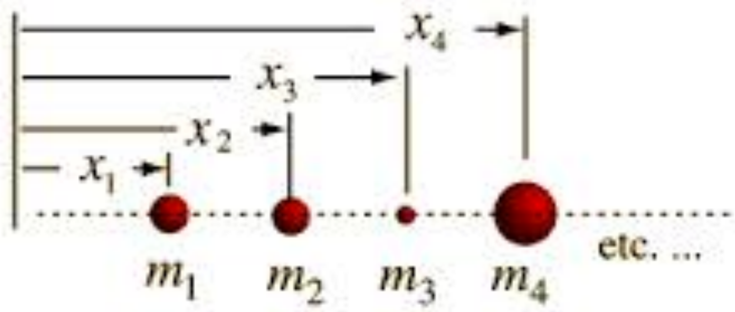
$$= -5g \, x_2 - 0.5g = 0$$

$$\Rightarrow 5g \, x_2 = -0.5g$$

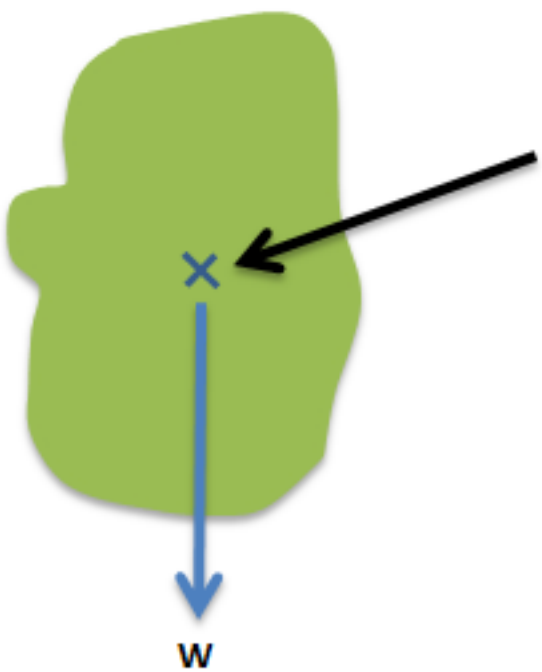
$$\Rightarrow x_2 = -0.1 \text{ m}$$

$$= \boxed{-10 \text{ cm}}$$

Center of Mass / Center of Gravity



$$x_{cm} = \frac{\sum_{i=1}^{N} m_i x_i}{M}$$



Center of Mass

(This would also be **center of Gravity** since the object is in uniform gravitational field)

$$\tau = F \cdot x$$

$$\tau_i = F_i \cdot x_i \quad \text{for } i = 1, 2, \ldots$$

$$\tau_{total} = \Sigma \tau_i$$

$$= \Sigma F_i x_i$$

if $F_i = m_i g$

$$\tau_{total} = \Sigma m_i g x_i$$

but $\Sigma m_i x_i = M x_{cm}$

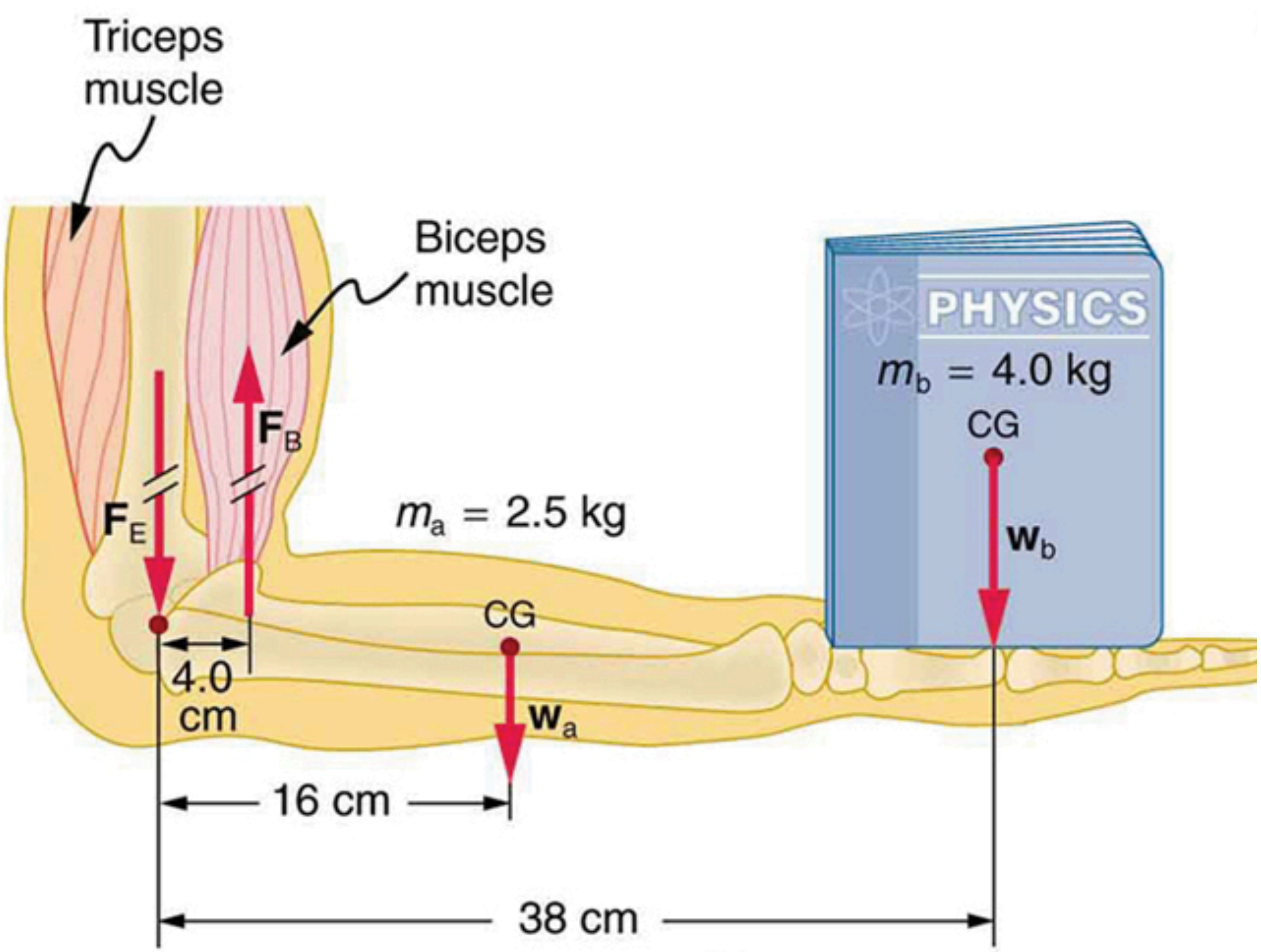
$$\Rightarrow \tau_{total} = M g x_{cm}$$

Acts as if all mass @ center of mass

Gravitational Torque on a Solid Object

- Torque = Force F * Lever Arm L
  - Torque on a portion of an object m_i at position x_i
    - $\tau_i = F_i * x_i$
  - Total torque
    - $\tau = \Sigma F_i x_i = \Sigma m_i a_i x_i = M*(\Sigma m_i a_i x_i/M) = M*g*L_c$
    - Since gravitational acceleration g is the same for every portion of the object:
      - Can treat object as if total mass M at center of mass L_c
- Use CM/CG with translational force/acceleration

Torque and Center of Mass



(a)

Pick elbow as pivot.

$\tau_E = F_E r_E = 0$

$\tau_B = F_B \cdot r_B = F_B \cdot 0.04$

$\tau_a = -m_a g \cdot r_a = -2.5 \cdot g \cdot 0.16$

$\tau_b = -m_b g \cdot r_b = -4 g \cdot 0.38$

$\Sigma \tau = 0$

$= F_B \cdot 0.04 - 2.5 \cdot g \cdot 0.16 - 4 g \cdot 0.38$

$= F_B \cdot 0.04 - 3.92 - 14.9$

$= F_B \cdot 0.04 - 18.82 \text{ Nm}$

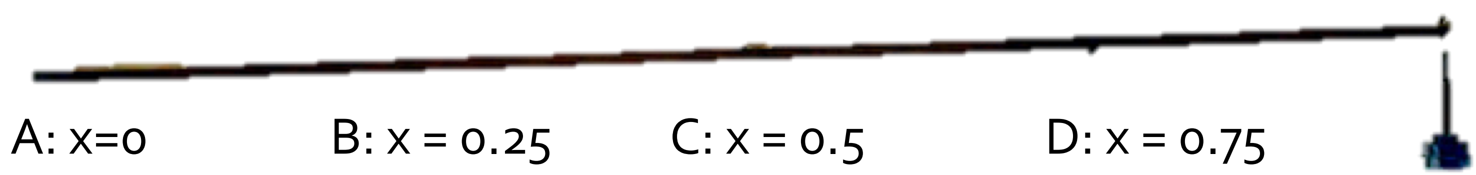
$\Rightarrow F_B \cdot 0.04 = 18.82$

or $\boxed{F_B = 470 \text{ N}}$

– Much bigger than weight since lever arm so small

Concept Check

- Imagine that we balance a meter stick weighing 160 g with a suspended weight on one end that also weighs 160 g. Where should the balance point be located?



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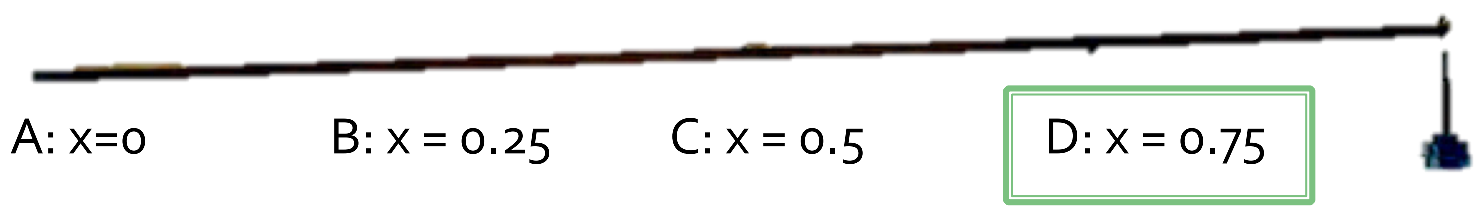
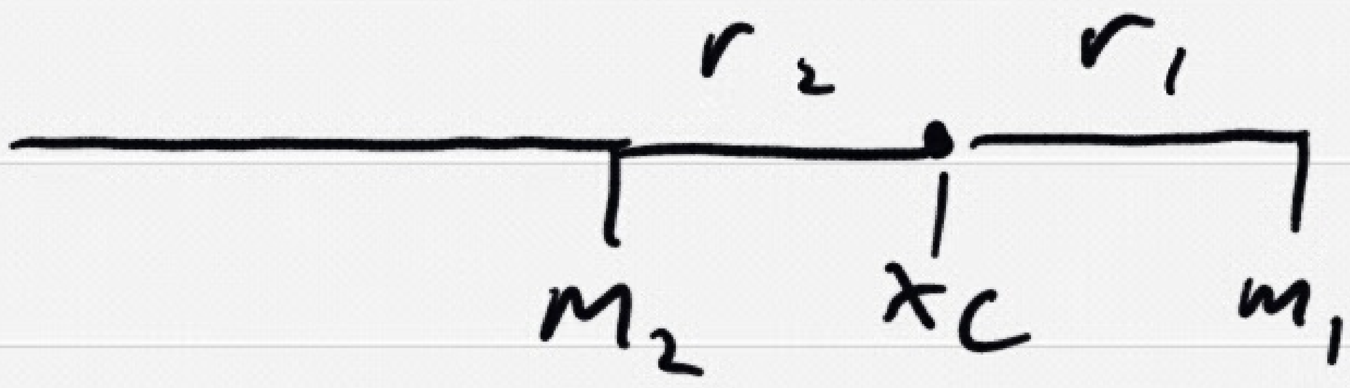


Treat meter stick as if all mass @ CM.

- CM is at $x = 0.5$
- Put pivot @ $x = x_c$

so 

$m_1 = m_2 = 160\,g$

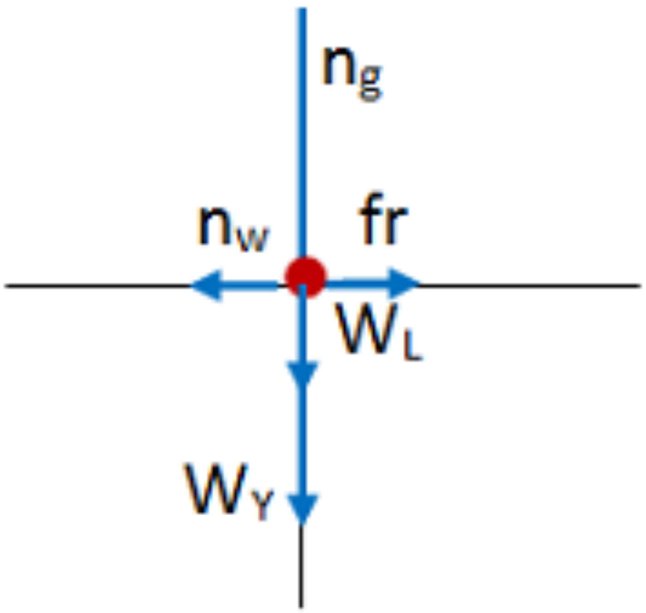
$$r_1 = 1 - x_c$$
$$r_2 = x_c - 0.5$$

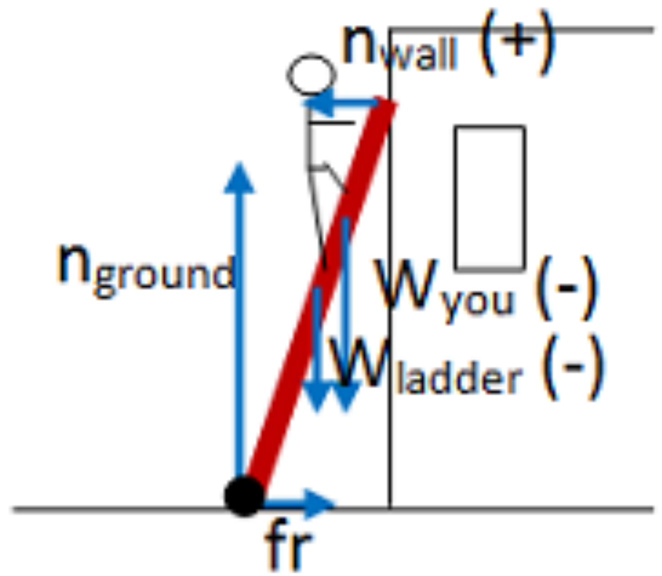
$$|\tau_1| = |\tau_2|$$

$$m_1 (1 - x_c) = m_2 (x_c - 0.5)$$

$$1 - x_c = x_c - 0.5$$
$$1 = 2x_c - 0.5$$
$$1.5 = 2x_c$$
$$\boxed{x_c = 0.75}$$

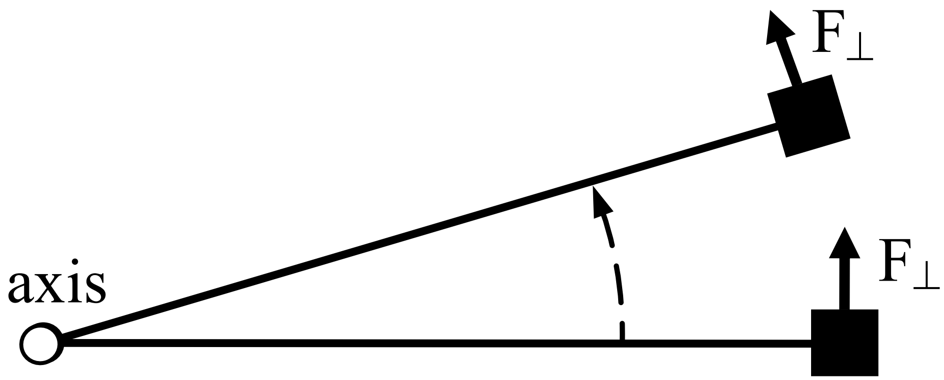
Non-Equilibrium





$\Sigma F = ma$ $\Sigma \tau = I\alpha$

Moment of Inertia



$I = mr^2$ = “Moment of Inertia”

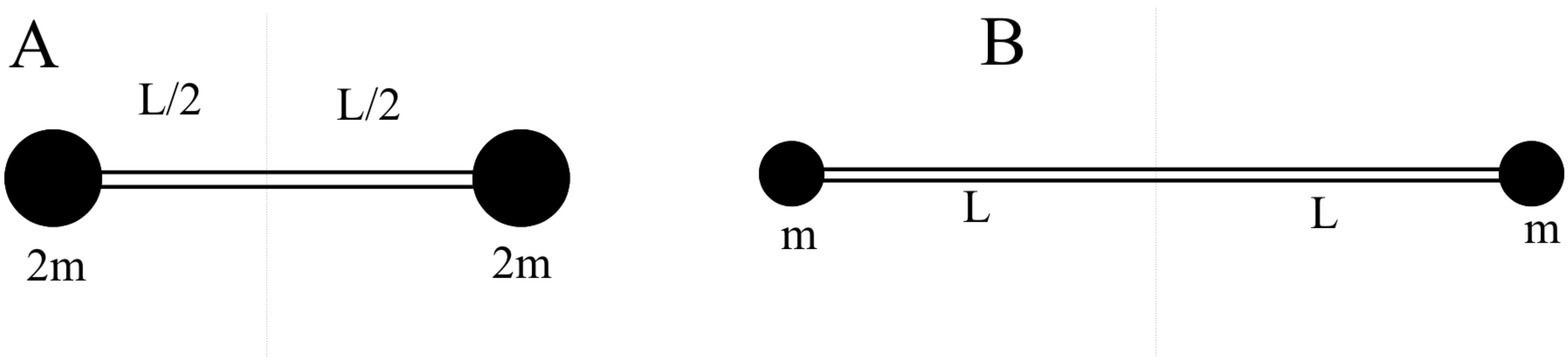
Moment of Inertia

$$I = \sum_{i=1}^{N} m_i r_i^2 \qquad N \text{ point particles}$$

$$= \int_0^M r^2\, dm \qquad \text{Solid of mass } M$$

Concept Check

Consider two masses, each of size 2m at the ends of a light rod of length L with the axis of rotation through the center of the rod. The rod is doubled in length and the masses are halved. What happens to I?



A: I_A is bigger

B : I_B is bigger

C: $I_A = I_B$

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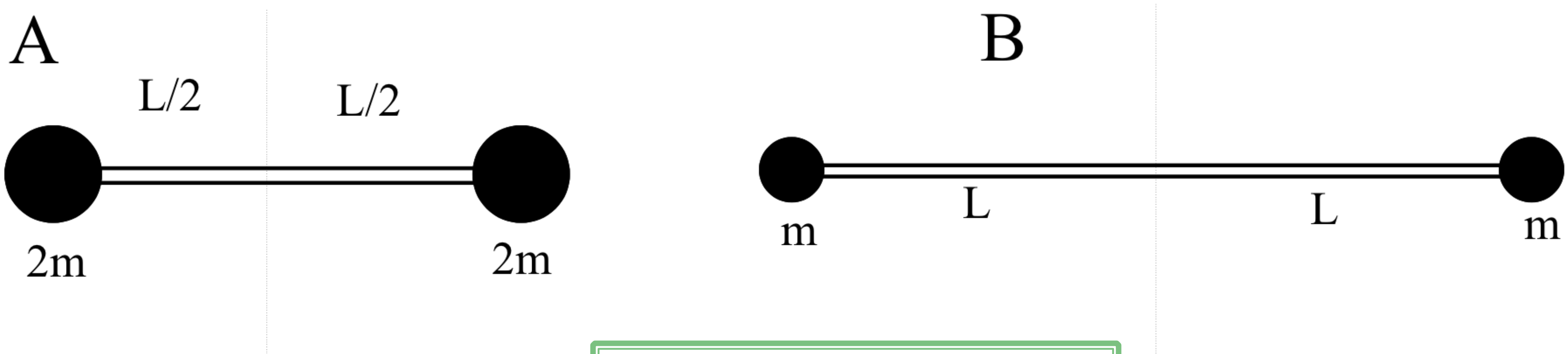

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$$I = \Sigma m r^2$$

$$I_1 = 2m \cdot (L/2)^2 + 2m(L/2)^2$$

$$= 2m L^2/4 + 2m L^2/4$$

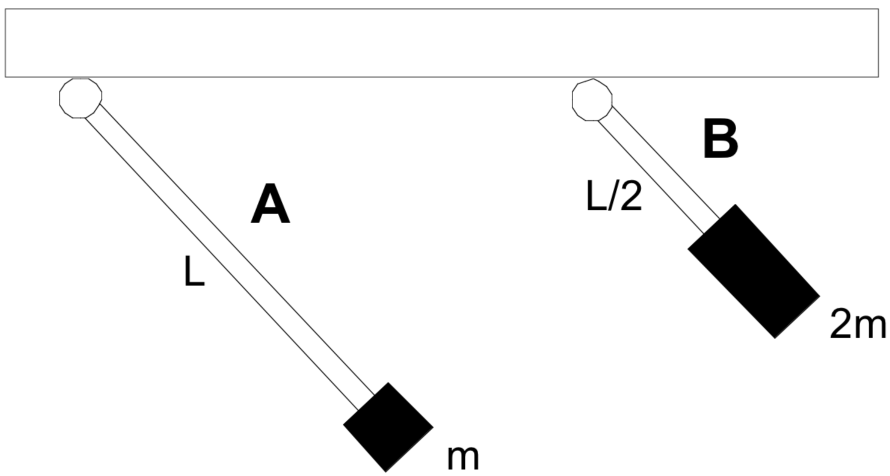
$$= mL^2$$

$$I_2 = mL^2 + mL^2$$

$$= 2mL^2$$

Concept Check

Two light (massless) rods, labeled A and B, each are connected to the ceiling by a frictionless pivot.
Rod A has length L and has mass m at the end of the rod.
Rod B has length L/2 and has a mass 2m at its end.
Both rods are released from rest in a horizontal position.



Which one experiences the larger torque?

A: A B: B C: Both have the same size τ.

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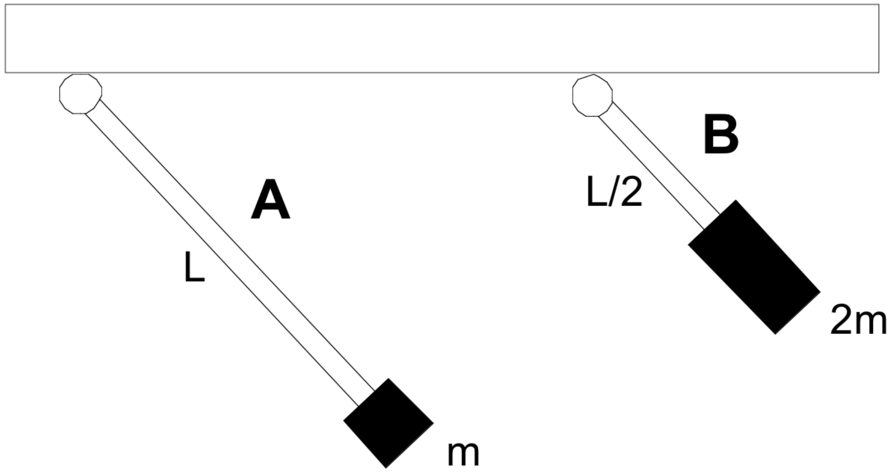

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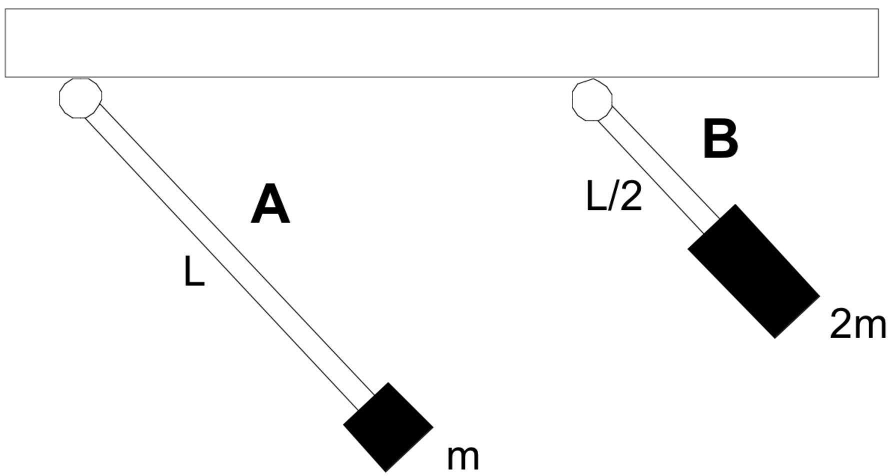

Which one falls to the vertical position fastest?

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(Hint: $\alpha = \frac{\tau}{I}$)

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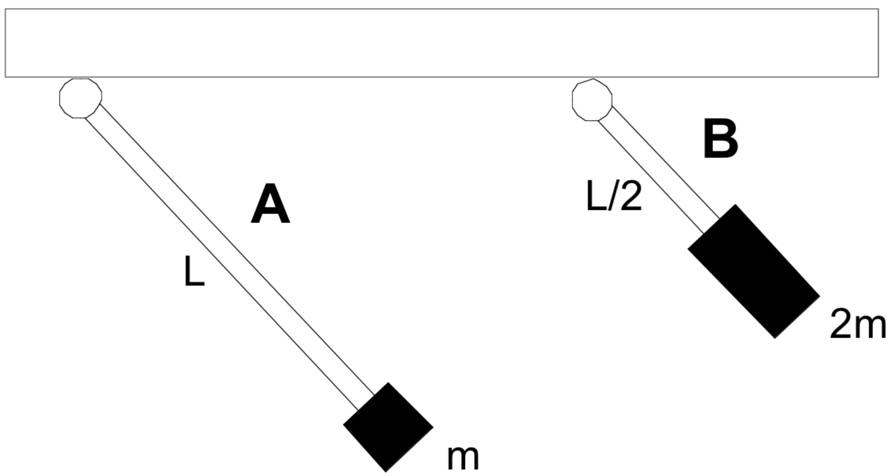

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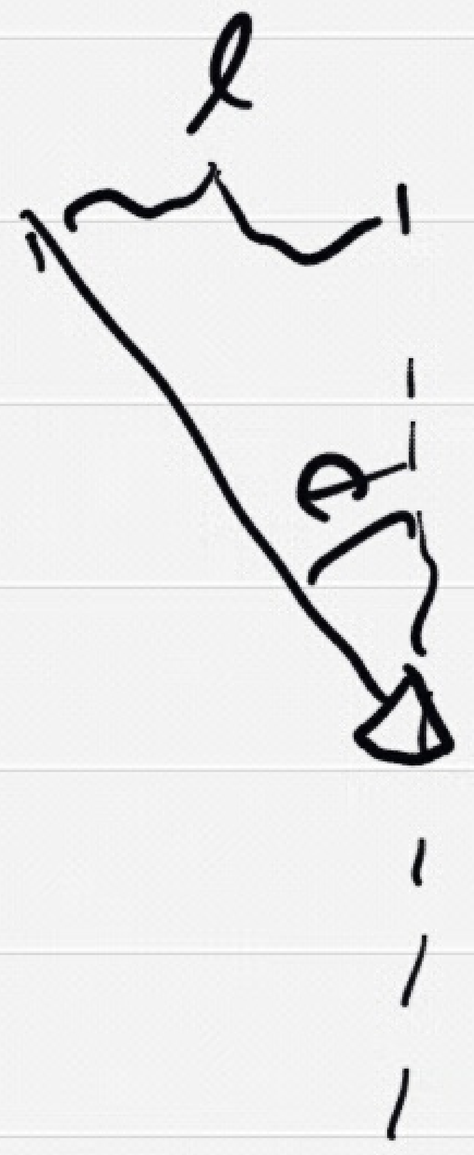

$l = L\sin\theta$

$\tau = Fl$
$= mgL\sin\theta$

$I = mL^2$

$\alpha = \tau / I$

$= \dfrac{mgL\sin\theta}{mL^2}$

$= \dfrac{g\sin\theta}{L}$

$l = L/2 \sin\theta$

$\tau = Fl$
$= 2mg \cdot L/2 \sin\theta$
$= mgL\sin\theta$

$I = 2m(L/2)^2$
$= \frac{1}{2}mL^2$

$\alpha = \tau / I$

$= \dfrac{mgL\sin\theta}{\frac{1}{2}mL^2}$

$= \dfrac{2g\sin\theta}{L}$