College Physics I: 1511 Mechanics & Thermodynamics

Professor Jasper Halekas
Van Allen Lecture Room 1
MWF 8:30-9:20 Lecture

Announcements

- Rescheduled office hours this week
  - No Tuesday office hours
  - Wednesday office hours extended 9:30-11:00
  - Thursday office hours extended 3:30-5:00

Rotational Quantities VI: Quantities Related to Moment of Inertia

Moment of Inertia

I

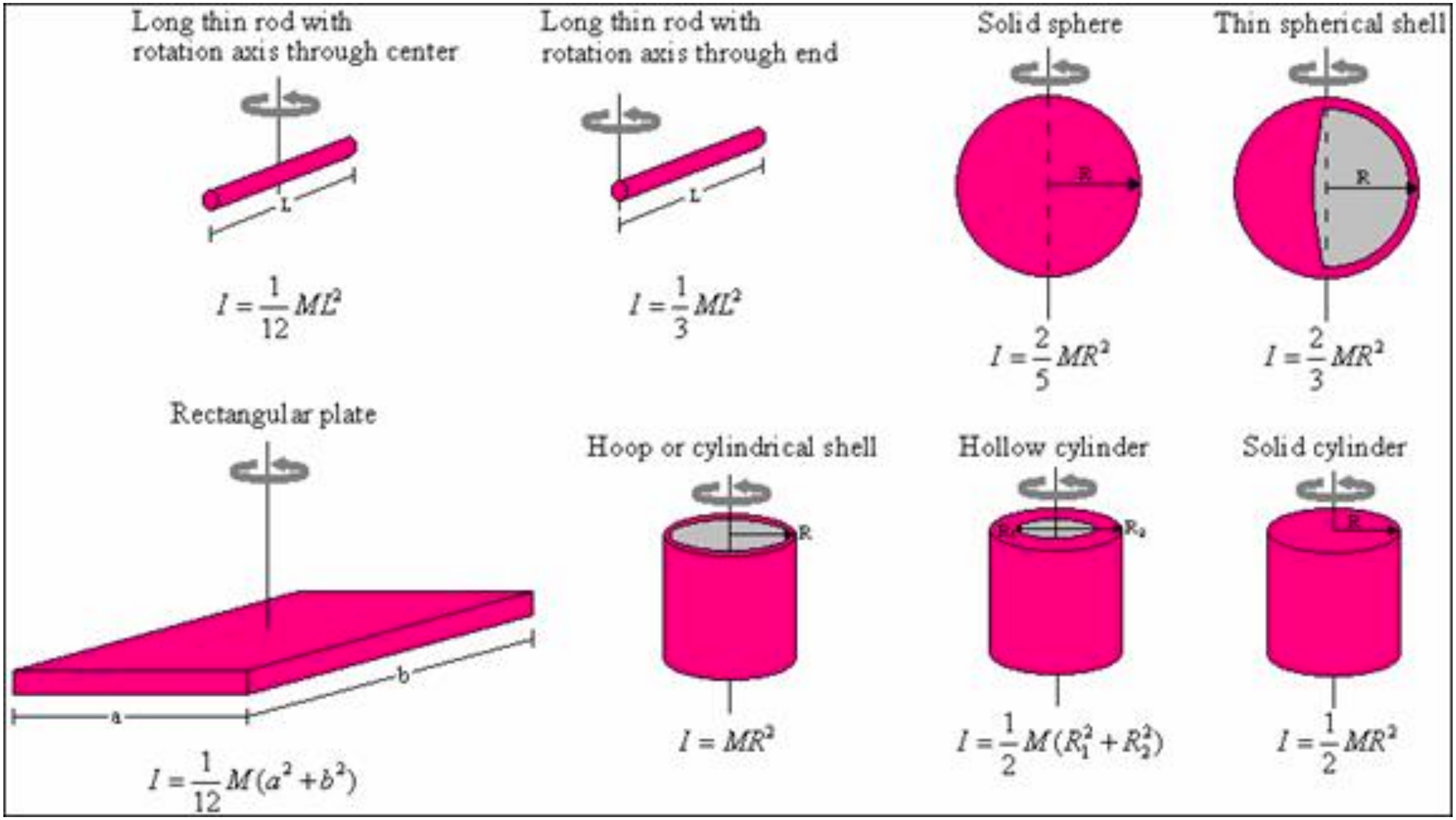
	Linear	Angular
Newton's Second Law	$F = ma$	$\tau = I\alpha$
Momentum	$p = mv$	$L = I\omega$
Kinetic Energy	$KE = \frac{1}{2}mv^2$	$KE = \frac{1}{2}I\omega^2$
Work-Energy	$F_{net}d = \Delta\left(\frac{1}{2}mv^2\right)$	$\tau_{net}\theta = \Delta\left(\frac{1}{2}I\omega^2\right)$

Linear Vs. Rotational Quantities

Units	Linear Motion		Rotational Motion		Units
meters	Position	x	θ	Angular position	radians
m/s	Velocity	$v = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$	Angular velocity	rads/s
m/s^2	Acceleration	$a = \frac{dv}{dt}$	$\alpha = \frac{d\varpi}{dt}$	Angular acceleration	$rads/s^2$
kg	Mass	m	I	Moment of inertia	kg m^2
Newtons	Force	$F = ma$	$T = I\alpha$	Torque	N m
kg m/s	Momentum	$p = mv$	$L = I\omega$	Angular Momentum	kg m^2/s
Joules	Work	$W = Fdx$	$W = Td\theta$	Work	Joules
Joules	Kinetic Energy	$K = \frac{1}{2}mv^2$	$K = \frac{1}{2}I\omega^2$	Kinetic Energy	Joules
Watts	Power	$P = Fv$	$P = T\omega$	Power	Watts

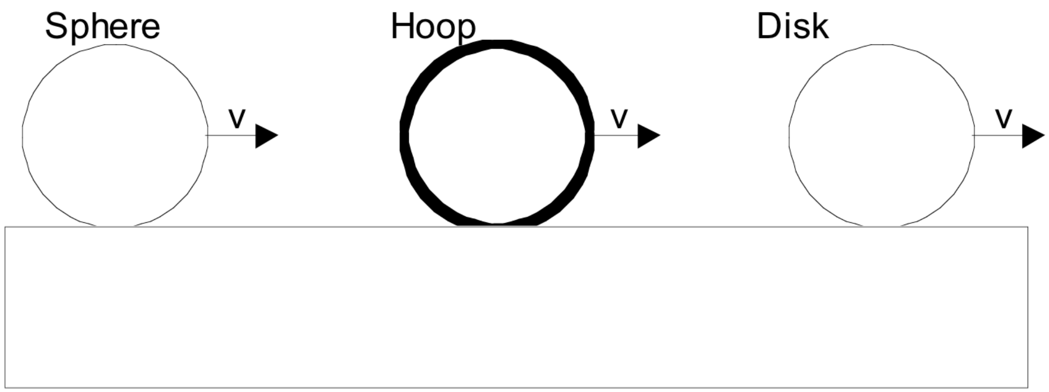
Same Units Whether Linear or Rotational

Moment of Inertia for Different Mass Distributions



Concept Check

A sphere, a hoop, and a cylinder, all with the same mass M and same radius R, are rolling along, all with the same speed v.



Which has the most kinetic energy?

A: Sphere B: Hoop C: Disk

D: All have the same KE.

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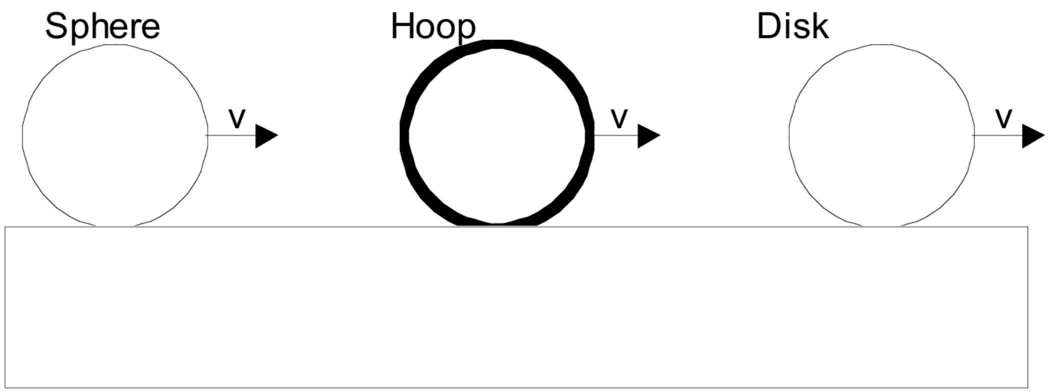

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$$KE = KE_{trans} + KE_{rot}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

for rolling $\omega = v/r$

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}I\, v^2/r^2$$

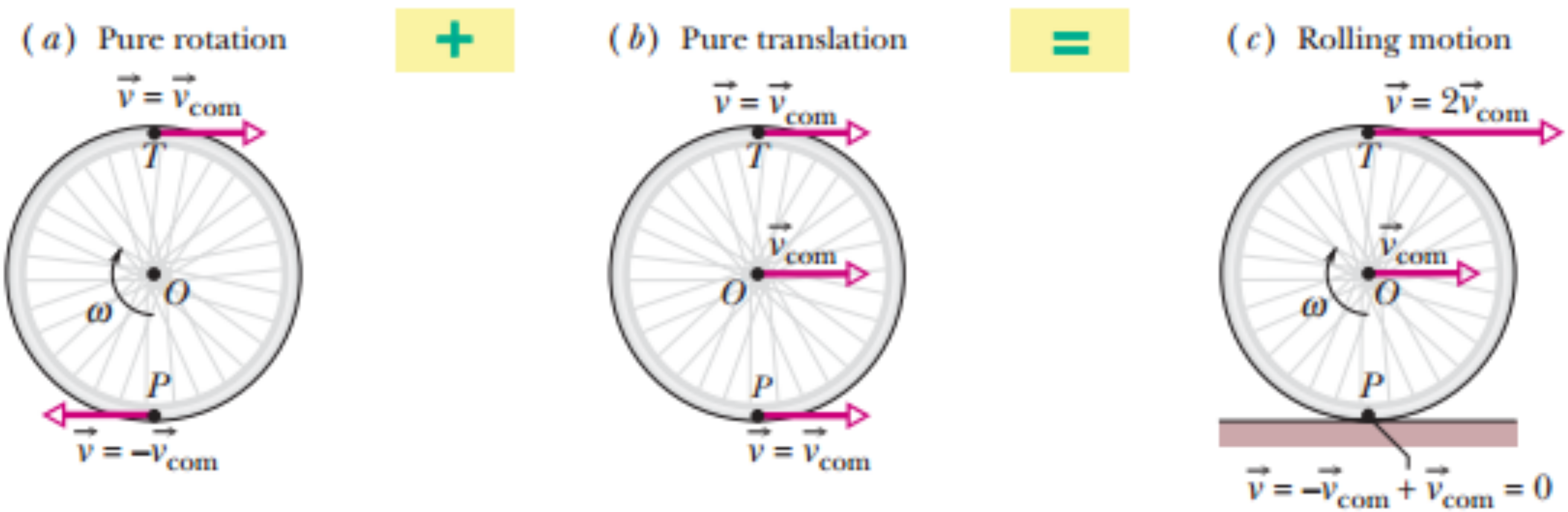
for constant v,
biggest $I \rightarrow$ biggest KE

$$I_{hoop} = mr^2$$

$$I_{cyl} = \frac{1}{2}mr^2$$

$$I_{sph} = \frac{2}{5}mr^2$$

Mixed Application: Kinetic Energy of Rolling Wheel



- Must combine translational and rotational kinetic energy
  - Translational kinetic energy = $1/2\ mv^2$
  - Rotational kinetic energy = $1/2\ I\omega^2 = 1/2\ mr^2\,(v/r)^2 = 1/2\ mv^2$
    - (Assuming all mass concentrated at the rim)
  - Total kinetic energy = $1/2\ mv^2 + 1/2\ mv^2 = mv^2$

What About a Rolling Cylinder?

- Same problem but now $I = 1/2\ m\ r^2$
  - Translational kinetic energy = $1/2\ mv^2$
  - Rotational kinetic energy = $1/2\ I\omega^2 = 1/2\ 1/2\ mr^2\ (v/r)^2 = 1/4\ mv^2$
  - Total kinetic energy = $1/2\ mv^2 + 1/4\ mv^2 = 3/4\ mv^2$

Conservation of Energy

- $E = KE + PE$

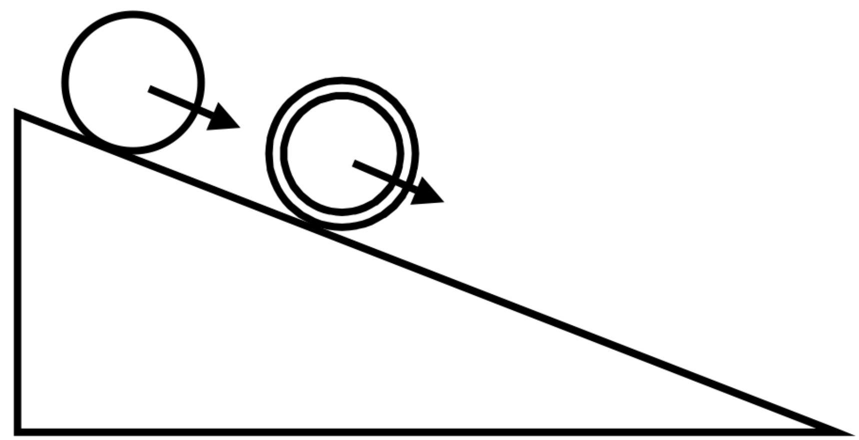
- $KE = 1/2\ mv^2 + 1/2\ I\omega^2$
- $PE = mgh + 1/2\ kx^2$

- All are forms of energy

Concept Check

A hoop and a disk, each with the same mass M and same radius R, race down a hill. Who wins?
(Assume they roll without slipping, and neglect rolling friction)

A: Hoop wins
B: Disk wins
C: Tie!



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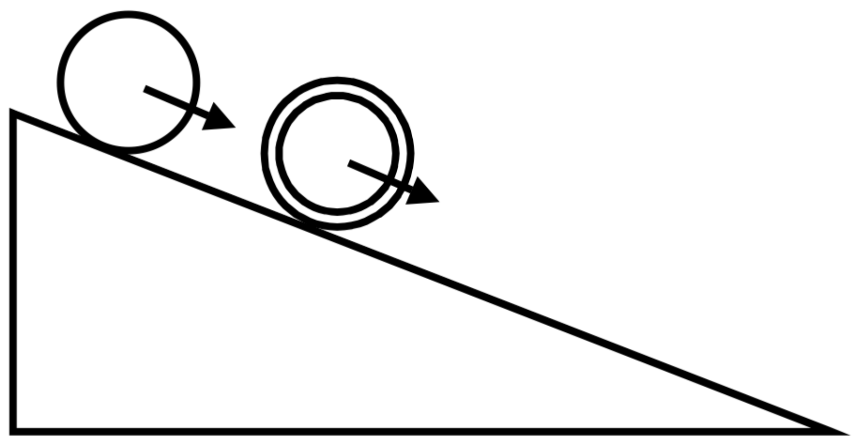


Race i

cons. energy

$PE + KE = \text{const.}$

$mgh + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \text{const.}$

$E_0 = mgh$

$E_f = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}I\,v^2/r^2$

$I_{hoop} = mr^2 \qquad I_{disk} = \frac{1}{2}mr^2$

$E_{fh} = \frac{1}{2}mv^2 + \frac{1}{2}mr^2\,v^2/r^2$

$= m v_h^2$

$E_{fd} = \frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{1}{2}mr^2\,v^2/r^2$

$= \frac{3}{4}\,m v_d^2$

$E_{fd} = E_{fh} \Rightarrow m v_h^2 = \frac{3}{4} m v_d^2$

$\Rightarrow v_h^2 = \frac{3}{4} v_d^2$

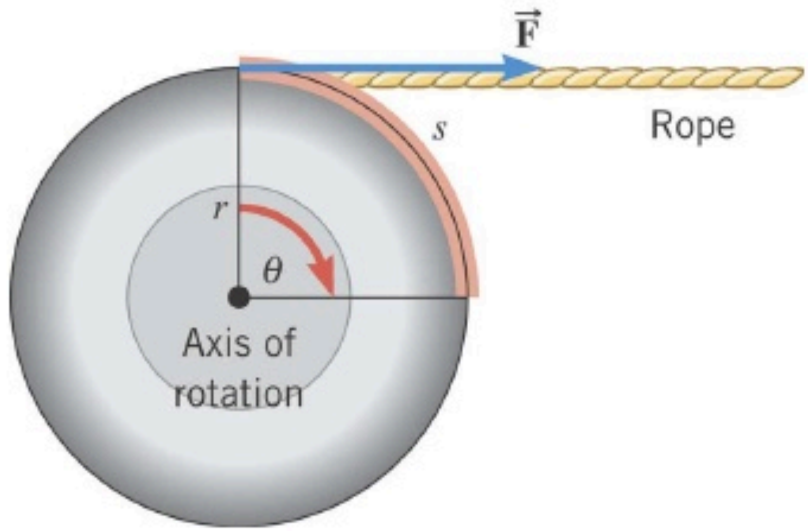
Rotational Work

$$s = r\theta$$

$$W = F \cdot s = Fr\theta$$

$$\tau = F \times r$$

$$W = \tau\theta$$



Sample Problem

- You apply your brakes to slow your 2000 kg car, which has four 10 kg wheels (10 cm radius), from traveling at 10 m/s to rest. How much work do your brakes do to stop the car and its wheels?

Braking car

Work-energy:

$$\Delta KE_{trans} = W_{trans}$$

$$= \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_0^2$$

$$= 0 - \tfrac{1}{2} m v_0^2$$

$$= -\tfrac{1}{2} \cdot 2000 \cdot 10^2$$

$$= -1000 \cdot 100$$

$$= -100{,}000 \text{ J}$$

$$\Delta KE_{rot} = W_{rot}$$

$$= \tfrac{1}{2} I \omega_f^2 - \tfrac{1}{2} I \omega_0^2$$

$$= -\tfrac{1}{2} I \omega_0^2$$

$$= -\tfrac{1}{2} m_w r^2 \cdot (v_0/r)^2$$

↑ if all mass @ rim

$$= -\tfrac{1}{2} m_w v_0^2$$

$$= -\tfrac{1}{2} \cdot 10 \cdot 10^2$$

$$= -500 \text{ J} \quad \text{per wheel}$$

$$W_{tot} = -100{,}000 - 4 \cdot 500$$

$$= \boxed{-102{,}000 \text{ J}}$$

If I stop in 1000 m, what are force & torque?

$$W_{trans} = F \Delta x$$
$$= -100{,}000 \text{ J}$$
$$= F \cdot 1000 \text{ m}$$

$$\Rightarrow F = -100{,}000/1000$$
$$= \boxed{-100 \text{ N}}$$

$$W_{rot} = \tau \Delta\theta = -2000 \text{ J}$$

$$r\Delta\theta = s = \Delta x$$
$$\Rightarrow \Delta\theta = \Delta x / r$$

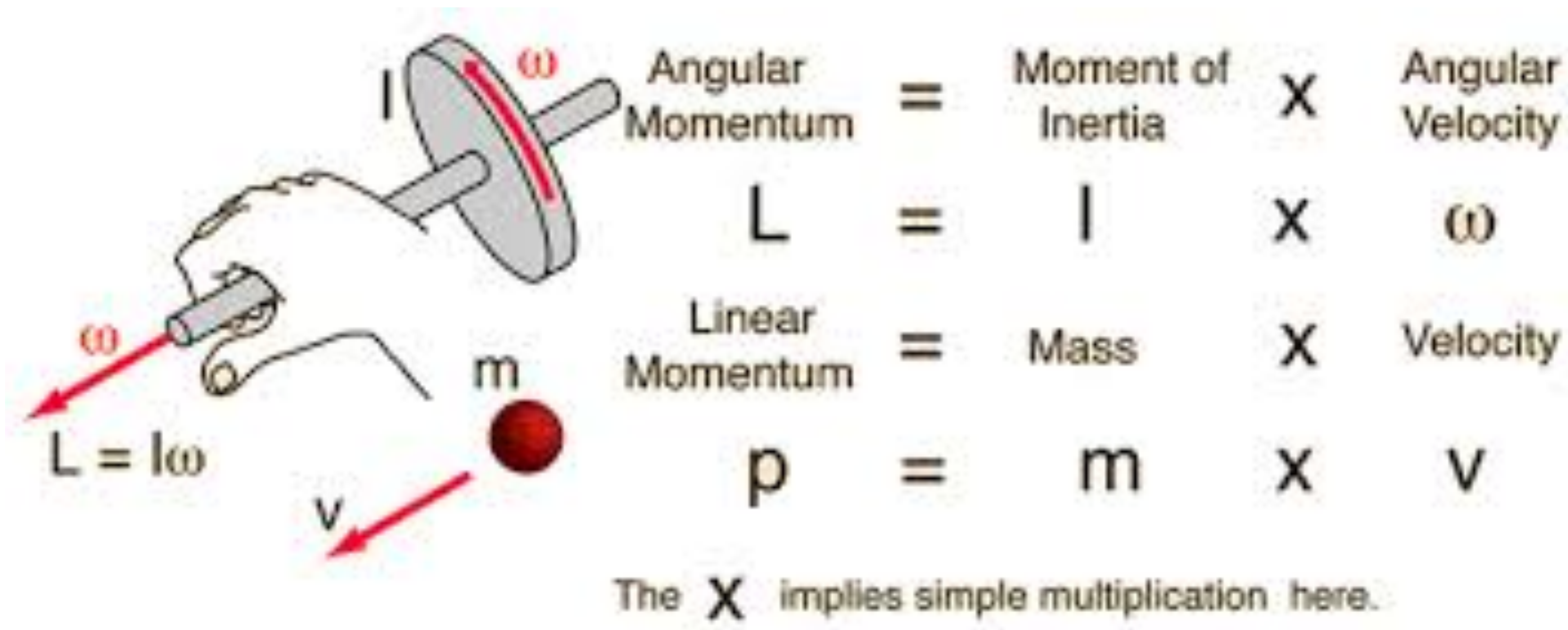
$$= 1000/0.1$$
$$= 10{,}000 \text{ rad}$$

$$\tau \cdot 10{,}000 = -2000$$

$$\tau = -2000/10{,}000$$

$$= \boxed{-0.2 \text{ Nm}}$$

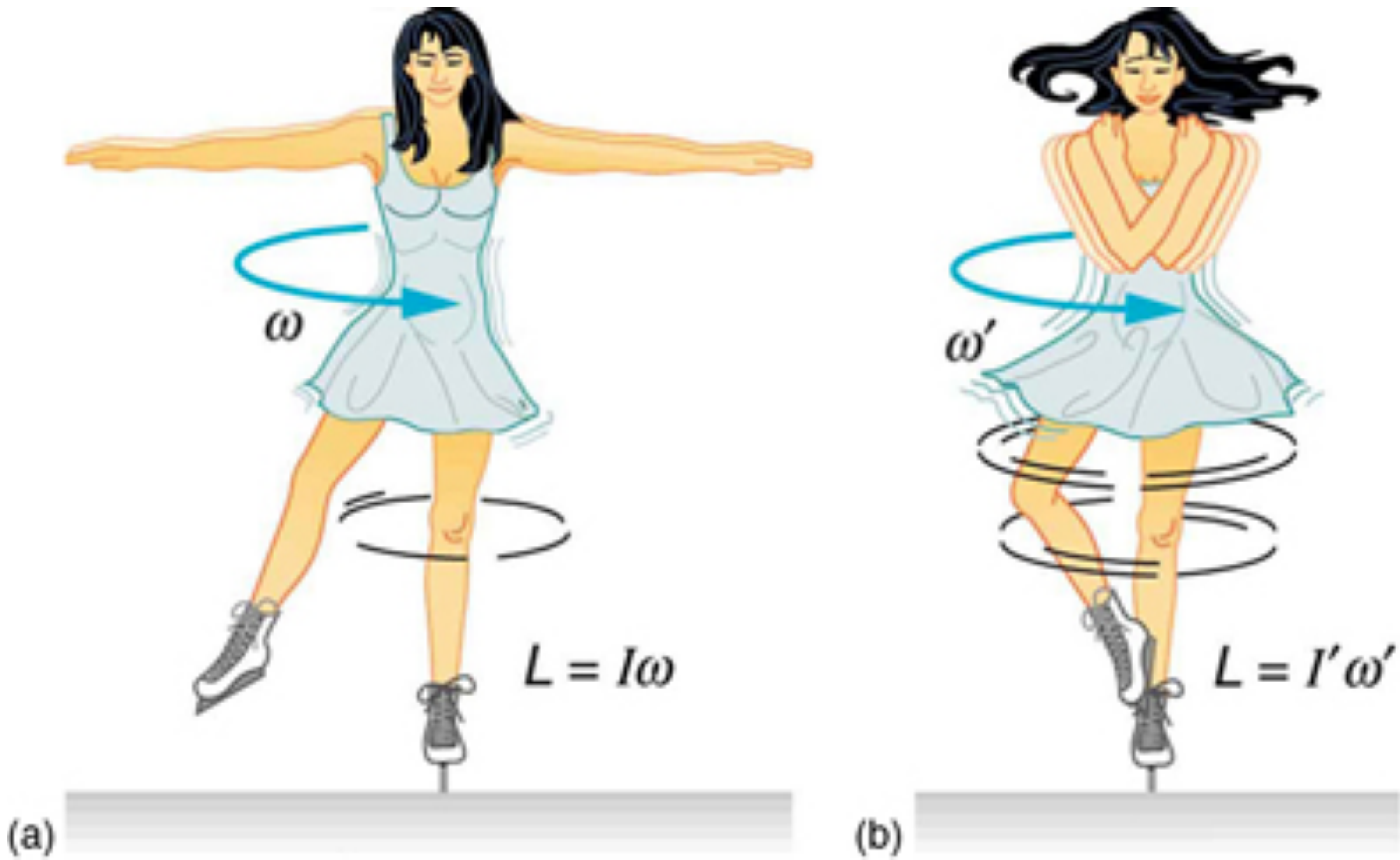
Angular Momentum



Conservation of Angular Momentum

- Just as with linear momentum:
  - The total angular momentum of a system of bodies is conserved, as long as no net external torque acts upon them

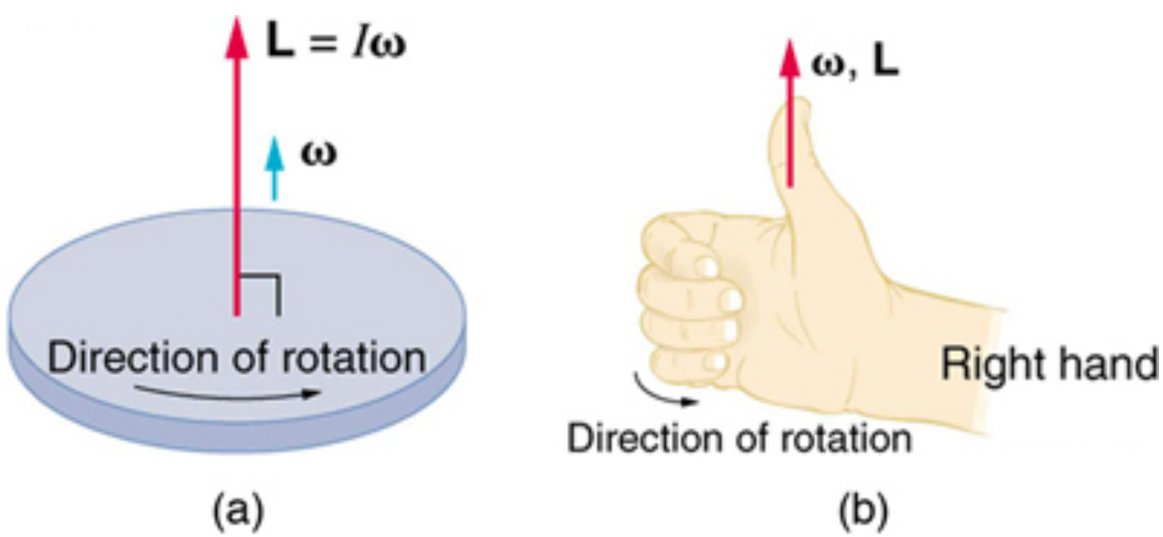
Angular Momentum: Change in Moment of Inertia



Angular Momentum: Transfer Between Bodies

- $I_1\omega_{1i} + I_2\omega_{2i} = I_1\omega_{1f} + I_2\omega_{2f}$

- This is secretly a vector equation, because the direction of the spin axes matters



Concept Check

- Imagine I hold a spinning wheel on a stool, and start with the system at rest
- What happens if I flip the wheel over?

A. Nothing
B. I spin in same direction as the wheel (in its new orientation)
C. I spin in opposite direction as the wheel (in its new orientation)
D. I fall off the stool

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