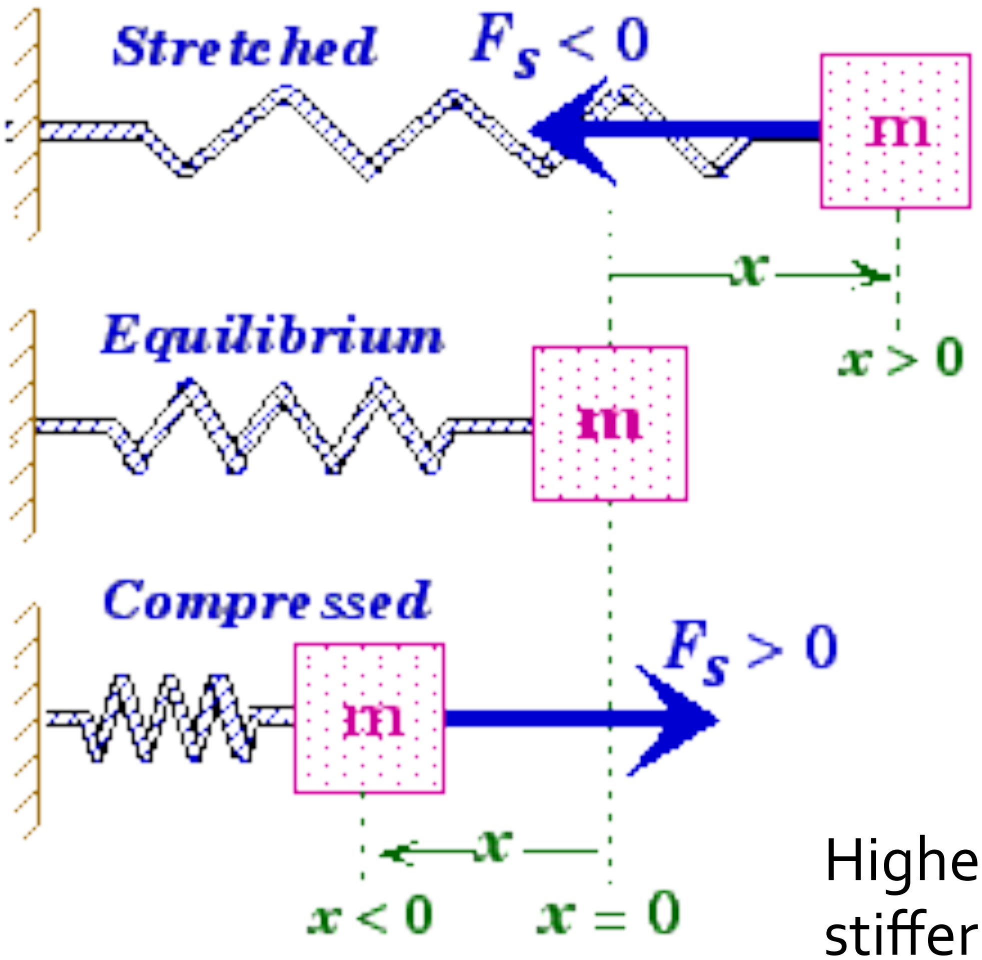
College Physics I: 1511 Mechanics & Thermodynamics

Professor Jasper Halekas
Van Allen Lecture Room 1
MWF 8:30-9:20 Lecture

Announcements

- We will only be covering part of Ch. 10
  - We already covered part of 10.1 (springs)
  - This week we will cover the remainder of 10.1, 10.3, and 10.4
  - We will not cover 10.2 or 10.5-10.8

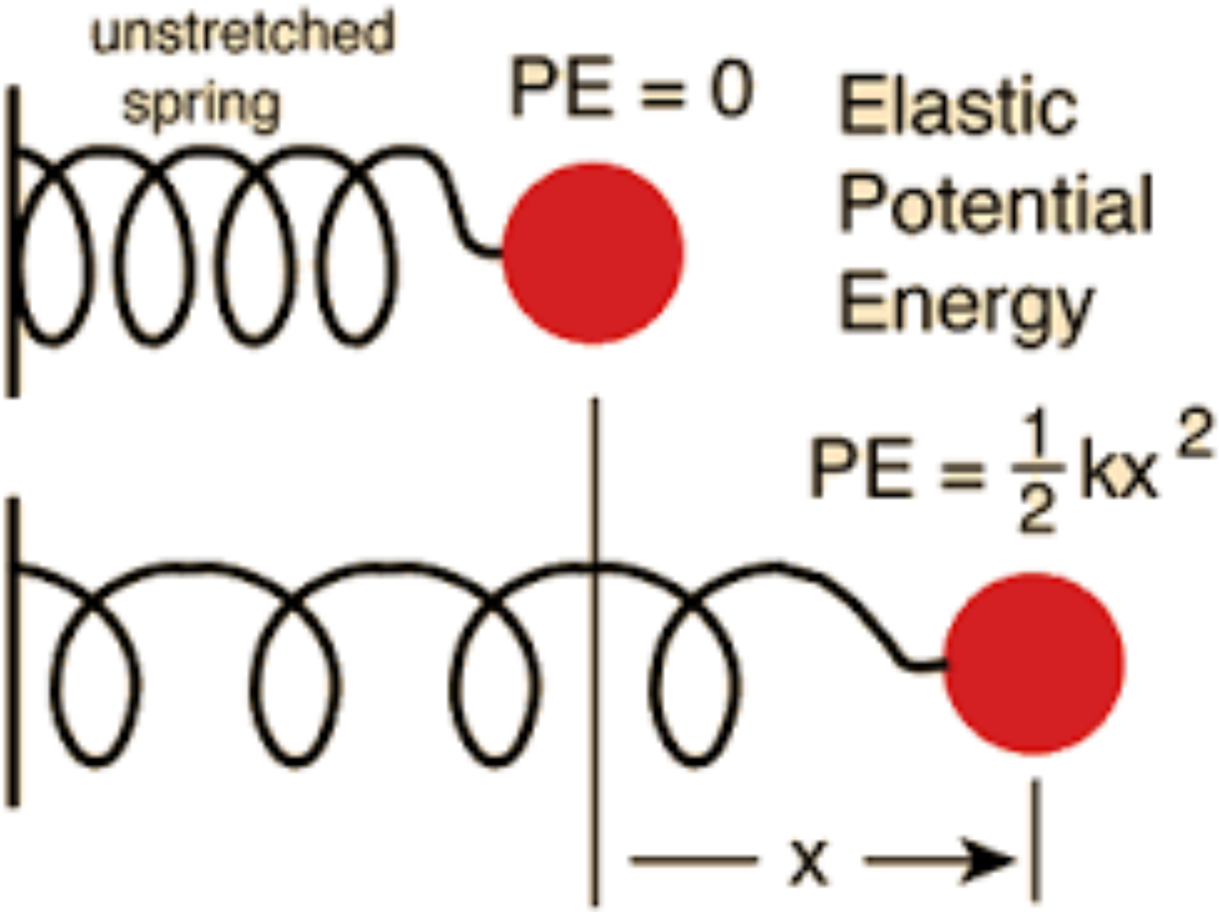
Spring Force (Hooke's Law)



$$F_s = -k\,x$$

Higher spring constant k = stiffer spring

Spring Potential Energy



Conservation of Energy

$$mgh + \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \text{constant}$$

Gravitational potential energy

spring potential energy

Kinetic energy

Concept Check

- You stretch a mass on a spring a distance x_m from its equilibrium and let go of it. The mass reaches a maximum speed v_m. You then stretch the spring a distance $2x_m$. What is the maximum speed of the mass this time?

A. $v_m/\sqrt{2}$
B. v_m
C. $\sqrt{2}\, v_m$
D. $2v_m$
E. $4v_m$

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$$E = \frac{1}{2} k x^2 + \frac{1}{2} m v^2 = \text{const.}$$

$$E_0 = \frac{1}{2} k x_m^2$$

$$E_f = \frac{1}{2} m v_m^2$$

$$\frac{1}{2} m v_m^2 = \frac{1}{2} k x_m^2$$

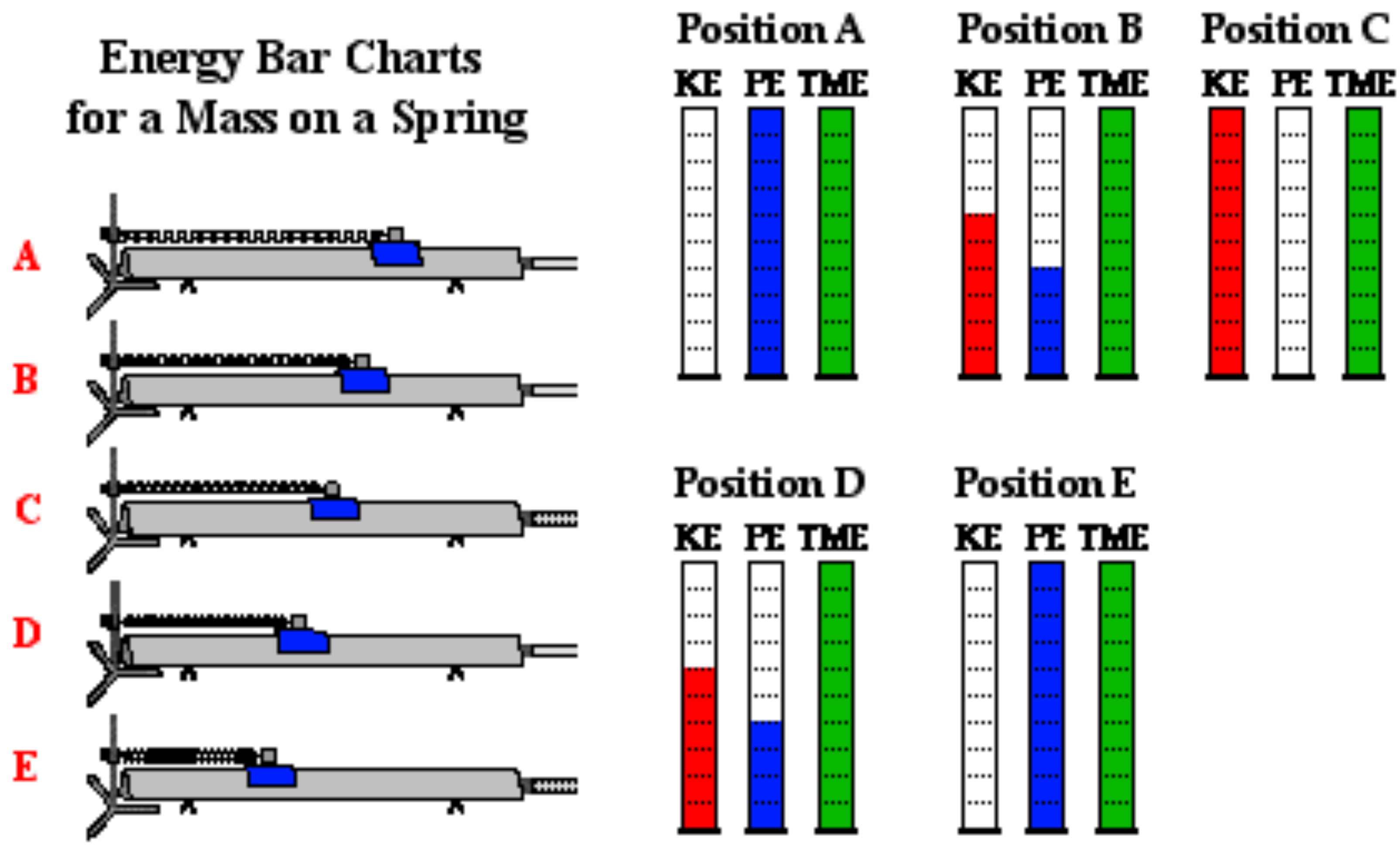
$$m v_m^2 = k x_m^2$$

$$v_m^2 = \frac{k}{m} x_m^2$$

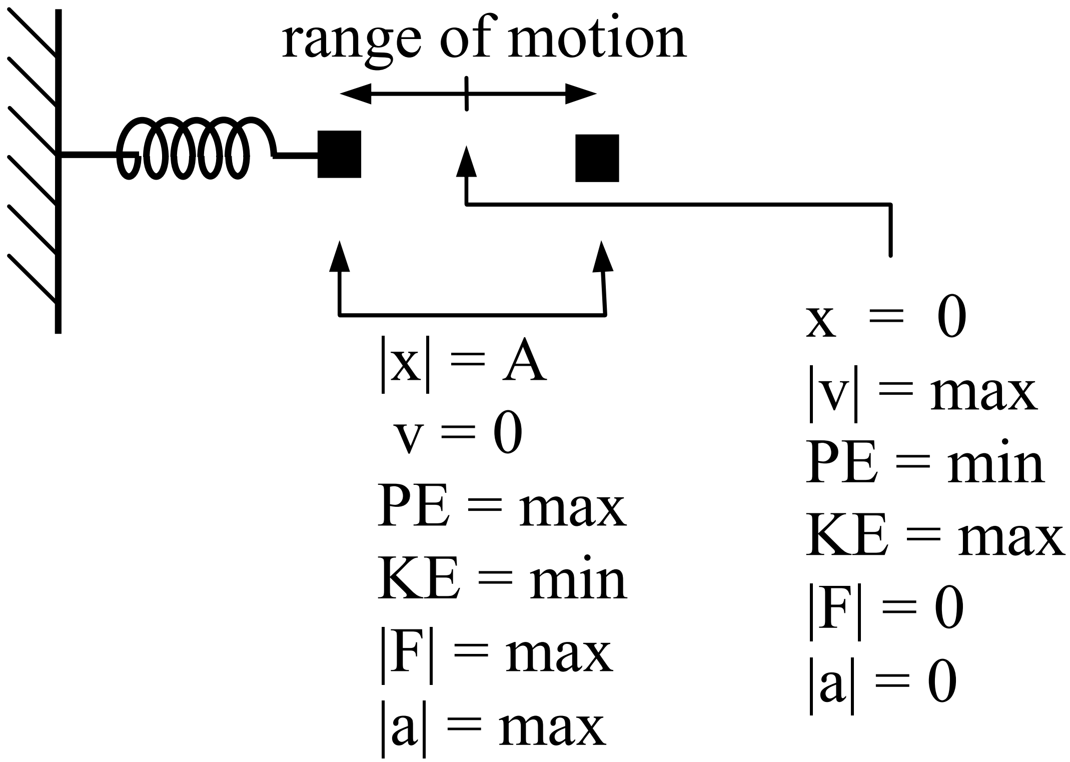
$$\boxed{v_m = \sqrt{\frac{k}{m}}\, x_m}$$

Double x_m $\rightarrow$ Double v_m

Spring Potential and Kinetic Energy



Spring Potential and Kinetic Energy



Harmonic Oscillator

$|X_m| = A$

$|V_m| = \sqrt{k/m}\,|X_m| = \sqrt{k/m}\,A$

$PE_{max} = \frac{1}{2} k X_m^2 = \frac{1}{2} k A^2$

$KE_{max} = \frac{1}{2} m V_m^2 = \frac{1}{2} k A^2$

$|F_{max}| = k|X_m|$

$= kA$

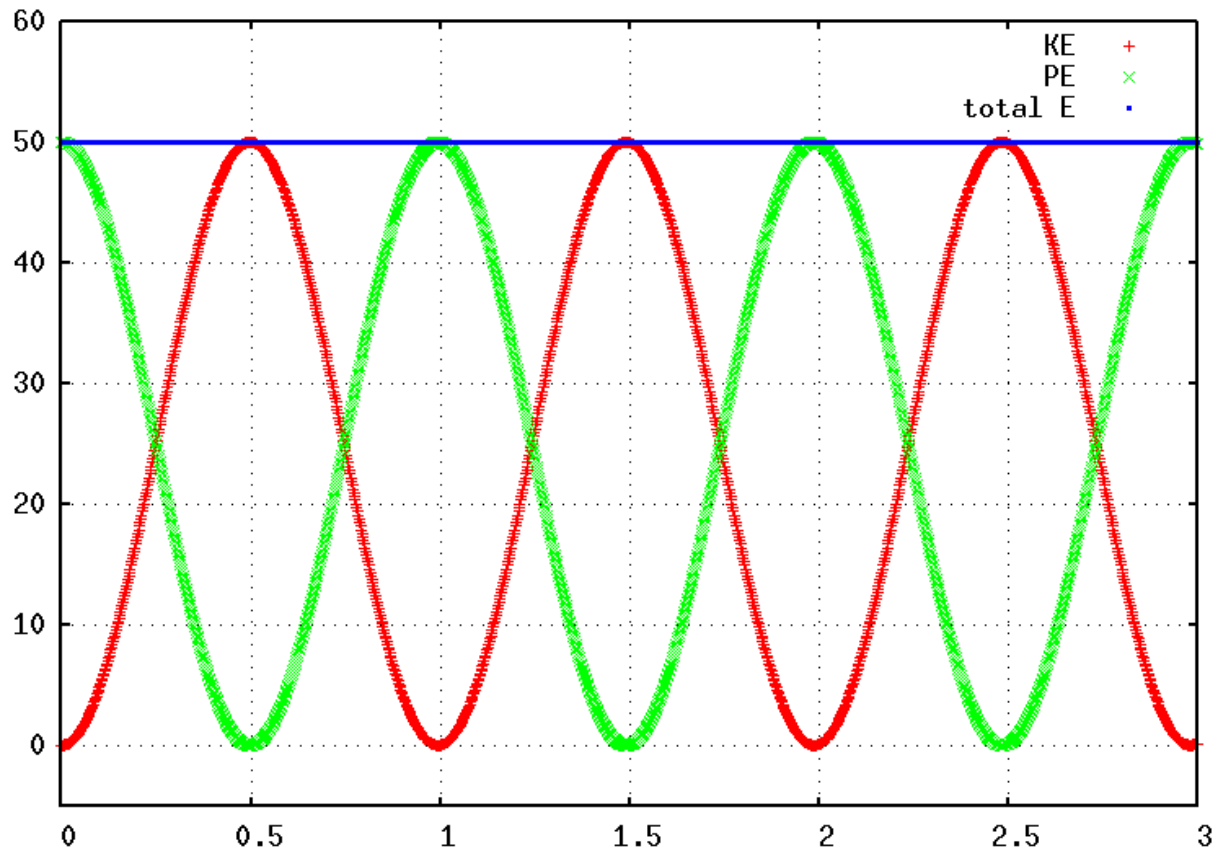
$a_{max} = |F_{max}|/m$

$= k/m\, A$

$= \left(\sqrt{k/m}\right)^2 A$

$= \sqrt{k/m}\,|V_m|$

Spring as a Harmonic Oscillator

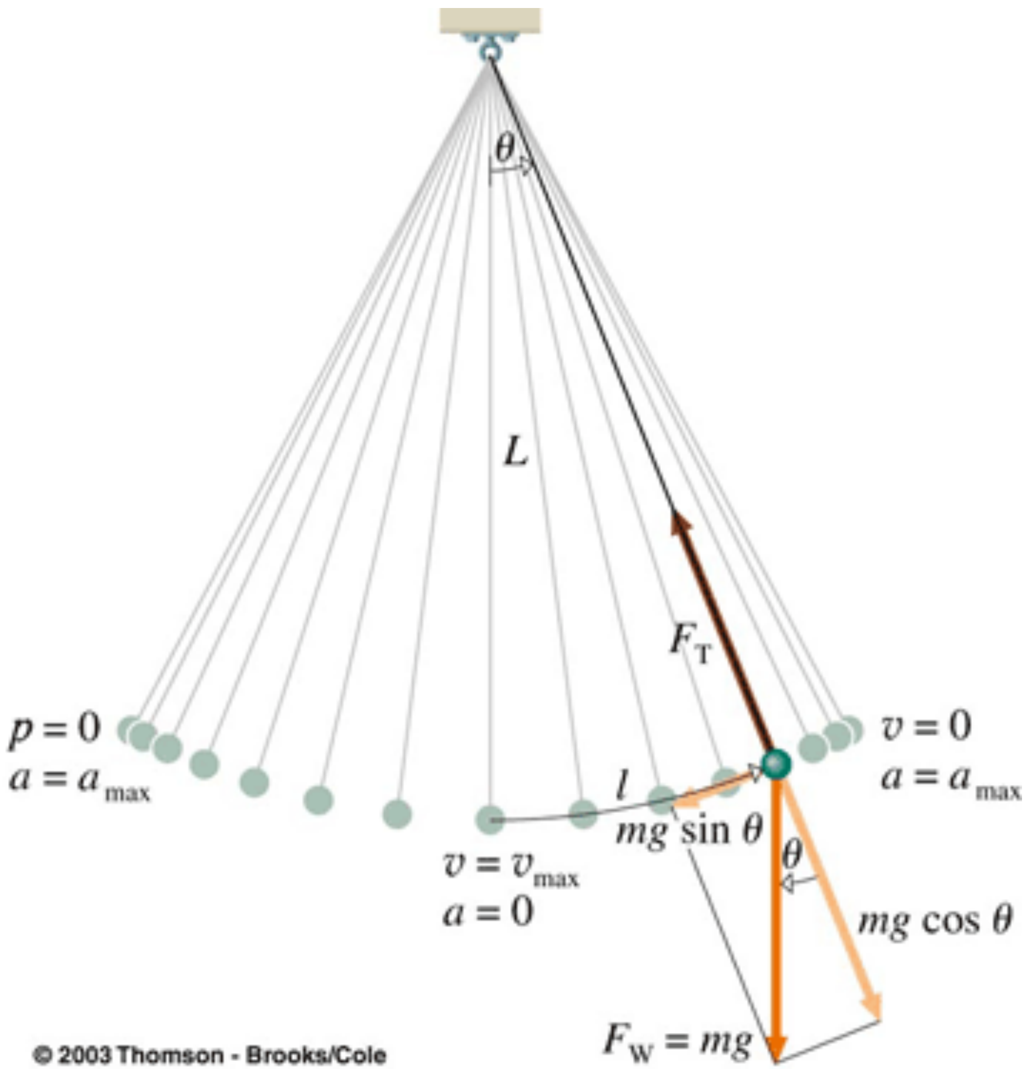


Frequency of oscillation $f = 1/(2\pi)\sqrt{(k/m)}$ with units [cycles]/[second]

What is a Harmonic Oscillator?

- The system has a restoring force proportional to the displacement from equilibrium: $F \propto -x$
  - The potential energy is proportional to the square of the displacement: $PE \propto x^2$
  - The position *x*, the velocity *v*, and the acceleration *a* all vary sinusoidally in time.
  - The period T or frequency $f = 1 / T$ is independent of the amplitude of the motion.

Forces on Simple Pendulum



Pendulum

$F = -mg \sin\theta$

for small angles:

$\sin\theta \sim \theta$

so $F \sim -mg\theta$

$\theta = s/L$

so $F \sim -\frac{mg}{L} s$

same form as

$F \sim -kx$

w/ $k \sim mg/L$

so $f = 1/(2\pi) \sqrt{k/m}$

$= \boxed{1/(2\pi) \sqrt{g/L}}$

Pendulum

$$\tau = F_{\perp} r$$

$$= -mg\theta L$$

$$= -\kappa_{rot}\theta$$

w/ $\kappa_{rot} = mgL$

$$f = 1/(2\pi) \sqrt{\frac{\kappa_{rot}}{I}}$$

$$= 1/(2\pi) \sqrt{\frac{mgL}{mL^2}}$$

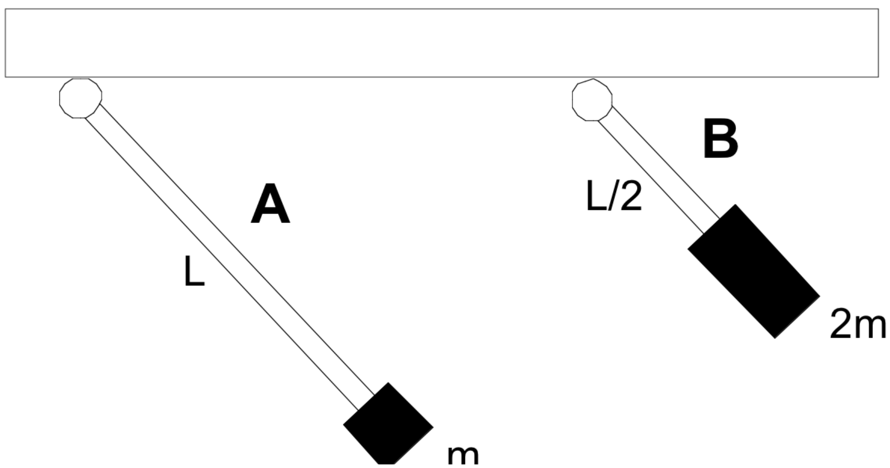
$$= \boxed{1/(2\pi) \sqrt{g/L}}$$

Pendulum as Harmonic Oscillator

- For small angles, the pendulum satisfies the condition for being a harmonic oscillator
  - $F = -mg\theta = -mg\ S/L$
  - $\tau = -mg\theta L = -gI/L\ \theta$
- **=> The period T or frequency f = 1 / T is independent of the amplitude of the motion**
  - For a pendulum the frequency of the motion is:
    - $1/(2\pi) * \sqrt{(g/L)}$

Concept Check Revisited

Two light (massless) rods, labeled A and B, each are connected to the ceiling by a frictionless pivot.
Rod A has length L and has mass m at the end of the rod.
Rod B has length L/2 and has a mass 2m at its end.
Both rods are released from rest in a horizontal position.



Which one falls to the vertical position fastest?

A: A   B: B   C: Both fall at the same rate.

Concept Check

- Your beautiful antique grandfather clock is gaining time (it is running too fast). To fix it, you could…

- **A)** Use a file to shorten the pendulum
- **B)** Hang something off the bottom to lengthen the pendulum
- **C)** Use a file to scrape some mass off the sides of the pendulum
- **D)** None of these things could possibly help.

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