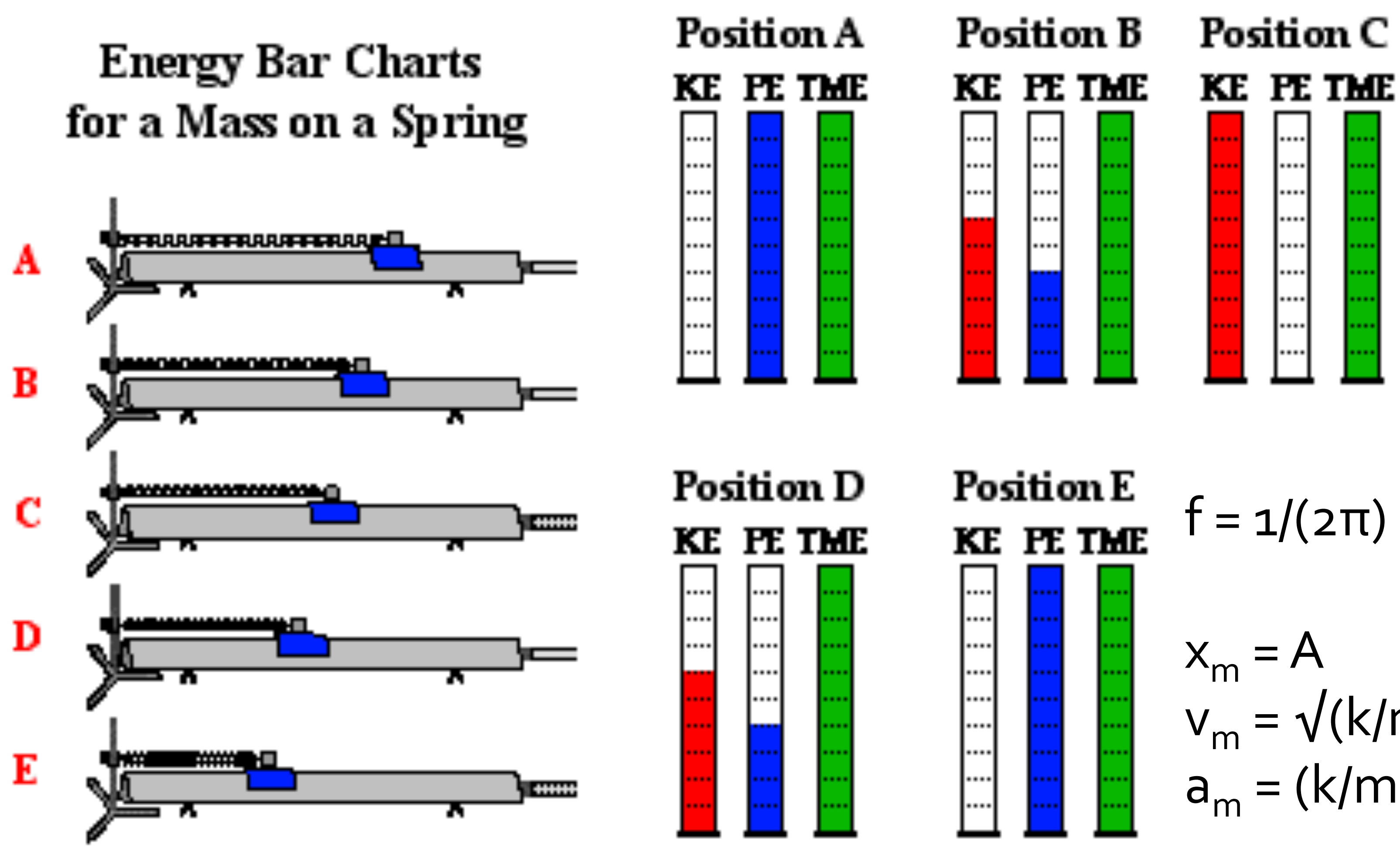
College Physics I: 1511 Mechanics & Thermodynamics

Professor Jasper Halekas
Van Allen Lecture Room 1
MWF 8:30-9:20 Lecture

Harmonic Oscillator: Spring



$f = 1/(2\pi)\sqrt{(k/m)}$

$x_m = A$

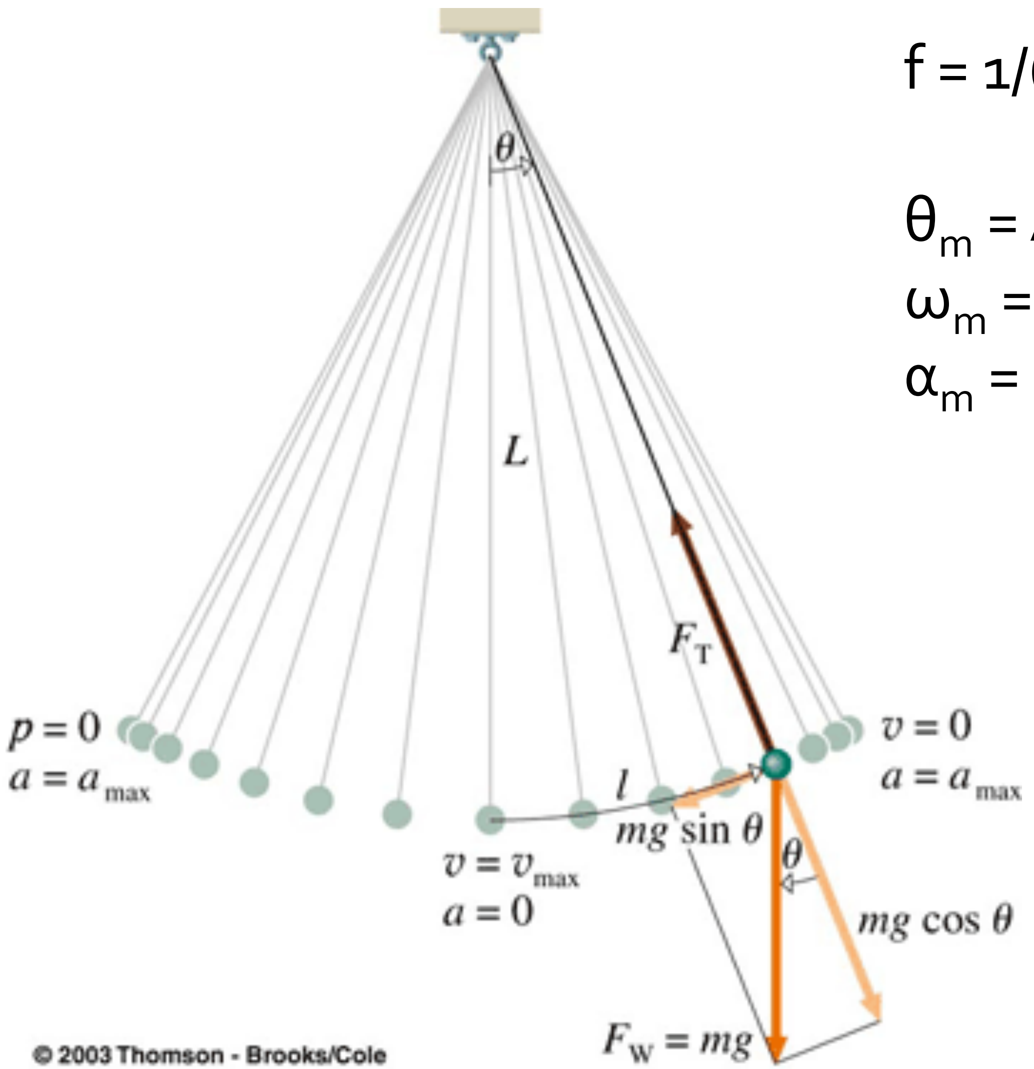
$v_m = \sqrt{(k/m)}\,A$

$a_m = (k/m)\,A$

Harmonic Oscillator: Pendulum

$k = mg/L$
$k/m = g/L$

$k_{rot} = mgL$
$k_{rot}/\mathbb{I} = g/L$



$f = 1/(2\pi)\sqrt{(g/L)}$

$\theta_m = A$
$\omega_m = \sqrt{(g/L)}\,A$
$\alpha_m = (g/L)\,A$

General Harmonic Oscillator

- Why is the frequency $1/(2\pi)\sqrt{(k/m)}$?
- Set by the ratio between the restoring force constant and the inertia
- In the case of a pendulum, force and inertia each have a factor of mass, so m cancels out

General Harmonic Oscillator

- Given a restoring force $F = -kx$:
  - Frequency = $1/(2\pi)\sqrt{(k/m)}$
- Works with angular variables too, so given $\tau = -k_{rot}\theta$:
  - Frequency = $1/(2\pi)\sqrt{(k_{rot}/\mathbb{I})}$
  - For pendulum $k_{rot} = mgL$, $\mathbb{I} = mL^2$, so $f = 1/(2\pi)\sqrt{(g/L)}$

Concept Check

- Imagine you form a harmonic oscillator with a mass m on a spring with constant k, and set it oscillating with amplitude A. As the mass reaches its greatest displacement ($x = A$), you attach a second mass m. What happens to the maximum velocity v_m of the subsequent motion?

A. v_m stays the same
B. v_m doubles
C. v_m is cut in half
D. v_m is cut by a factor $\sqrt{2}$

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$$PE_{max} = \frac{1}{2} K X^2$$
$$= \frac{1}{2} K A^2$$

$$= KE_{max}$$
$$= \frac{1}{2} m V_m^2$$

$$\frac{1}{2} m V_m^2 = \frac{1}{2} (2m) V_m'^2$$

$$V_m^2 = 2 V_m'^2$$

$$V_m'^2 = \frac{1}{2} V_m^2$$

$$\boxed{V_m' = \frac{1}{\sqrt{2}} V_m}$$

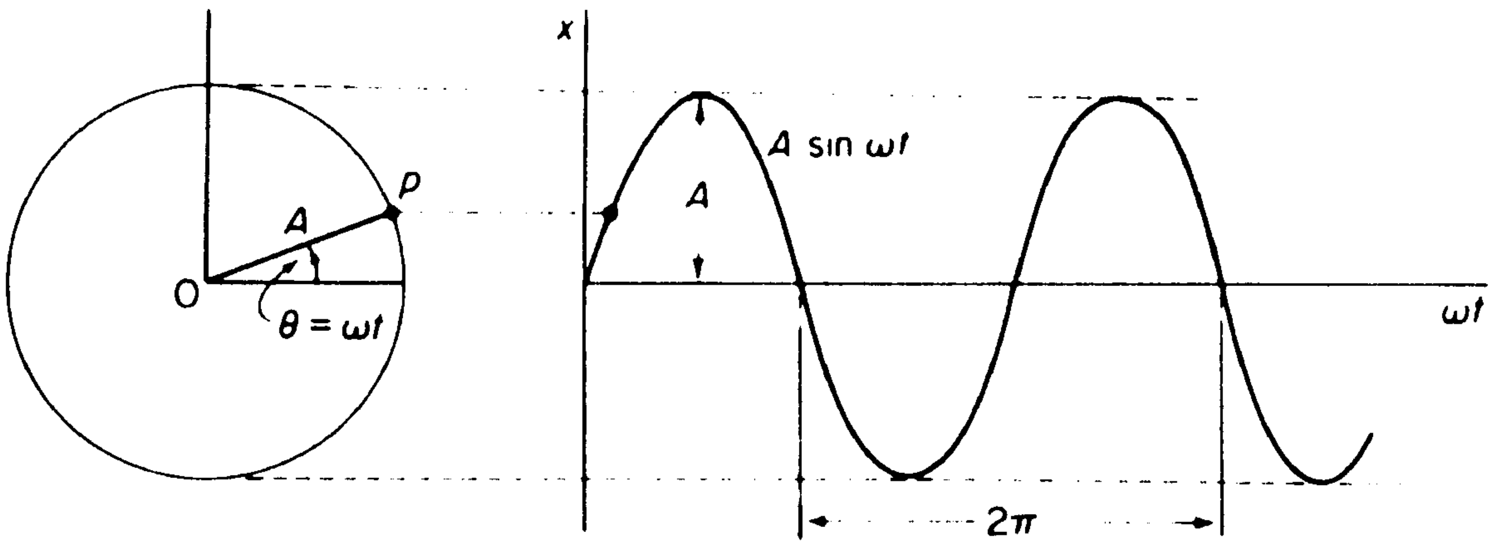
Also see $V_m = \sqrt{\frac{K}{m}} A$

$$V_m' = \sqrt{\frac{K}{2m}} A$$

$$= \frac{1}{\sqrt{2}} V_m$$

Harmonic And Rotational Motion

"Reference Circle"



ω = 2πf = 2π/T for both kinds of motion

Frequency Vs. Angular Frequency: A Tale of Two Omegas

- Whatever the type of harmonic oscillator, it's useful to think about its frequency as an angular frequency (similar to angular velocity, and same units, but more general):
  - $\omega_h = 2\pi f$ (applicable to any cyclical oscillation)
  - For general harmonic oscillator:
    - $\omega_h = \sqrt{(k/m)}$
  - For pendulum:
    - $\omega_h = \sqrt{(g/L)}$

$$\omega_n = 2\pi f$$

$$= \sqrt{k/m} \text{ for spring}$$

$$\Rightarrow \quad v_m = \omega_n x_m = \omega_n A$$

$$a_m = \omega_n^2 A$$

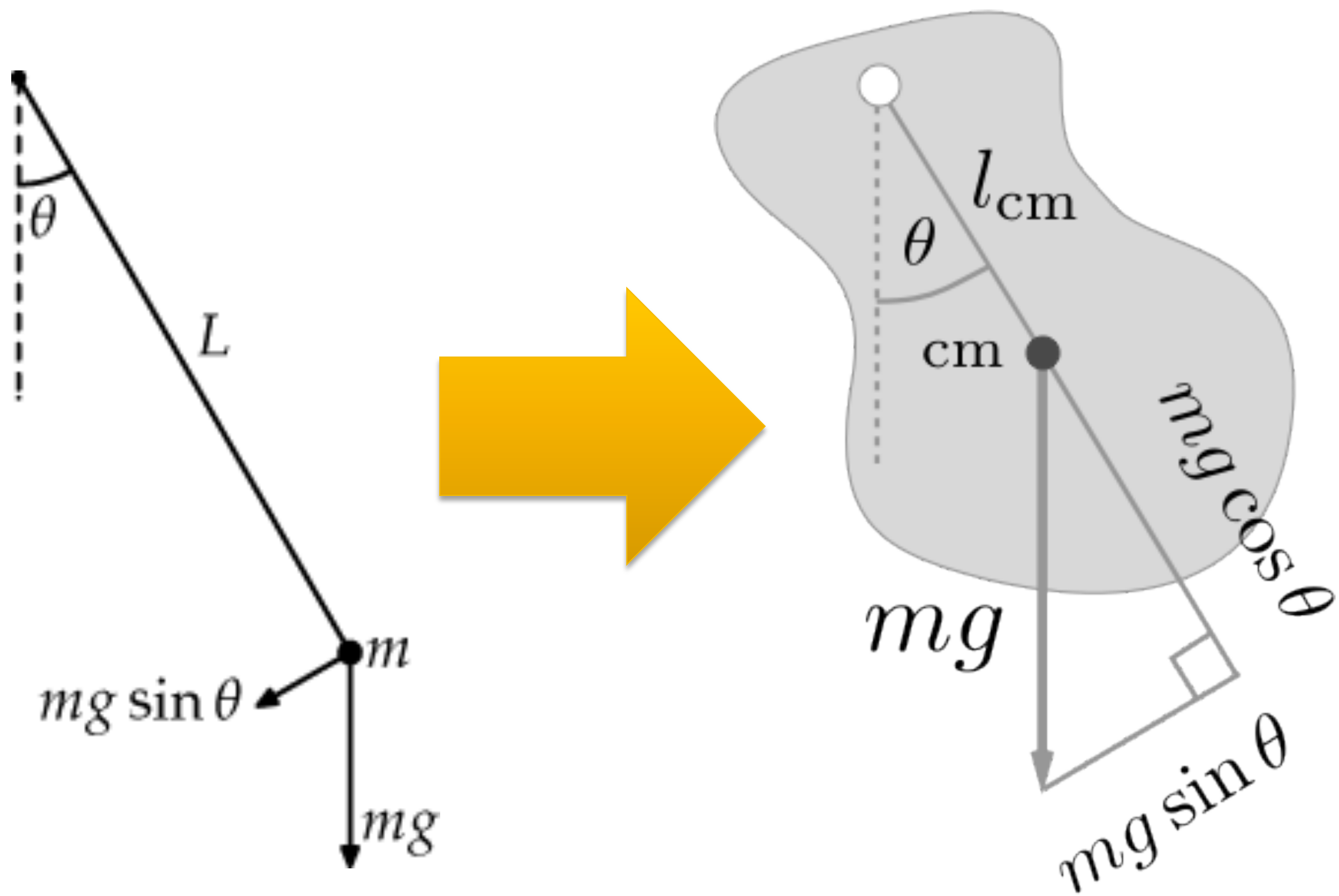
Conservation of Energy

- $E = PE_{tot} + KE_{tot}$
  - $= PE_{grav} + PE_{spring} + KE_{trans} + KE_{rot}$
  - $= mgh + 1/2kx^2 + 1/2mv^2 + 1/2I\omega^2$
  - = Conserved in the absence of friction etc.
- Be careful!!!
  - We have two meanings for the symbol ω
    - One is angular velocity for circular motion
    - One is angular frequency for harmonic motion ω_h
    - They are related, but not always interchangeable
    - Be careful to only use angular velocity in $1/2I\omega^2$

Period Independent of Amplitude

- The period $T = 2\pi/\omega_h = 2\pi\sqrt{(m/k)}$
  - Does not depend on amplitude
  - How can this be?
  - Maximum velocity and acceleration both depend on the amplitude of the motion
    - Bigger displacement -> faster motion
  - $x_{max} = A$
  - $v_{max} = \omega_h A$
  - $a_{max} = \omega_h^2 A$

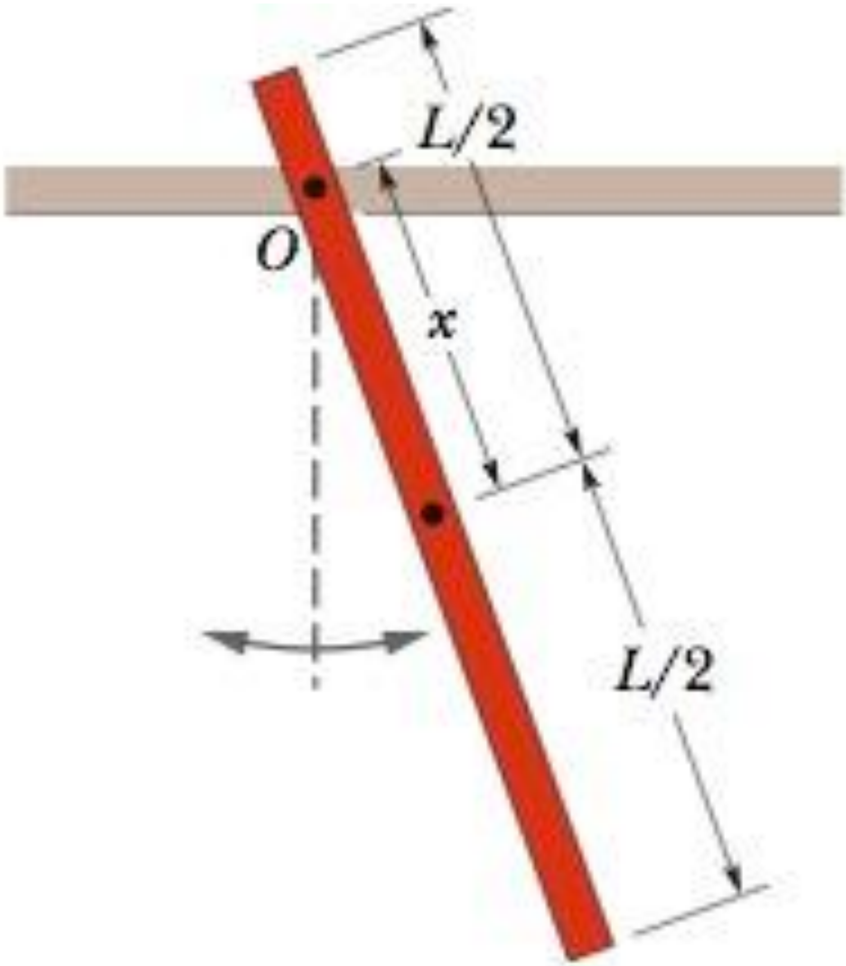
Physical Pendulum



Physical Pendulum

- Found that angular frequency of oscillation of a simple pendulum was:
  - $\omega_h = \sqrt{(mgL/\mathbb{I})} = \sqrt{(mgL/mL^2)} = \sqrt{(g/L)}$
- For a physical pendulum almost the same formula applies:
  - $\omega_h = \sqrt{(mgL_{CM}/\mathbb{I})}$
  - But now the details depend on the center of mass L_{CM} and the moment of inertia $\mathbb{I}$

Example of physical pendulum



$L_{CM} = L/2$

$I = 1/3\ ML^2$

$\omega_h = \sqrt{(mgL/2/(1/3ML^2))}$
$= \sqrt{(3g/2L)}$

Faster than simple pendulum since moment of inertia smaller

Concept Check

- A person swings on a swing. When the person sits still, the swing oscillates back and forth at its natural frequency. If, instead, the person *stands* on the swing, the natural frequency of the swing is..

- **A:** greater.
- **B:** the same.
- **C:** smaller.

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