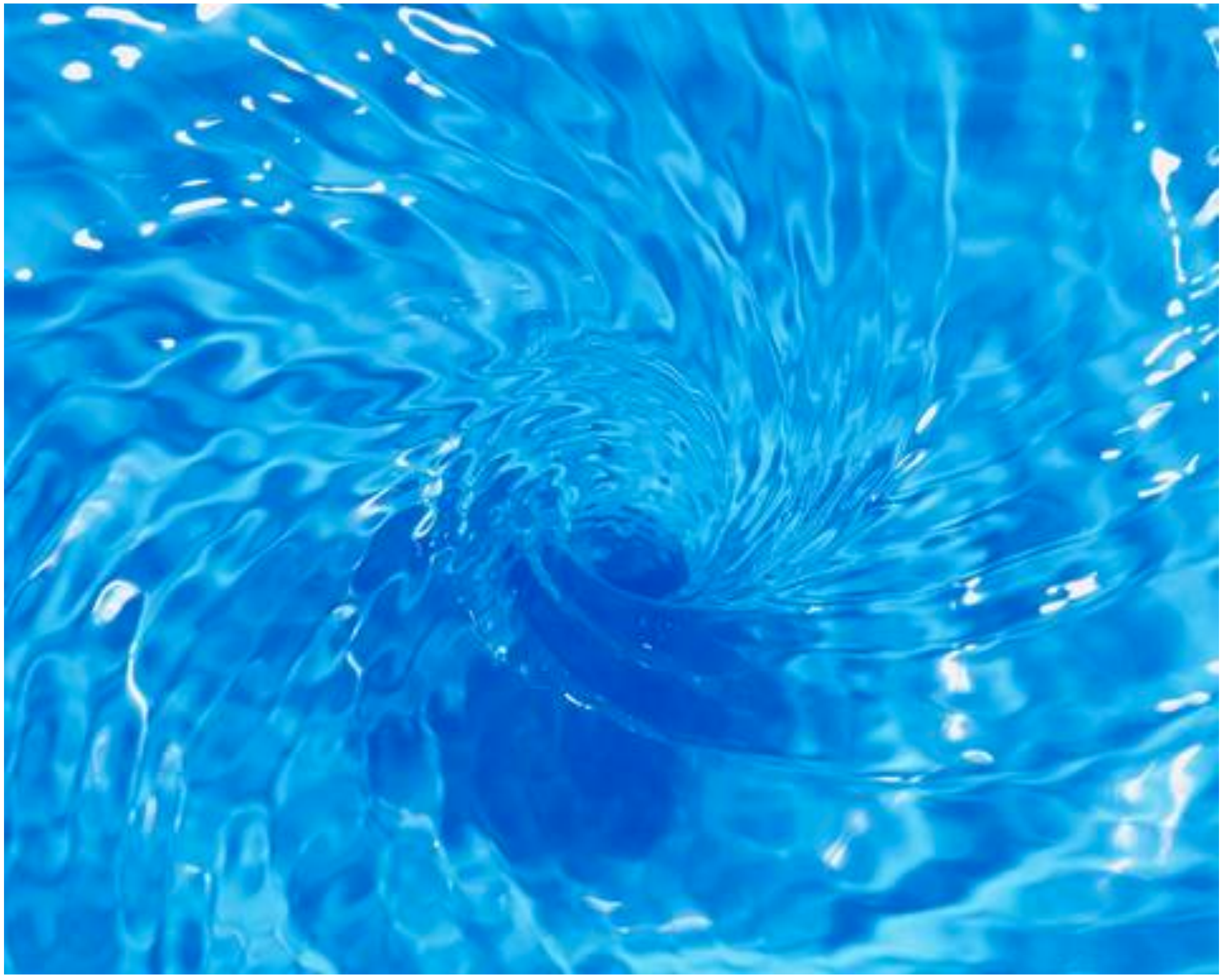
College Physics I: 1511 Mechanics & Thermodynamics

Professor Jasper Halekas
Van Allen Lecture Room 1
MWF 8:30-9:20 Lecture

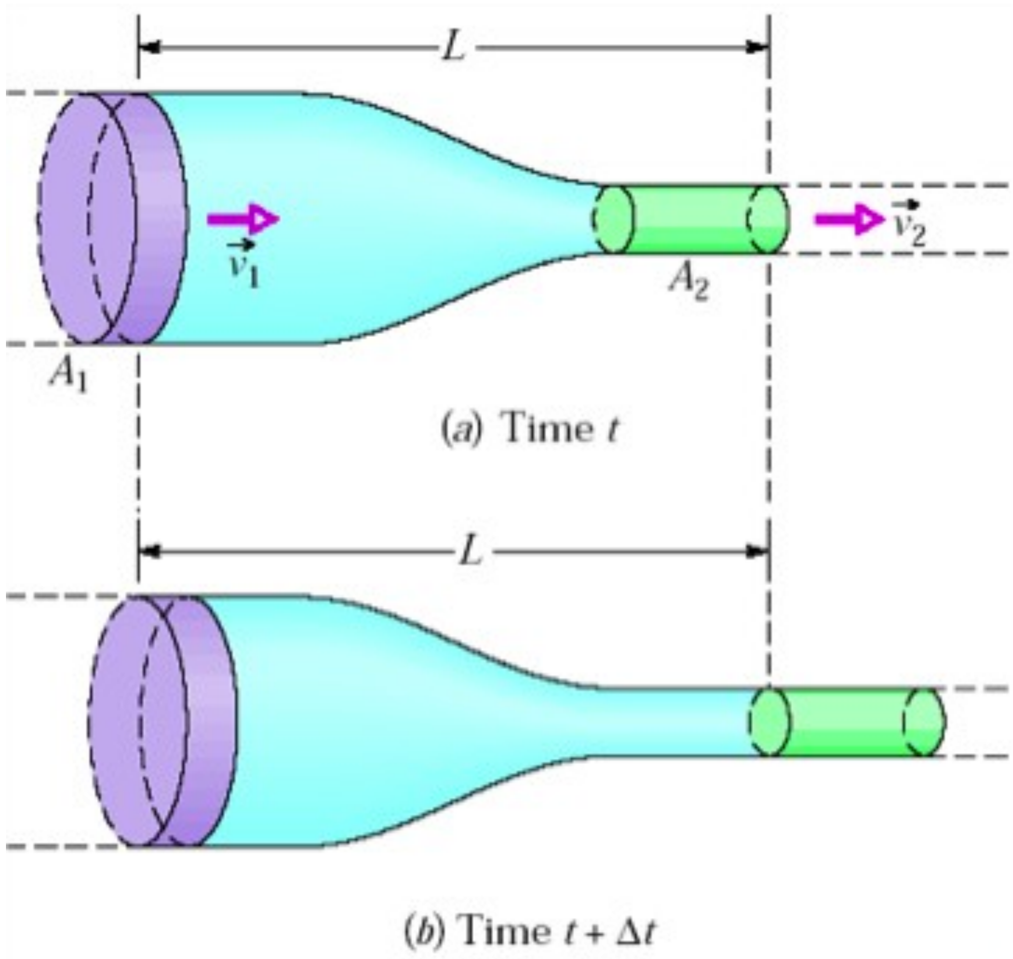
Announcements

- Office hours schedule next week
  - Monday 9:30-11:00 am
  - Tuesday none (out of office)
  - Wednesday 9:30-11:00 am
  - Thursday 12:00-1:00 pm
  - Thursday 3:30-5:00 pm
- No labs or homework next week
- Midterm #2 is in class Friday

From Static to Flowing Fluid



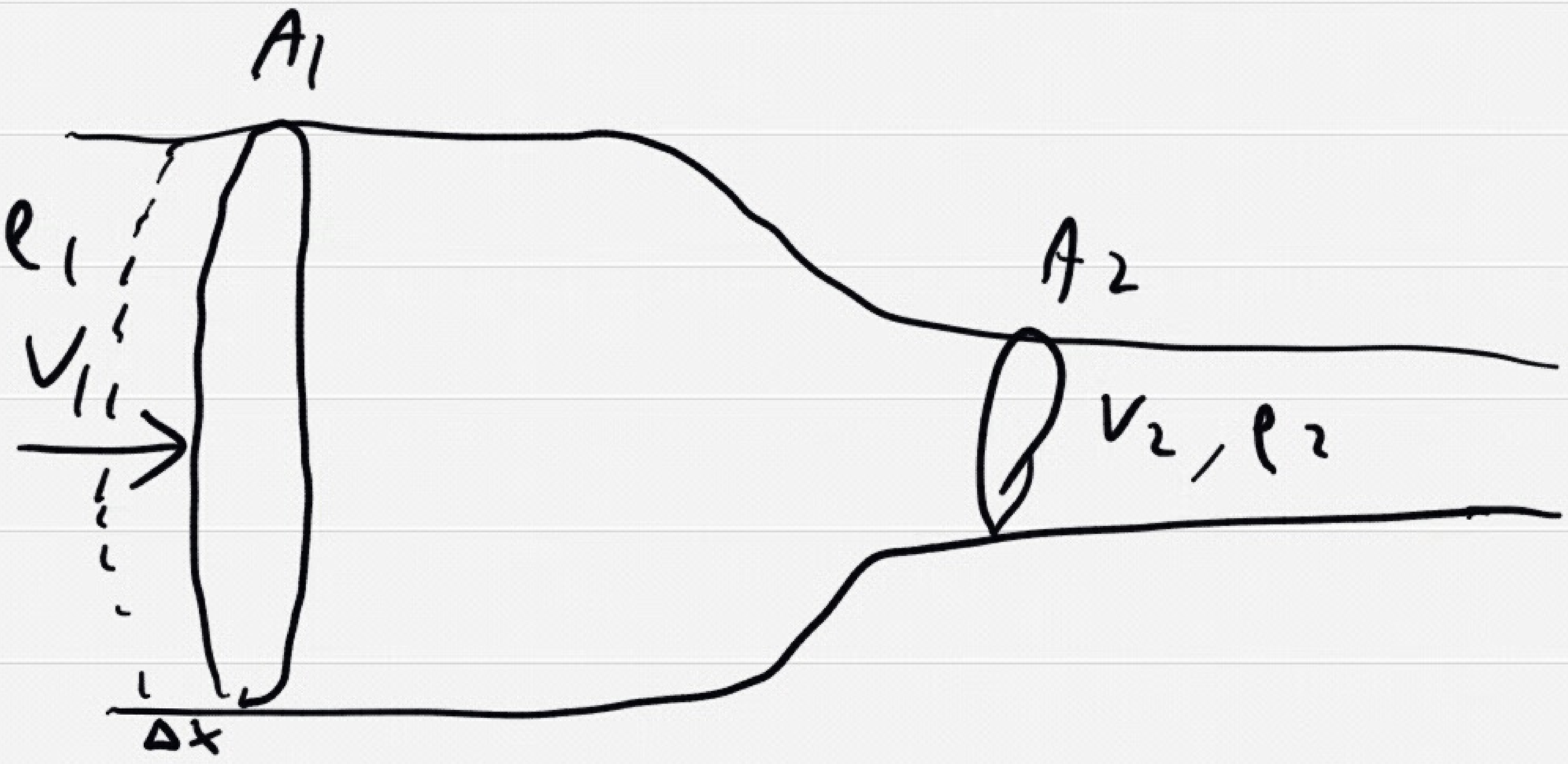
Continuity of Fluid Flow



Continuity

- Assuming steady-state:
  - Amount of mass entering pipe per unit time must equal amount of mass leaving pipe per unit time
  - Mass entering in time Δt = $\rho_1 A_1 \Delta x_1$
    - But $v_1 = \Delta x_1/\Delta t$
    - So $\Delta x_1 = v_1 \Delta t$
    - So mass entering per unit time = $\rho_1 A_1 v_1$
  - Similarly mass exiting per unit time = $\rho_2 A_2 v_2$

Continuity



Mass past A_1 in time Δt

$$\Delta m = \rho_1 A_1 \Delta x_1$$

$$V_1 = \Delta x_1 / \Delta t$$

$$\Rightarrow \Delta x_1 = V_1 \Delta t$$

$$\Delta m = \rho_1 A_1 V_1 \Delta t$$

$$\Delta m / \Delta t = \rho_1 A_1 V_1$$

similarly $\Delta m / \Delta t = \rho_2 A_2 V_2$
@ point 2

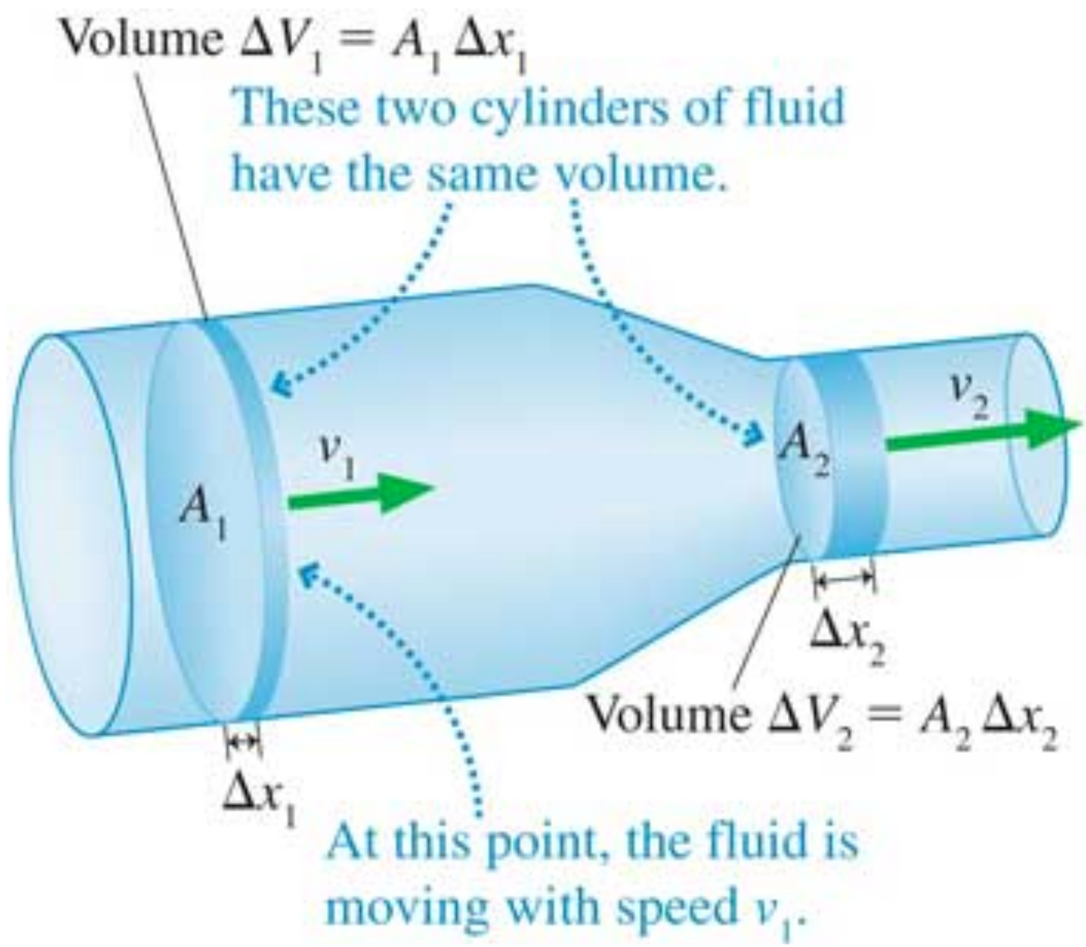
$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

Continuity Equation

- $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

- For incompressible fluids $\rho_1 = \rho_2$
  - So $A_1 v_1 = A_2 v_2$ if incompressible

Continuity for Incompressible Fluids

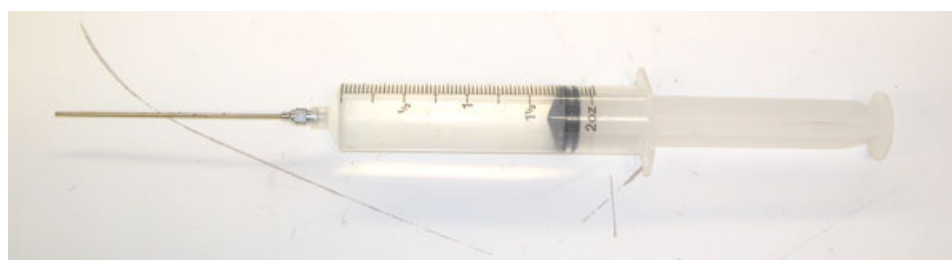


Continuity is Equivalent to Conservation of Volume if Incompressible

Concept Check

- What is the velocity of water coming out the end of the syringe, if you depress the syringe at a rate of 1 cm/s, assuming the diameter of the barrel is 1 cm, and the diameter of the nozzle is 0.05 cm?

A. 10 cm/s
B. 20 cm/s
C. 40 cm/s
D. 400 cm/s



Concept Check

- What is the velocity of water coming out the end of the syringe, if you depress the syringe at a rate of 1 cm/s, assuming the diameter of the barrel is 1 cm, and the diameter of the nozzle is 0.05 cm?

A. 10 cm/s
B. 20 cm/s
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Syringe Outflow Velocity

- $A_1v_1 = A_2v_2$

- $A_1 = \pi * r_1^2$
- $A_2 = \pi * r_2^2$

- $v_2 = A_1v_1/A_2 = r_1^2/r_2^2 * v_1 = 400\ v_1$

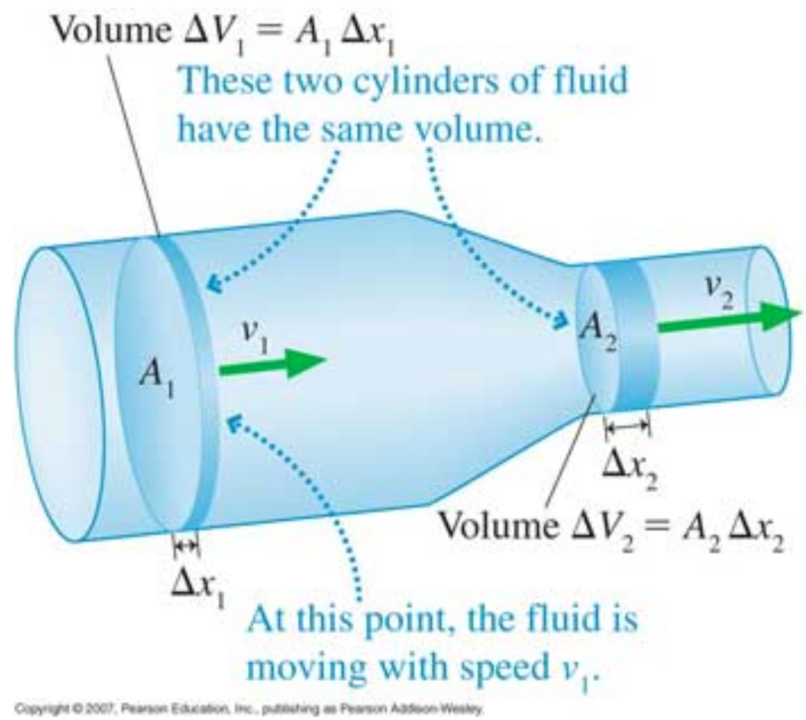
- 400 times faster than plunger speed!

Continuity Equation and Pressure

- For incompressible flow, look at implications of continuity for pressure

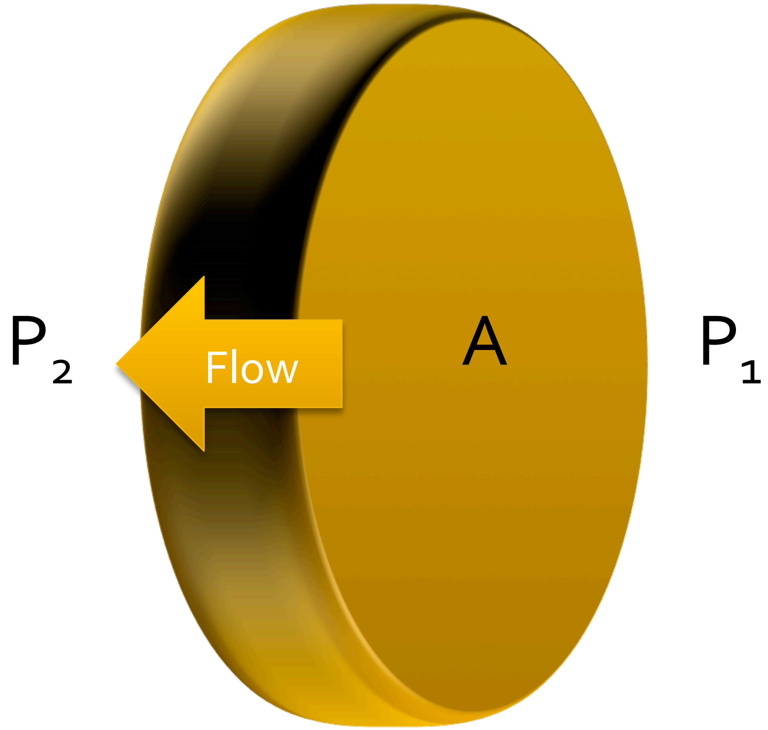
- F = ma for every fluid element
- Since the fluid accelerates, it must be subject to an unbalanced force

- The only way this can be provided is if the pressure is non-uniform



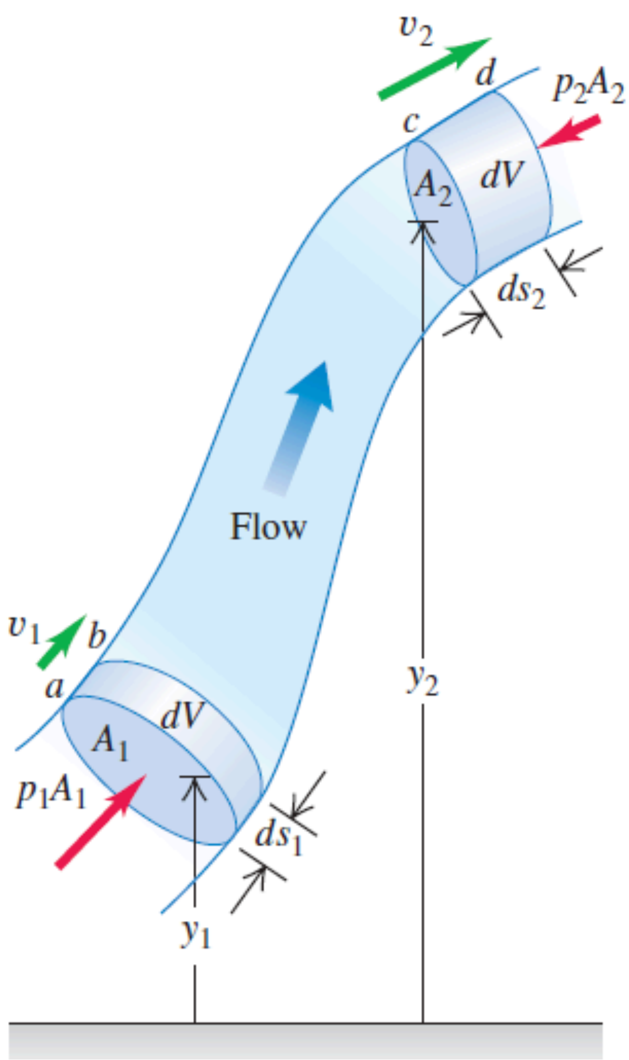
Work Done On Fluid Element

$F = (P_2 - P_1)A$ to the right



$W_{1to2} = -F\Delta x = -(P_2 - P_1)*A*\Delta x = (P_1 - P_2)*Volume$

Work-Energy Theorem Applied to Fluids



Deriving Bernoulli's equation. The net work done on a fluid element by the pressure of the surrounding fluid equals the change in the kinetic energy
plus the change in the gravitational potential
energy.

Bernoulli

$$W_{1\,to\,2} = (P_1 - P_2)\,Volume$$

$$W_{1\,to\,2} = W_{nc} = \Delta E$$

$$= E_2 - E_1$$

$$= KE_2 + PE_2 - (KE_1 + PE_1)$$

$$KE = \tfrac{1}{2} m v^2$$
$$PE = mgy$$

$$(P_1 - P_2)\cdot Vol = \tfrac{1}{2} m v_2^2 + mgy_2 - \tfrac{1}{2} m v_1^2 - mgy_1$$

$$\Rightarrow P_1 - P_2 = \tfrac{1}{2} \rho v_2^2 + \rho g y_2 - \tfrac{1}{2} \rho v_1^2 - \rho g y_1$$

or

$$P_1 + \tfrac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2} \rho v_2^2 + \rho g y_2$$

Work-Energy Theorem Applied to Fluids

- Work done in moving fluid element from region 1 to region 2
  - $W_{nc} = (P_1 - P_2)V = E_2 - E_1$
- Total mechanical energy $E = 1/2mv^2 + mgy$
- $(P_1 - P_2)V = 1/2mv_2^2 + mgy_2 - (1/2mv_1^2 + mgy_1)$
- Divide both sides by volume to get…

Bernoulli's Equation

- $(P_1 - P_1) = 1/2\rho v_2^2 + \rho g y_2 - (1/2\rho v_1^2 + \rho g y_1)$
- Usually rearranged in following way:
  - $P_1 + 1/2\rho v_1^2 + \rho g y_1 = P_2 + 1/2\rho v_2^2 + \rho g y_2$

Bernoulli's Equation: Equal Depth

- $P_1 + 1/2\rho v_1^2 = P_2 + 1/2\rho v_2^2$
- Pressure plus kinetic energy density = constant

Concept Check

- You blow air between two balloons. What happens?

A. Nothing
B. They spread apart
C. They come together
D. They pop

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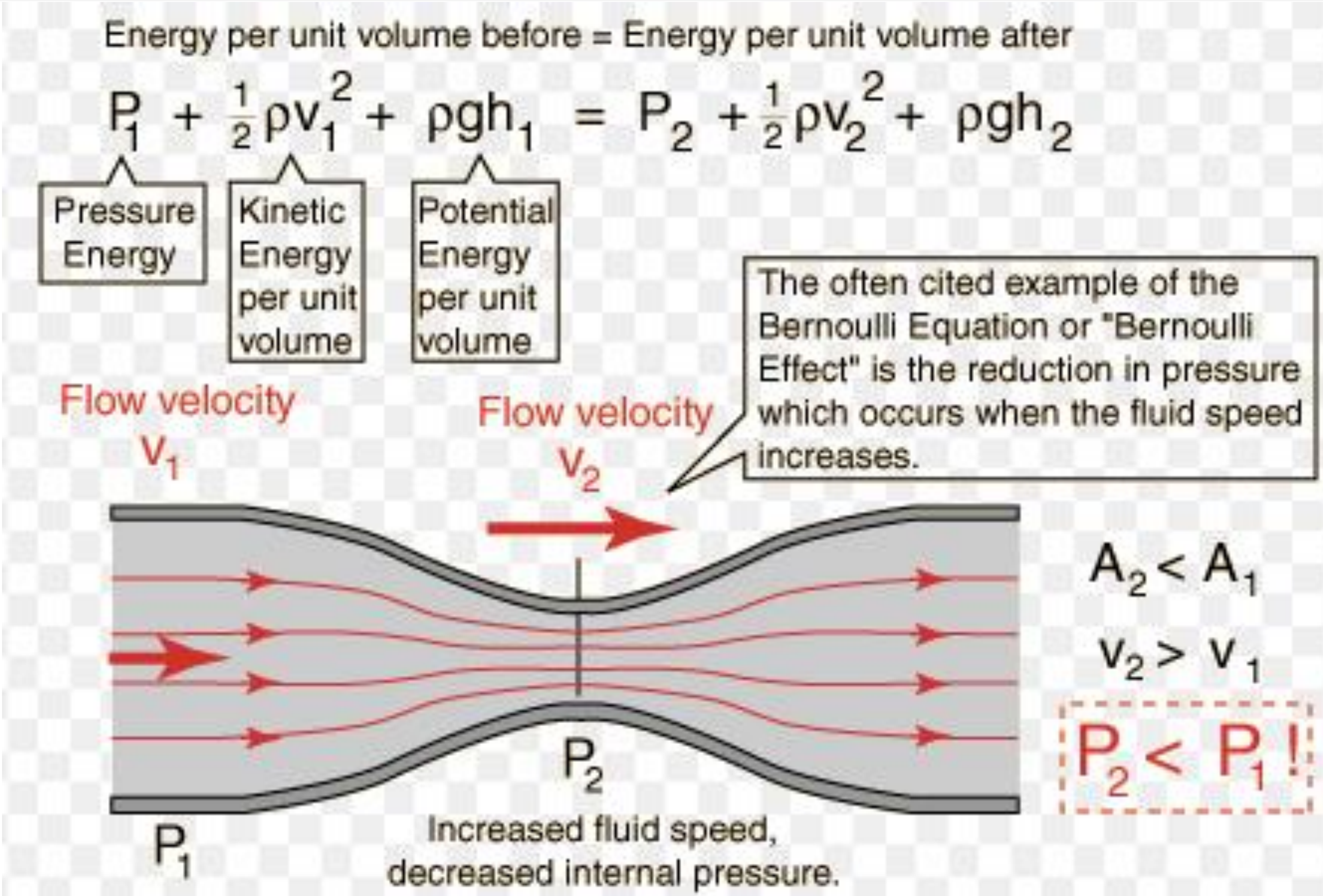
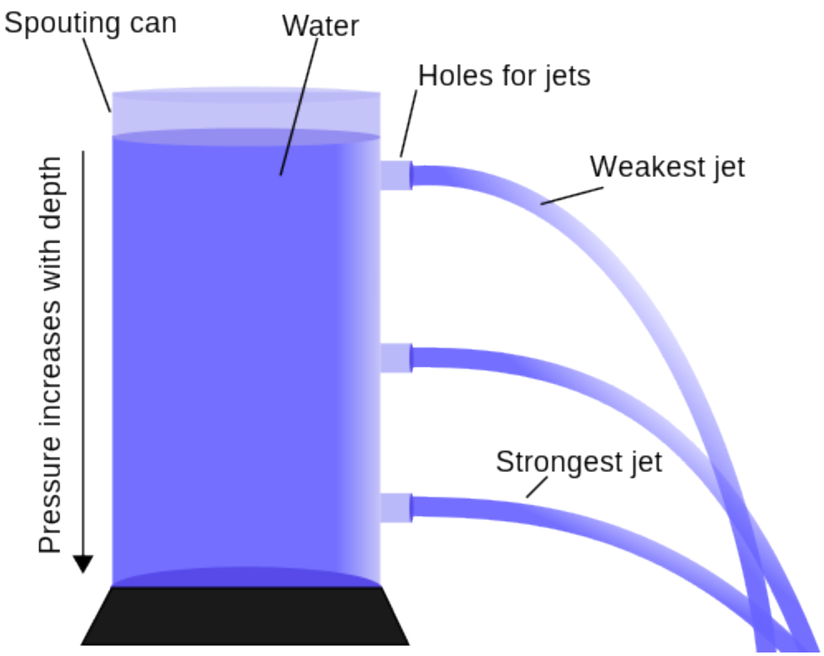
Bernoulli's Equation



Bernoulli's Equation: Unequal Depth but Equal Pressure



- $1/2\rho v_1^2 + \rho g y_1 = 1/2\rho v_2^2 + \rho g y_2$
- Velocity $v_1 = 0$ at top
- $1/2\rho v_2^2 = \rho g y_1 - \rho g y_2$
- $v_2 = \sqrt{(2g\Delta y)}$
  - (Same result as classical projectile)

Beyond Bernoulli

- To derive Bernoulli's equations, we assumed:
  - Steady-state
  - Incompressible
  - Non-Viscous Flow
- All sorts of interesting things happen when these conditions are lifted (as they often are in real life)

Turbulent Flow