

College Physics I: 1511

Mechanics & Thermodynamics

Professor Jasper Halekas
Van Allen Lecture Room 1
MWF 8:30-9:20 Lecture

Announcements

- Office hours schedule this week
 - Today 9:30-11:00 am
 - Tuesday none (out of office)
 - Wednesday 9:30-11:00 am
 - Thursday 12:00-1:00 pm
 - Thursday 3:30-5:00 pm
- No labs or homework this week
- Midterm #2 is in class Friday

Midterm Details

- Will Cover Chs. 7-11
 - Not cumulative, but many concepts rely on previous material ($F = ma!$)
 - No questions on the following material:
 - 8.7 (vector nature of angular variables)
 - 10.5-10.8 (driven/damped oscillators, stress, strain)
 - 11.9-11.11 (Bernoulli's equation, viscous flow)
 - Fifteen questions: Four on Ch. 7, five on Chs. 8-9, three on Ch. 10, three on Ch. 11

Equation Sheet: Top

Trigonometry (For right triangle with sides Adjacent, Opposite, and Hypoteneuse):

$$\begin{array}{lllll} \sin(\theta) = O/H & \cos(\theta) = A/H & \tan(\theta) = O/A & H^2 = O^2 + A^2 & A_{\text{circle}} = \pi r^2 \\ \sin(30^\circ) = \cos(60^\circ) = 1/2 & \sin(60^\circ) = \cos(30^\circ) = \sqrt{3}/2 \sim 0.866 & \sin(45^\circ) = \cos(45^\circ) = \sqrt{2}/2 \sim 0.707 & & \\ \sin(0^\circ) = \cos(90^\circ) = 0 & \sin(90^\circ) = \cos(0^\circ) = 1 & & & \end{array}$$

Moment of Inertia

$$\begin{array}{ll} \text{Point mass or thin-walled wheel:} & I = mr^2 \\ \text{Thin rod pivoting around end:} & I = 1/3 mr^2 \end{array} \quad \begin{array}{ll} \text{Solid cylinder:} & I = 1/2 mr^2 \\ \text{Solid sphere:} & I = 2/5 mr^2 \end{array}$$

Kinematics:

$$\langle \vec{v} \rangle = \frac{\Delta \vec{r}}{\Delta t} \quad \langle \vec{a} \rangle = \frac{\Delta \vec{v}}{\Delta t} \quad \vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 \quad v(t)^2 = v_0^2 + 2\vec{a} \cdot \Delta \vec{r}(t)$$

Newton's Laws:

$$\sum \vec{F} = m\vec{a} \quad \vec{F}_{AB} = -\vec{F}_{BA}$$

Forces:

$$\begin{array}{lll} F_G = mg \text{ (@ surface)} & f_s^{MAX} = \mu_s F_N & f_k = \mu_k F_N \\ F_C = ma_c = \frac{mv^2}{r} & F_{\text{spring}} = -kx & F_{\text{Buoyant}} = m_{\text{fluid_displaced}} g \end{array}$$

Work & Energy:

$$\begin{array}{lll} KE_{\text{trans}} = \frac{1}{2}mv^2 & \Delta KE = W_{\text{net}} & PE_G = mgh \\ E = KE + PE & \Delta E = W_{\text{nc}} & PE_{\text{spring}} = \frac{1}{2}kx^2 \\ & & W = \vec{F} \cdot \Delta \vec{r} = |\vec{F}| |\Delta \vec{r}| \cos \theta_{Fdr} \end{array}$$

Equation Sheet: Bottom

Impulse & Momentum:

$$\vec{J} = \vec{F}\Delta t \quad \vec{p} = m\vec{v} \quad \Sigma \vec{J} = \Delta \vec{p} \quad \Sigma \vec{p}_f = \Sigma \vec{p}_i \text{ (if } F_{\text{ext}}=0)$$
$$\vec{v}_{cm} = \frac{\Sigma m_i \vec{v}_i}{\Sigma m_i} = \Sigma \vec{p} / M$$

Rotational Motion:

$$\theta = s/r \quad \langle \omega \rangle = \frac{\langle v_t \rangle}{r} = \frac{\Delta \theta}{\Delta t} \quad \langle \alpha \rangle = \frac{\langle a_t \rangle}{r} = \frac{\Delta \omega}{\Delta t}$$
$$\theta(t) = \theta_o + \omega_o t + \frac{1}{2} \alpha t^2 \quad \omega(t)^2 = \omega_o^2 + 2\alpha \Delta \theta(t)$$
$$\tau = rF \sin \theta_{rF} = F * \text{lever arm} \quad \Sigma \tau = I\alpha \quad L = mvr = I\omega$$
$$W_{rot} = \tau \Delta \theta \quad KE_{rot} = \frac{1}{2} I \omega^2 \quad r_{CM} = \frac{\Sigma m_i r_i}{\Sigma m_i}$$

Harmonic Motion:

$$\omega_h = 2\pi f_h = \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \quad x_{max} = A \quad v_{max} = A\omega_h \quad a_{max} = A\omega_h^2$$
$$\omega_{h_pendulum} = \sqrt{\frac{mg r_{CM}}{I}} = \sqrt{\frac{g}{L}} \text{ for simple pendulum of length } L$$

Fluids:

$$\rho = \text{mass}/\text{Volume} \quad P = F/A \quad P_2 = P_1 + \rho g d \quad F_B = W_{\text{fluid_displaced}}$$
$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \quad A_1 v_1 = A_2 v_2 \text{ (if } \rho_1 = \rho_2) \quad P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2$$

Impulse and Momentum

Impulse & Momentum:

$$\vec{J} = \vec{F} \Delta t \quad \vec{p} = m\vec{v} \quad \Sigma \vec{J} = \Delta \vec{p} \quad \Sigma \vec{p}_f = \Sigma \vec{p}_i \text{ (if } F_{ext}=0)$$
$$\vec{v}_{cm} = \frac{\Sigma m_i \vec{v}_i}{\Sigma m_i} = \Sigma \vec{p} / M$$

Impulse-Momentum Theorem

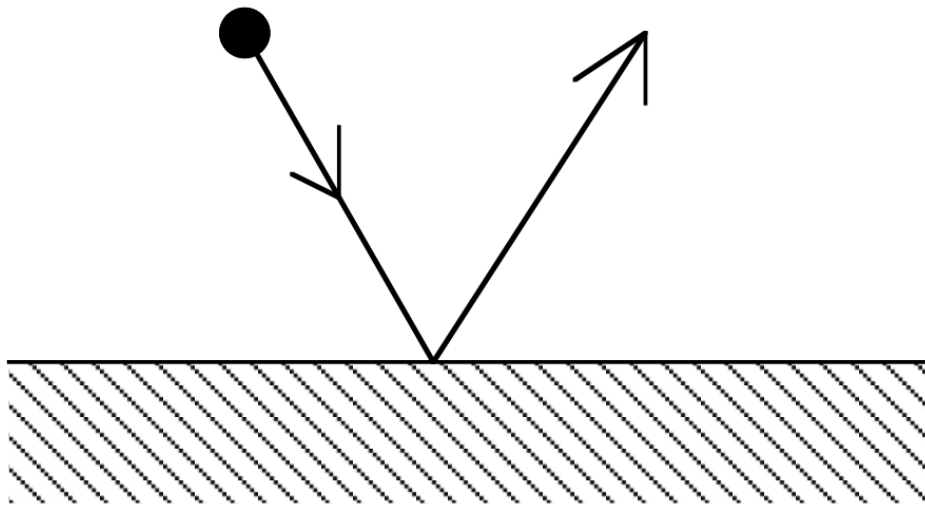
$$\boxed{F\Delta t} = \boxed{m\Delta v}$$

The Impulse The Change
in Momentum

Remember: Impulse-Momentum Theorem is just a restatement of $F = ma$.

Practice Question

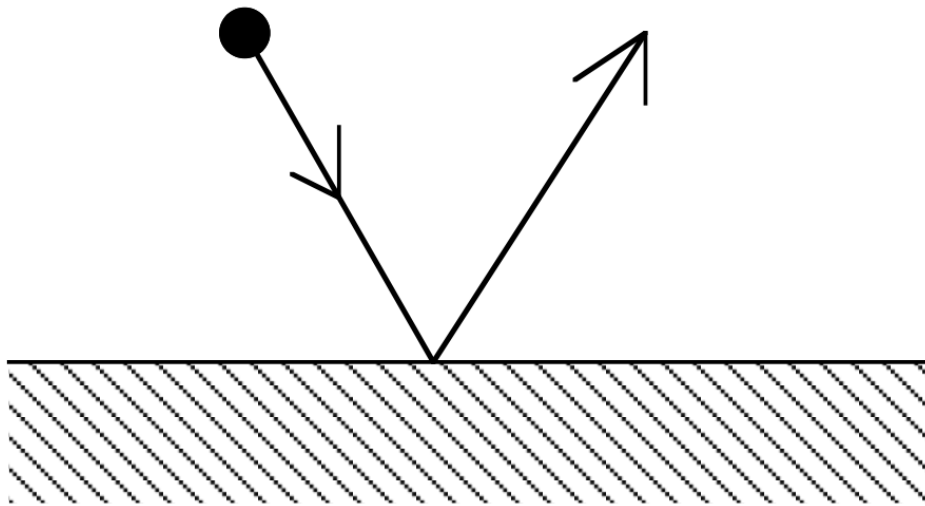
A ball bounces off the floor as shown. The direction of the impulse on the ball, $\Delta\mathbf{p}$, is ...



- A: straight up \uparrow
- B: straight down \downarrow
- C: to the right \rightarrow
- D: to the left \leftarrow

Practice Question

A ball bounces off the floor as shown. The direction of the impulse on the ball, $\Delta\mathbf{p}$, is ...



A: straight up \uparrow

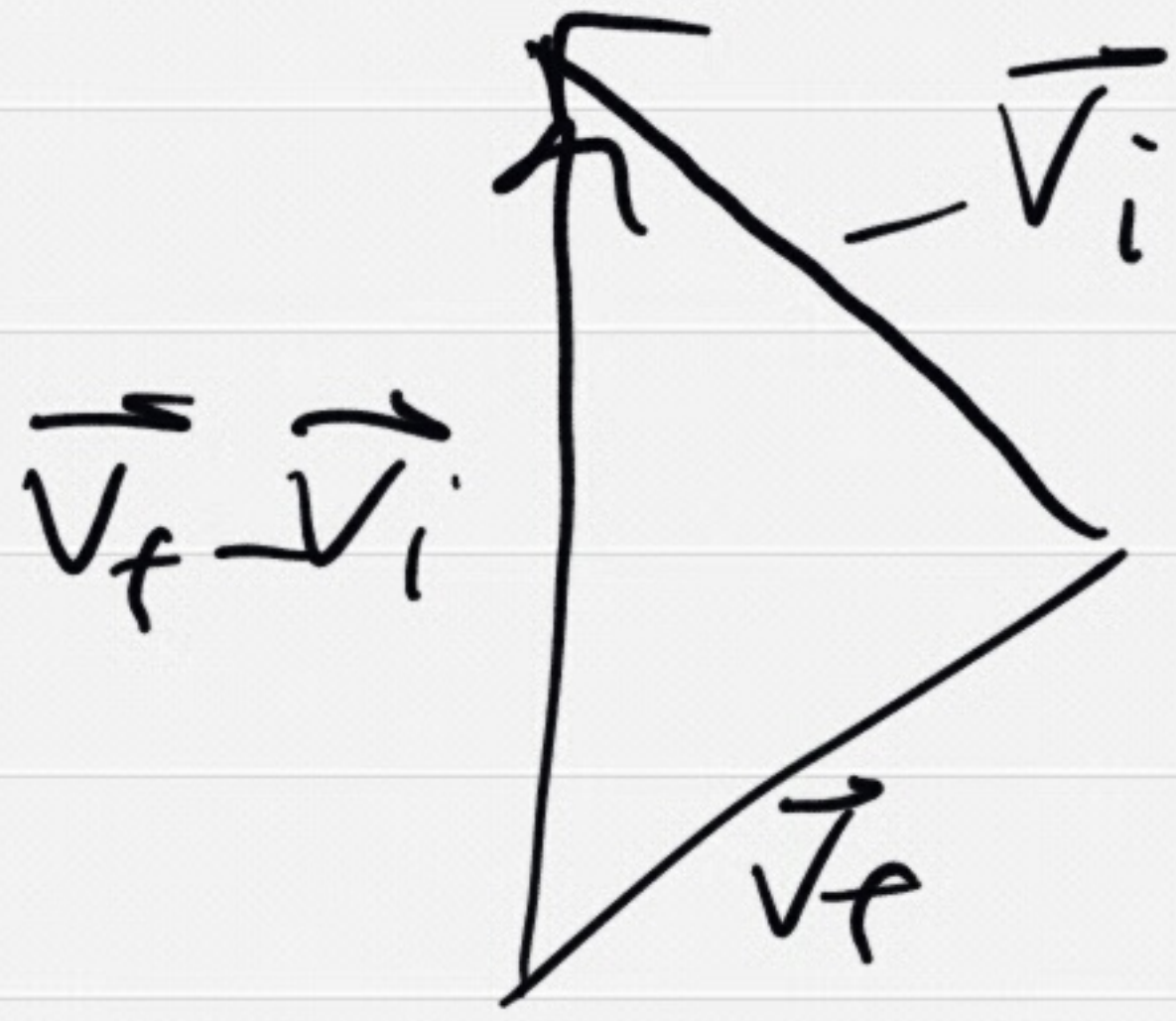
B: straight down \downarrow

C: to the right \rightarrow

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$$\begin{aligned}\Delta \vec{p} &= m\vec{v}_f - m\vec{v}_i \\ &= m(\vec{v}_f - \vec{v}_i)\end{aligned}$$



Conservation of Momentum

- If no external impulse:
 - Total momentum of an isolated system is conserved
 - Internal forces between different parts of an isolated system are equal and opposite, so total momentum is conserved
 - This is true even if mechanical energy is not conserved

Collisions/Explosions

Collisions

In all collisions where $\Sigma F_{\text{ext}} = 0$, momentum is conserved

Elastic Collisions

No deformation occurs.
Kinetic energy is also conserved.

Inelastic Collisions:

Deformation occurs.
Kinetic energy is **lost**.

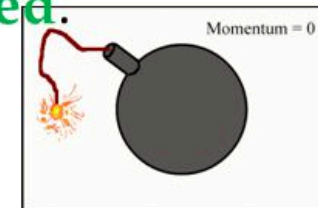
Perfectly Inelastic Collisions

Objects stick together,
kinetic energy is **lost**.

Explosions

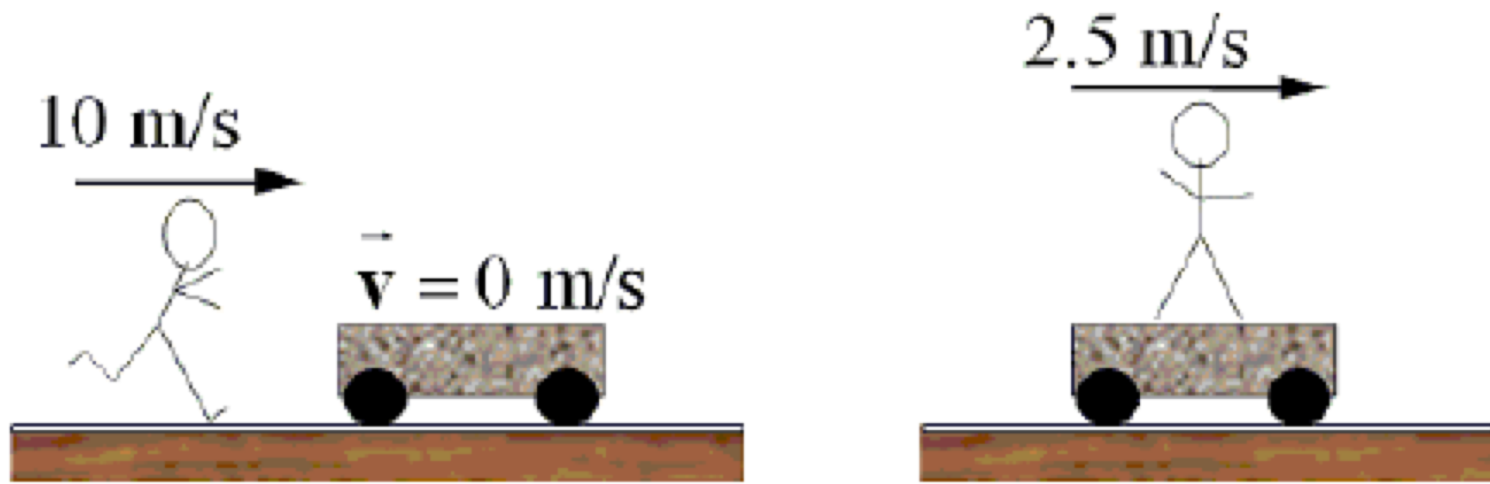
Reverse of perfectly inelastic collision, kinetic energy is **gained**.

$$KE = \frac{1}{2}mv^2$$



Practice Question

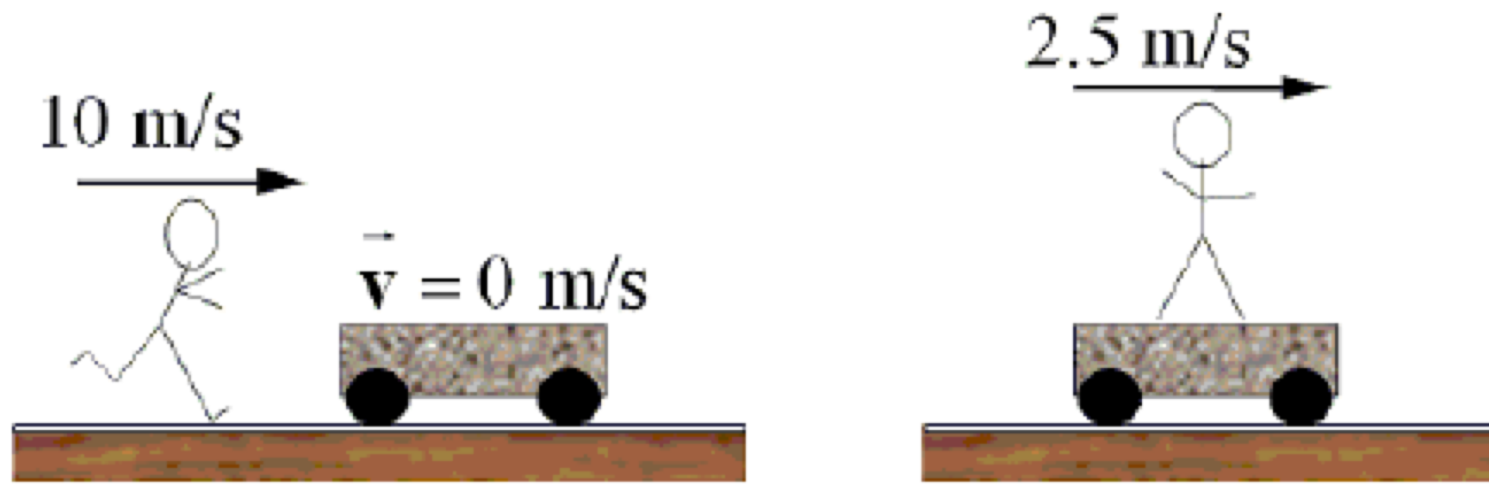
A 50.0-kg boy runs at a speed of 10.0 m/s and jumps onto a cart as shown in the figure. The cart is initially at rest. If the speed of the cart with the boy on it is 2.50 m/s, what is the mass of the cart?



- A. 260 kg
- B. 150 kg
- C. 175 kg
- D. 210 kg
- E. 300 kg

Practice Question

A 50.0-kg boy runs at a speed of 10.0 m/s and jumps onto a cart as shown in the figure. The cart is initially at rest. If the speed of the cart with the boy on it is 2.50 m/s, what is the mass of the cart?



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$$p_0 = m_1 v_{10} + m_2 v_{20}$$

$$= 50 - 10 + m_2 - 0$$

$$= 500 \text{ kg m/s}$$

$$p_f = p_0 = (m_1 + m_2) \cdot 2.5$$

$$= 125 + 2.5 m_2$$

$$\Rightarrow 2.5 m_2 = 375$$

$$\boxed{m_2 = 150 \text{ kg}}$$

Practice Question

- An object of mass $3m$, initially at rest, explodes breaking into two fragments of mass m and $2m$, respectively. Which one of the following statements concerning the fragments *after the explosion* is true?
 - A. They will fly off in the same direction.
 - B. They will fly off at right angles.
 - C. The smaller fragment will have twice the speed of the larger fragment.
 - D. The larger fragment will have twice the speed of the smaller fragment.
 - E. The smaller fragment will have four times the speed of the larger fragment.

Practice Question

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$$p_0 = 0 = p_f$$



$$2m \cdot v_2 - m v_1 = 0$$

$$2m v_2 = m v_1$$

$$v_2 = v_1 / 2$$

Rotational Motion

Rotational Motion:

$$\theta = s/r \quad \langle \omega \rangle = \frac{\langle v_t \rangle}{r} = \frac{\Delta \theta}{\Delta t}$$

$$\theta(t) = \theta_o + \omega_o t + \frac{1}{2} \alpha t^2$$

$$\tau = rF \sin \theta_{rF} = F * \text{lever arm}$$

$$W_{rot} = \tau \Delta \theta$$

$$\langle \alpha \rangle = \frac{\langle a_t \rangle}{r} = \frac{\Delta \omega}{\Delta t}$$

$$\omega(t)^2 = \omega_o^2 + 2\alpha \Delta \theta(t)$$

$$\sum \tau = I\alpha$$

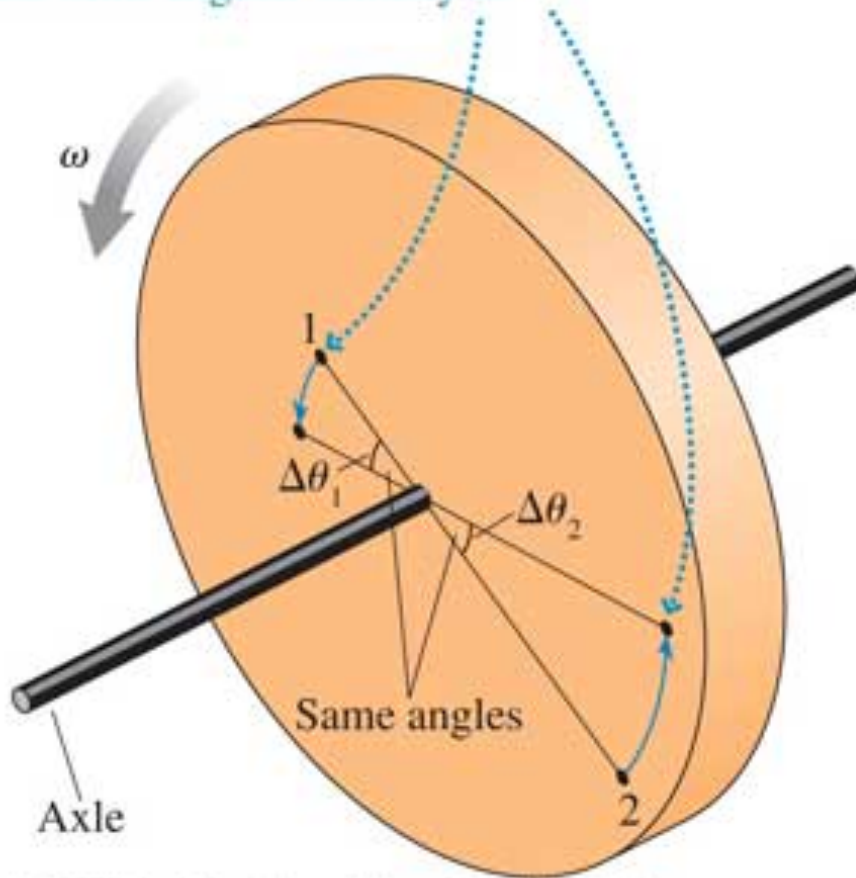
$$KE_{rot} = \frac{1}{2} I \omega^2$$

$$L = mvr = I\omega$$

$$r_{CM} = \frac{\sum m_i r_i}{\sum m_i}$$

Rotational Kinematics

Every point on the wheel undergoes circular motion with the same angular velocity ω .



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The angular velocity is *changing*, so the wheel has an angular acceleration.



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Practice Question

- During the spin-dry cycle of a washing machine, the motor slows from 5 rad/s to 3 rad/s while turning the drum through an angle of 4 radians. What is the magnitude of the angular acceleration of the motor?
 - A. 3 rad/s^2
 - B. 6 rad/s^2
 - C. 1 rad/s^2
 - D. 2 rad/s^2
 - E. 10 rad/s^2

Practice Question

- During the spin-dry cycle of a washing machine, the motor slows from 5 rad/s to 3 rad/s while turning the drum through an angle of 4 radians. What is the magnitude of the angular acceleration of the motor?
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Moment of Inertia

Moment of Inertia

Point mass or thin-walled wheel: $I = mr^2$

Solid cylinder: $I = \frac{1}{2} mr^2$

Thin rod pivoting around end: $I = \frac{1}{3} mr^2$

Solid sphere: $I = \frac{2}{5} mr^2$

- Moment of inertia is a measure of how hard it is to change the angular velocity of an object
- The highest possible moment of inertia is for a point-mass (or hoop or wheel with all mass at rim)
 - Any other object has some mass at smaller radius so has a lower moment of inertia

$$\omega^2 = \omega_0^2 + 2\alpha \Delta\theta$$

$$3^2 = 5^2 + 2\alpha \cdot 4$$

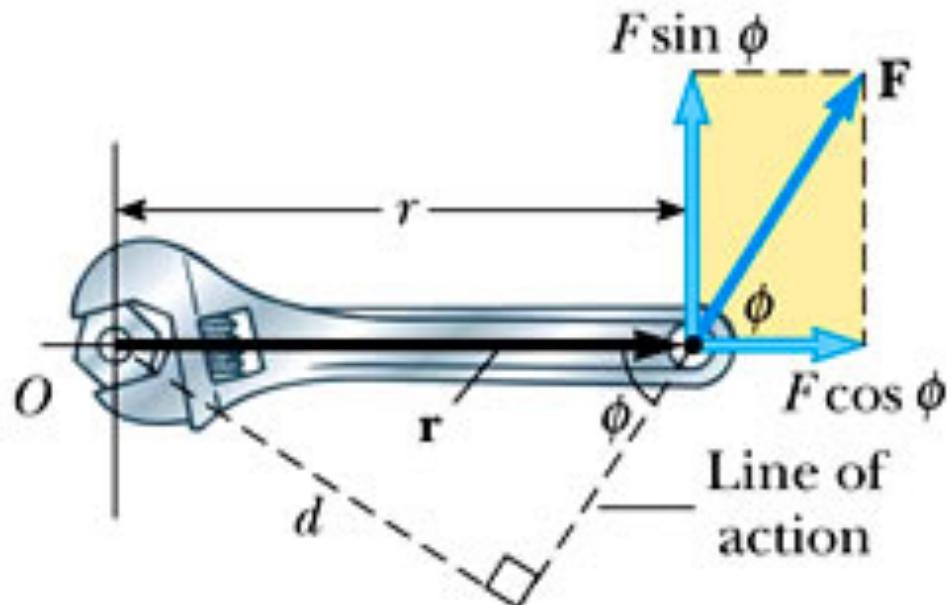
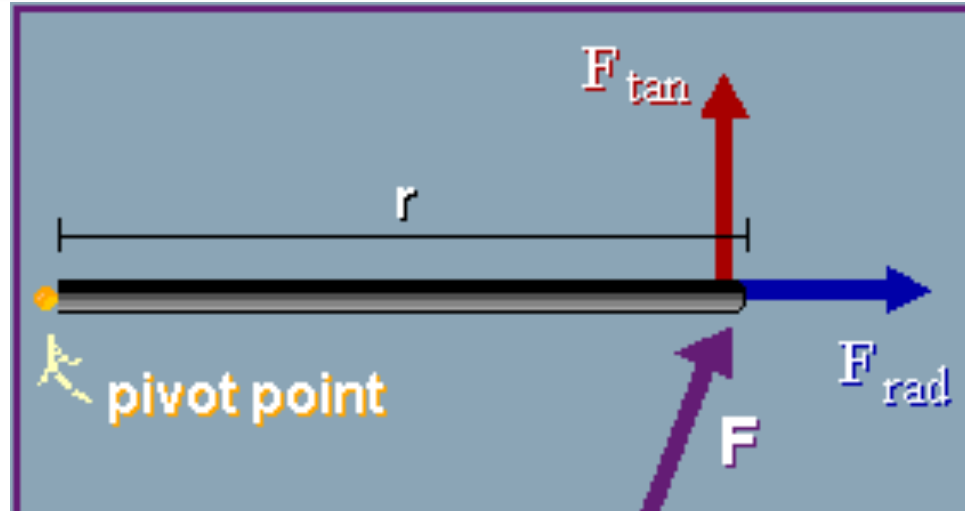
$$\rho = 25 + 8\alpha$$

$$-16 = 8\alpha$$

$$\alpha = -2 \text{ rad/s}^2$$

$$|\alpha| = 2 \text{ rad/s}^2$$

Torque

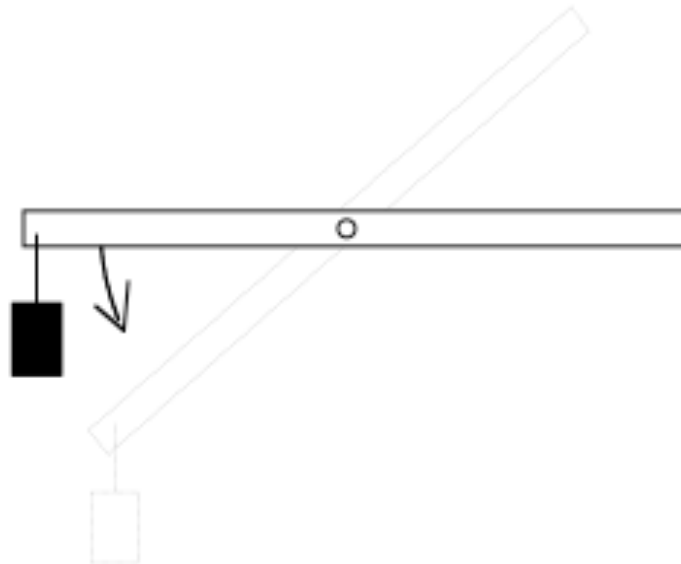


$d =$ lever arm
in this image

Practice Question

A mass is hanging from the end of a horizontal bar which pivots about an axis through its center, but it is being held stationary. The bar is released and begins to rotate. As the bar rotates from horizontal to vertical, the magnitude of the torque on the bar..

- A) increases B) decreases C) remains constant



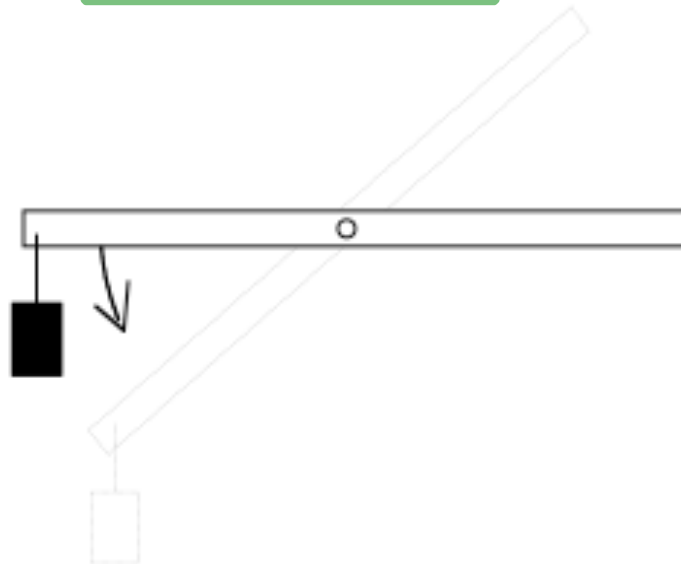
Practice Question

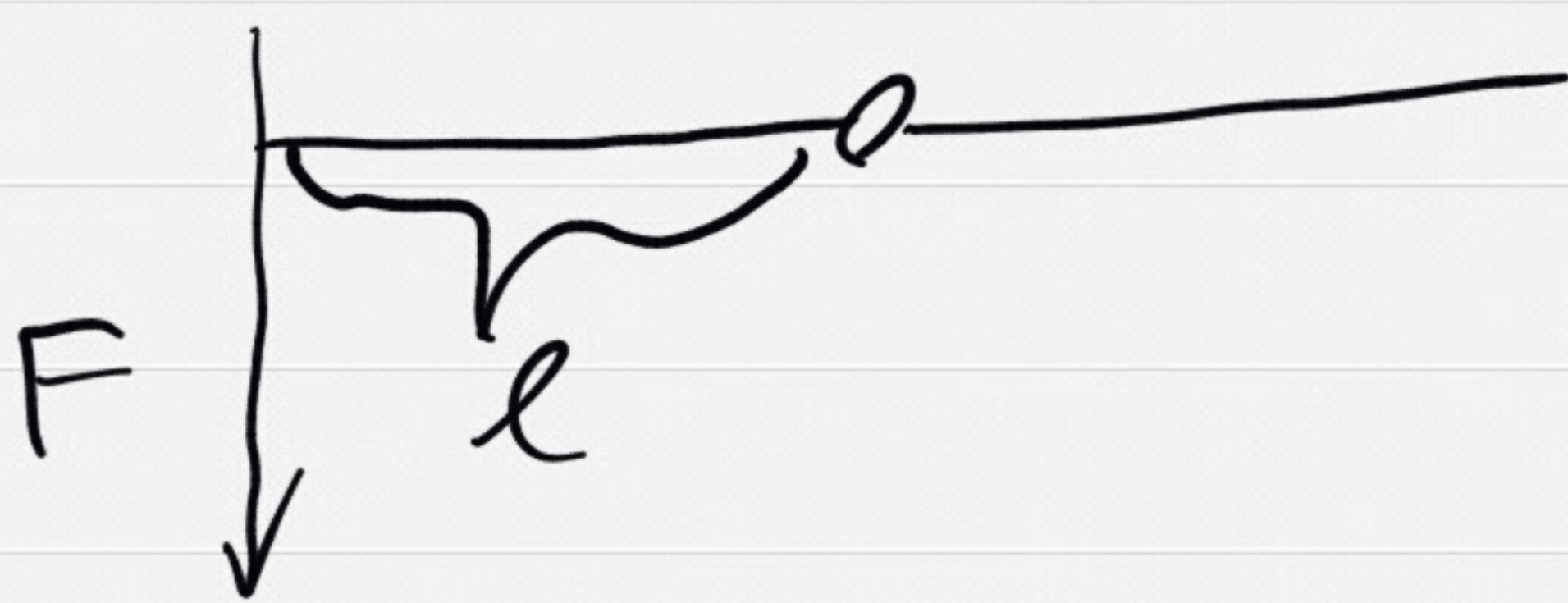
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A) increases

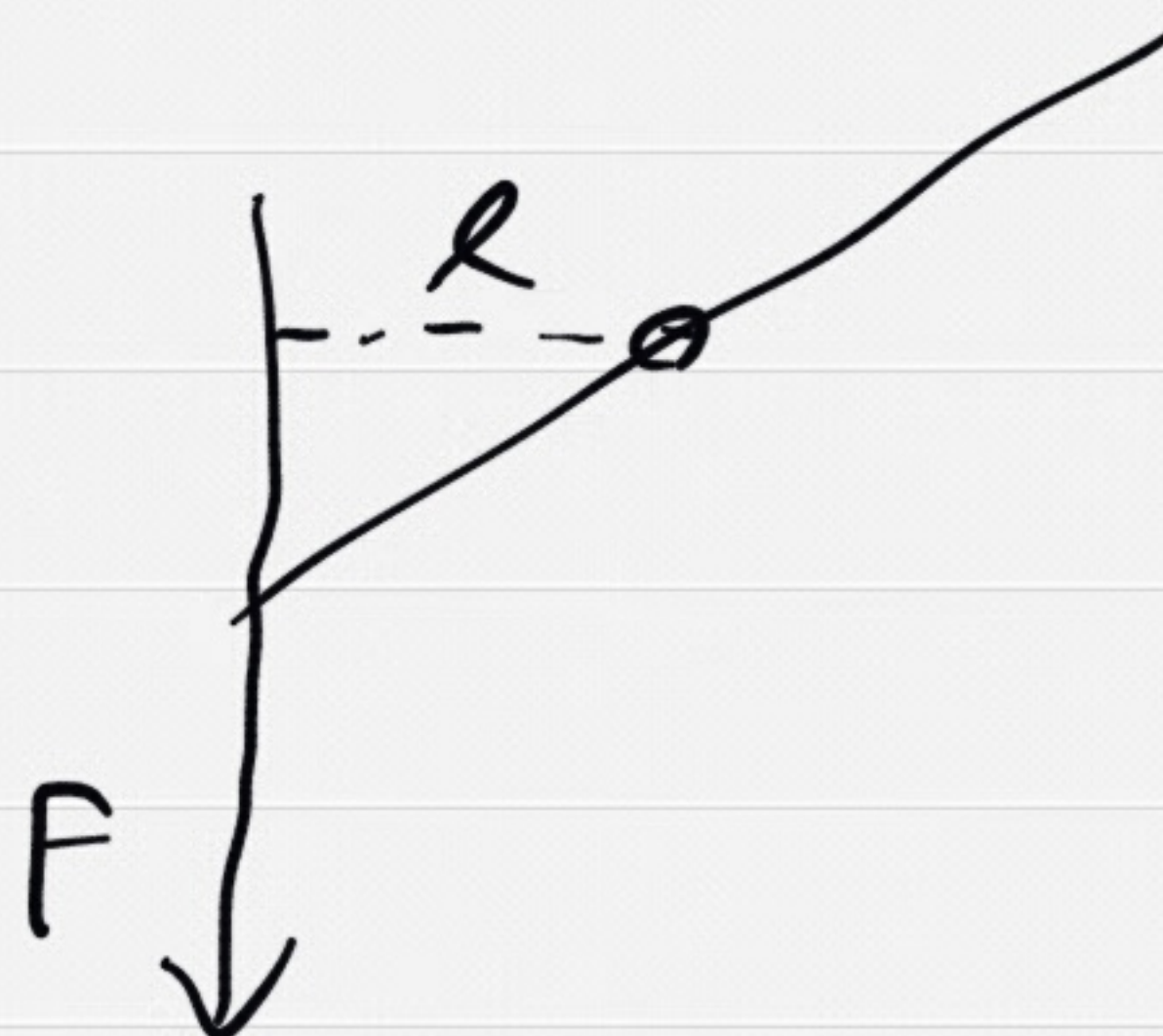
B) decreases

C) remains constant

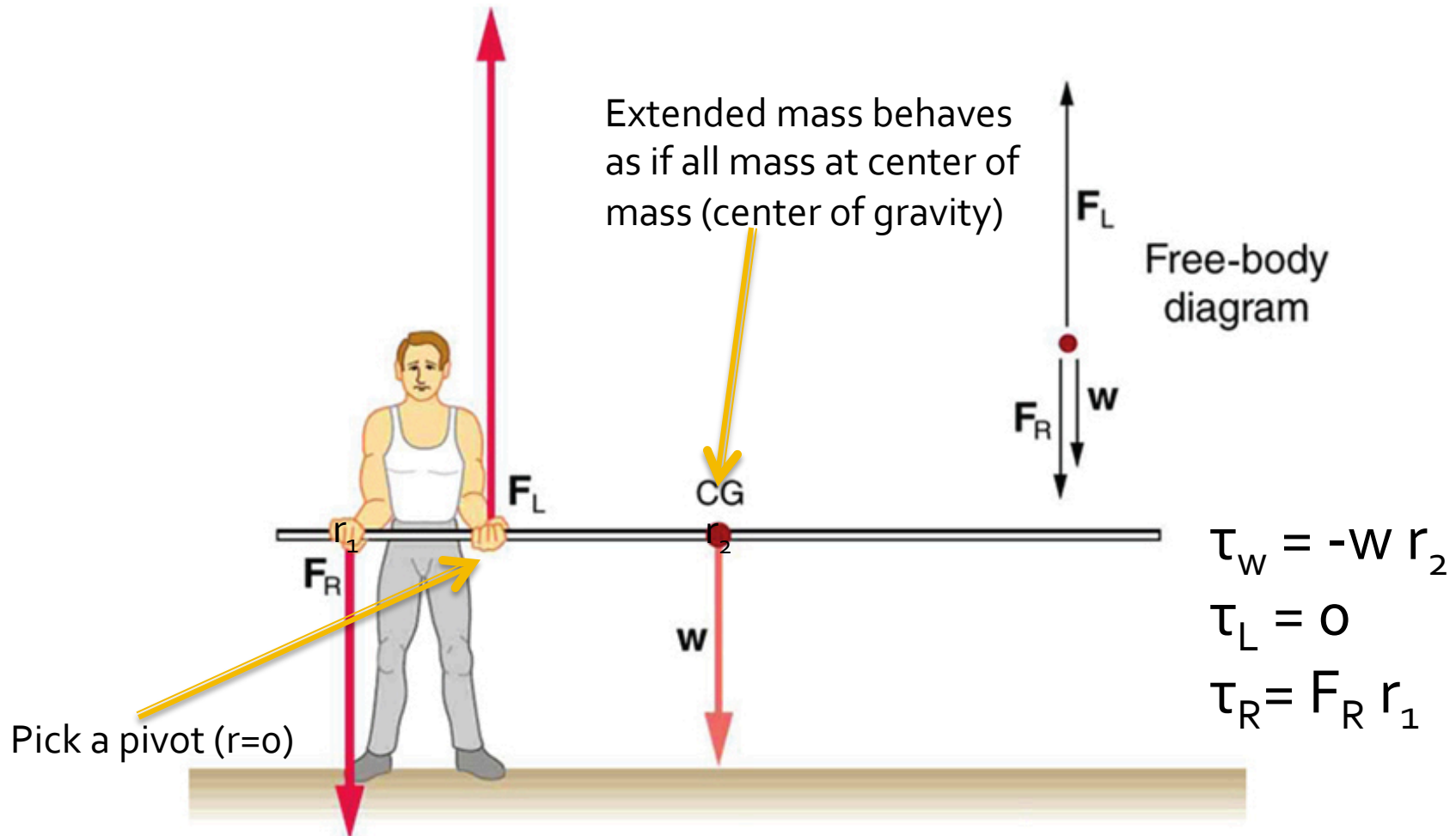




$T \sim$ decreases



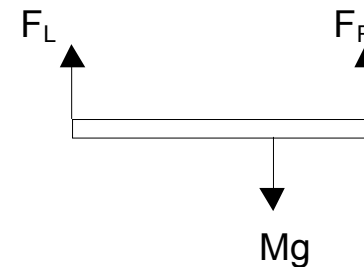
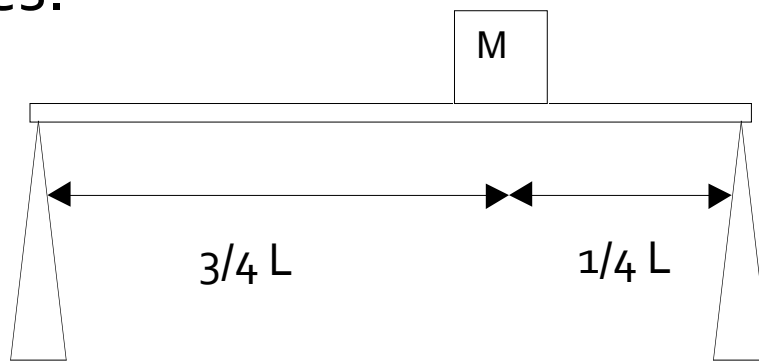
Equilibrium & Statics



$$F_R + F_L + F_w = 0 \text{ and } \tau_w + \tau_R + \tau_L = 0 \text{ in equilibrium}$$

Practice Question

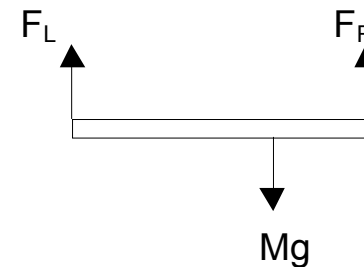
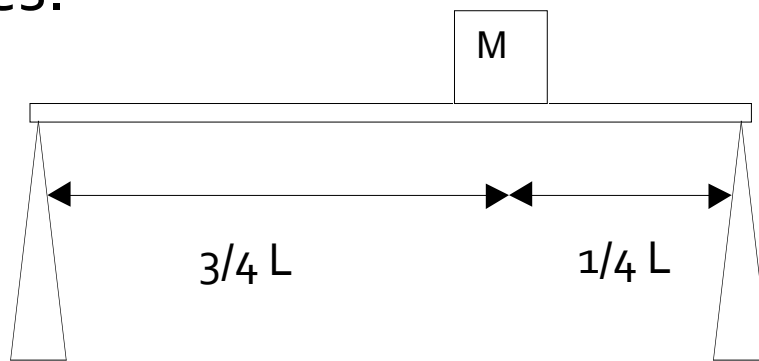
- A mass M is placed on a very light board supported at the ends, as shown. The free-body diagram shows directions of the forces, but not their correct relative sizes.



- What is the ratio F_R/F_L ?
- **A:** $3/4$ **B:** 4 **C:** $1/3$ **D:** 3
- **E:** some other answer

Practice Question

- A mass M is placed on a very light board supported at the ends, as shown. The free-body diagram shows directions of the forces, but not their correct relative sizes.



- What is the ratio F_R/F_L ?
- A: $3/4$
- B: 4
- E: some other answer

C: $1/3$

D: 3

Pivot @ M



$$\Sigma \tau = 0$$

$$\Rightarrow |\tau_L| = |\tau_R|$$

$$F_L \cdot 3L/4 = F_R \cdot L/4$$

$$\boxed{F_R = 3F_L}$$

Also $F_R + F_L = Mg$