Rotational Motion

Rotational Motion:

$\theta = s/r$ $\langle\omega\rangle = \frac{\langle v_t\rangle}{r} = \frac{\Delta\theta}{\Delta t}$ $\langle\alpha\rangle = \frac{\langle a_t\rangle}{r} = \frac{\Delta\omega}{\Delta t}$

$\theta(t) = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$ $\omega(t)^2 = {\omega_o}^2 + 2\alpha\Delta\theta(t)$

$\tau = rF\sin\theta_{rF} = F * lever\ arm$ $\sum\tau = I\alpha$ $L = mvr = I\omega$

$W_{rot} = \tau\Delta\theta$ $KE_{rot} = \frac{1}{2}I\omega^2$ $r_{CM} = \frac{\sum m_i r_i}{\sum m_i}$

Rotational Dynamics

$$F = ma$$

$$Fr = (ma)r$$

$$\tau = m(r\alpha)r = mr^2\alpha$$

$$\tau = I\alpha$$

Work and Energy

Translational	Rotational
$W = Fs\cos\theta$	$W_R = \tau\theta$ SI units: N·m = J
$KE = \frac{1}{2}mv^2$	$KE_R = \frac{1}{2}I\omega^2$ SI units: (kg·m^2) / s^2 = N·m = J

$$\underbrace{E}_{\text{Total mechanical energy}} = \underbrace{\frac{1}{2}mv^2}_{\text{Translational kinetic energy}} + \underbrace{\frac{1}{2}I\omega^2}_{\text{Rotational kinetic energy}} + \underbrace{mgh}_{\text{Gravitational potential energy}} + \underbrace{\frac{1}{2}kx^2}_{\text{Elastic potential energy}}$$

Practice Question

- A 1.0-kg solid disk rolls (without slipping) along a horizontal surface with a speed of 6.0 m/s. How much total work (linear and rotational) has to be done to slow the disk to rest?

A. 9.0 J
B. 18 J
C. 27 J
D. 36 J
E. 54 J

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$$W = \Delta KE$$
$$= \Delta KE_{rot} + \Delta KE_{trans}$$

$$|W| = \tfrac{1}{2} m v_0^2 + \tfrac{1}{2} I \omega_0^2$$

$I = \tfrac{1}{2} m r^2$ for disk
$\omega = v/r$ if not slipping

$$|W| = \tfrac{1}{2} m v^2 + \tfrac{1}{2} \cdot \tfrac{1}{2} m r^2 \cdot v^2/r^2$$
$$= \tfrac{1}{2} m v^2 + \tfrac{1}{4} m v^2$$

$$= \tfrac{3}{4} \cdot 6^2$$
$$= \boxed{27\ J}$$

Harmonic Motion Equations

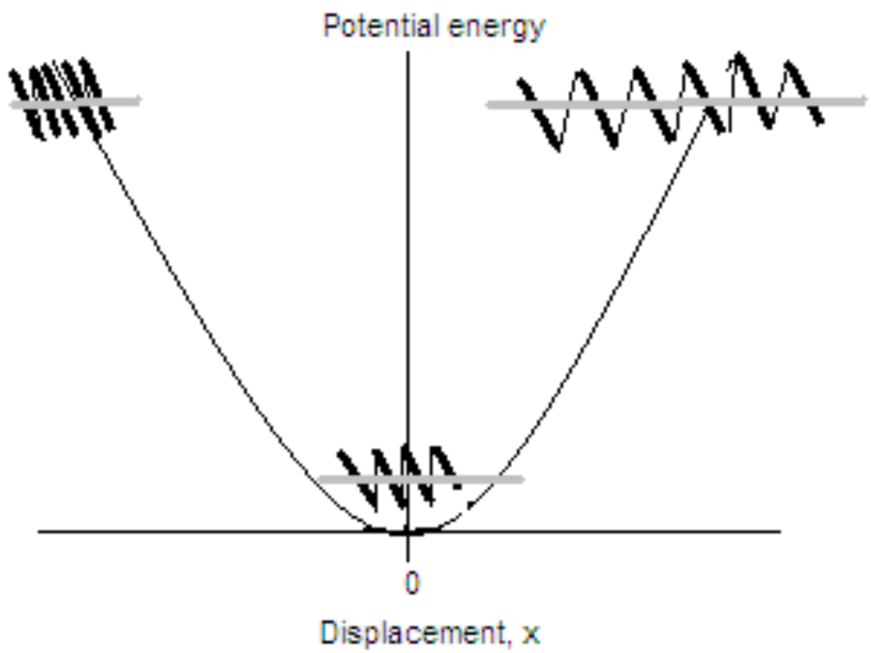
Harmonic Motion:

$\omega_h = 2\pi f_h = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$ $x_{max} = A$ $v_{max} = A\omega_h$ $a_{max} = A{\omega_h}^2$

$\omega_{h\_pendulum} = \sqrt{\frac{mgr_{CM}}{I}} = \sqrt{\frac{g}{L}}$ *for simple pendulum of length L*

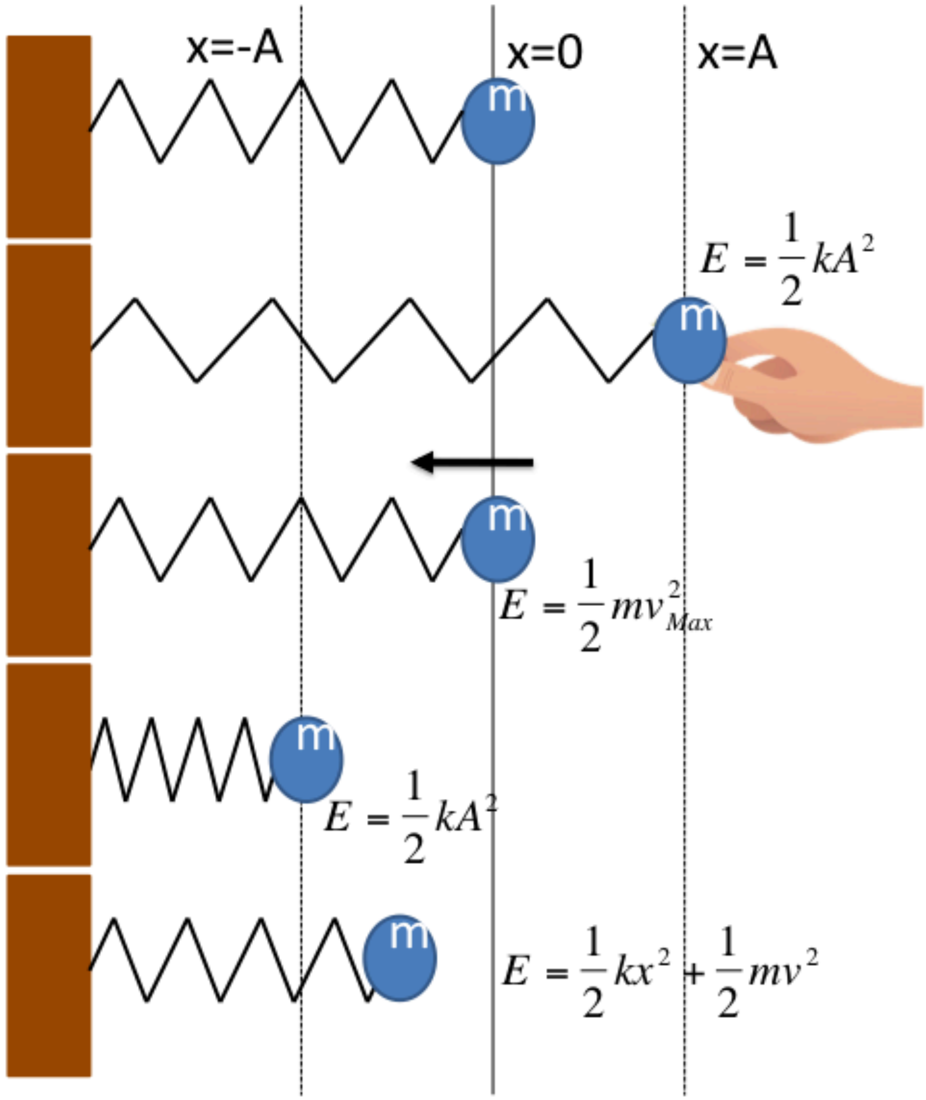
Harmonic Oscillator

- Any system with a linear restoring force
  - Any such system oscillates around an equilibrium, maintaining a constant mechanical energy
  - A linear restoring force is equivalent to having a U-shaped potential energy curve

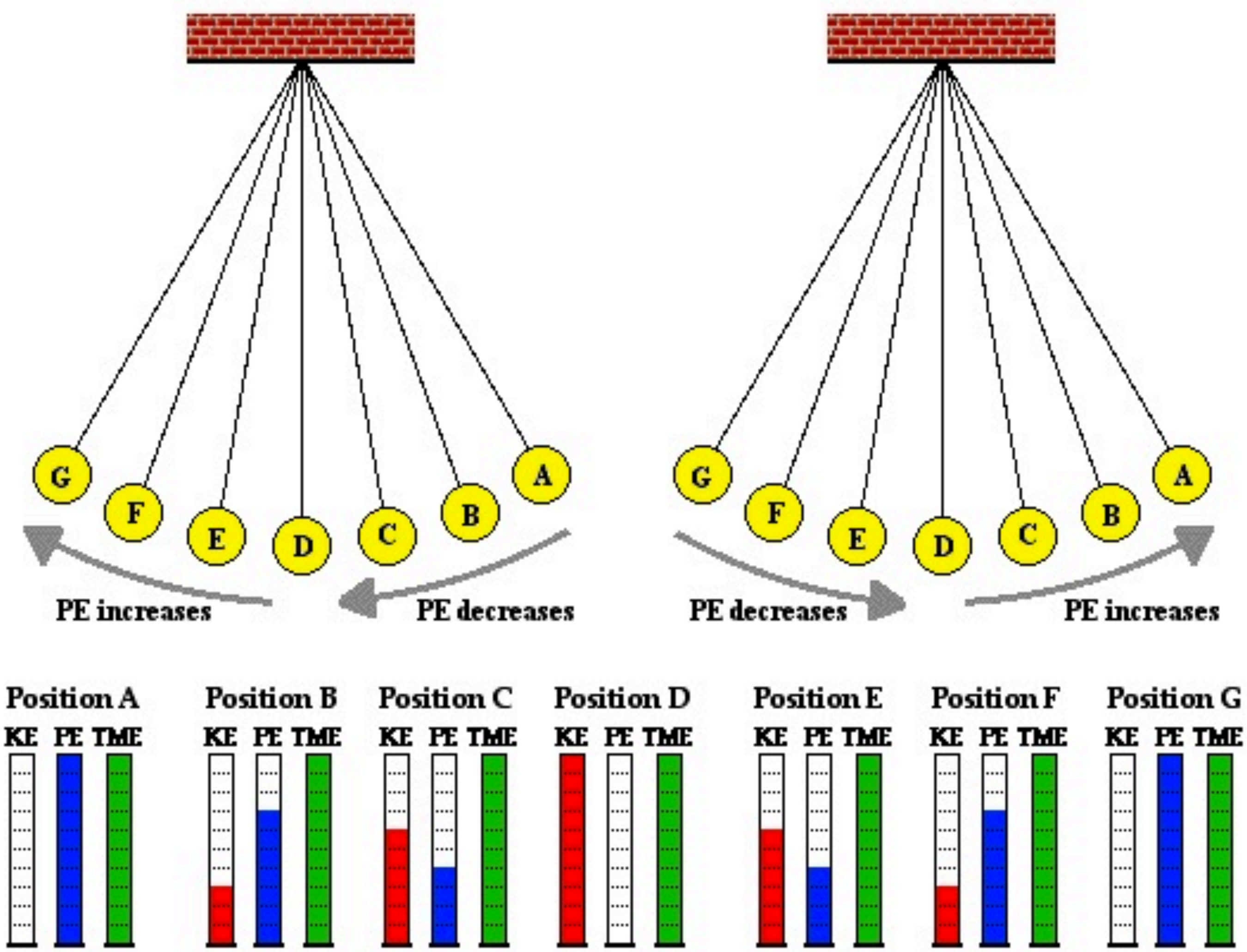


Harmonic Oscillator Energy

Mass-Spring System



Pendulum



G
F
E
D
C
B
A
PE increases
PE decreases
PE decreases
PE increases
Position A
KE PE TME
Position B
KE PE TME
Position C
KE PE TME
Position D
KE PE TME
Position E
KE PE TME
Position F
KE PE TME
Position G
KE PE TME

Practice Question

- A 4 kg mass on a spring (k = 4 N/m) is stretched 1 m from equilibrium and released. What is the maximum speed that the mass reaches during its oscillation?

A. 10 m/s
B. 6.66667 m/s
C. 5 m/s
D. 3.33333 m/s
E. 1 m/s

Practice Question

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$$v_m = \sqrt{k/m}\, A$$

$$= \sqrt{4/4} \cdot 1$$

$$= \boxed{1 \text{ m/s}}$$

Practice Question

- A mass on a spring oscillates in harmonic motion. What happens to the amplitude of the motion if at the equilibrium position of the spring, half of the mass falls off?

A. A increases by 2
B. A decreases by 2
C. A increases by $\sqrt{2}$
D. A decreases by $\sqrt{2}$
E. A stays the same

Practice Question

- A mass on a spring oscillates in harmonic motion. What happens to the amplitude of the motion if at the equilibrium position of the spring, half of the mass falls off?

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Dropping m @ equilibrium

changes KE

$E_0 = \frac{1}{2} m V_m^2$

$E_f = \frac{1}{2} \frac{m}{2} V_m^2$
$= E_0/2$

$E_0 = \frac{1}{2} k A^2$

$E_f = \frac{1}{2} k A_f^2 = E_0/2$

$\frac{1}{2} k A_f^2 = \frac{1}{2} \frac{1}{2} k A_0^2$
$A_f^2 = \frac{1}{2} A_0^2$
$\boxed{A_f = A_0/\sqrt{2}}$

Changing m @ X_m would not have had this effect.

Fluid Equations

Fluids:

$\rho = mass/Volume$ $\quad P = F/A$ $\quad P_2 = P_1 + \rho g d$ $\quad F_B = W_{fluid\_displaced}$

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$ $\quad A_1 v_1 = A_2 v_2 \ (if\ \rho_1 = \rho_2)$ $\quad P_1 + \frac{1}{2}\rho {v_1}^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho {v_2}^2 + \rho g y_2$

Practice Question

- Which exerts greater force on a 1 m^2 rug – the atmosphere or a 100 kg mass sitting on the rug? Assume $P_{atm} = 10^5$ Pa and $g = 10\ m/s^2$.

A. The atmosphere
B. The 100 kg mass
C. Both tie

Practice Question

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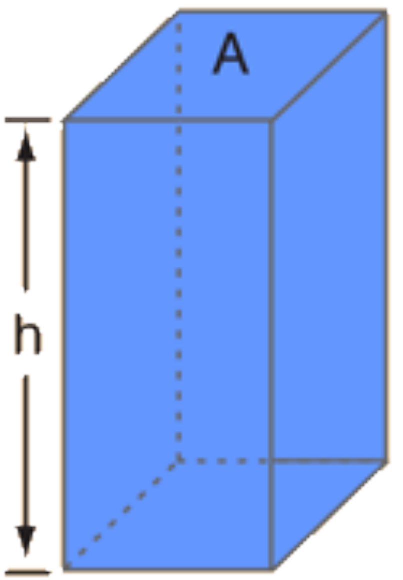
$$F_{atm} = P_{atm} \cdot A$$
$$= 10^5 \, N$$

$$F_{mass} = mg = 100 \cdot 10$$
$$= 1000 \, N$$

$$\boxed{F_{atm} \gg F_{mass}}$$

Fluid Pressure

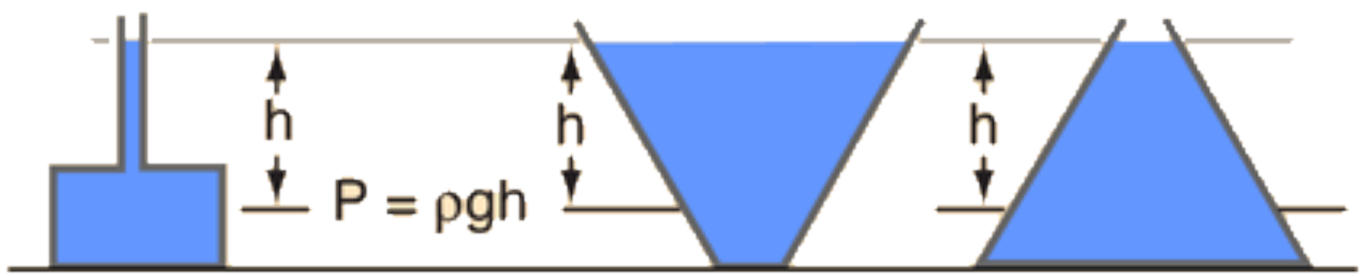


V = hA = volume

weight = mg

Static fluid pressure does not depend on the shape, total mass, or surface area of the liquid.

$$\text{Pressure} = \frac{\text{weight}}{\text{area}} = \frac{mg}{A} = \frac{\rho V g}{A} = \rho g h$$

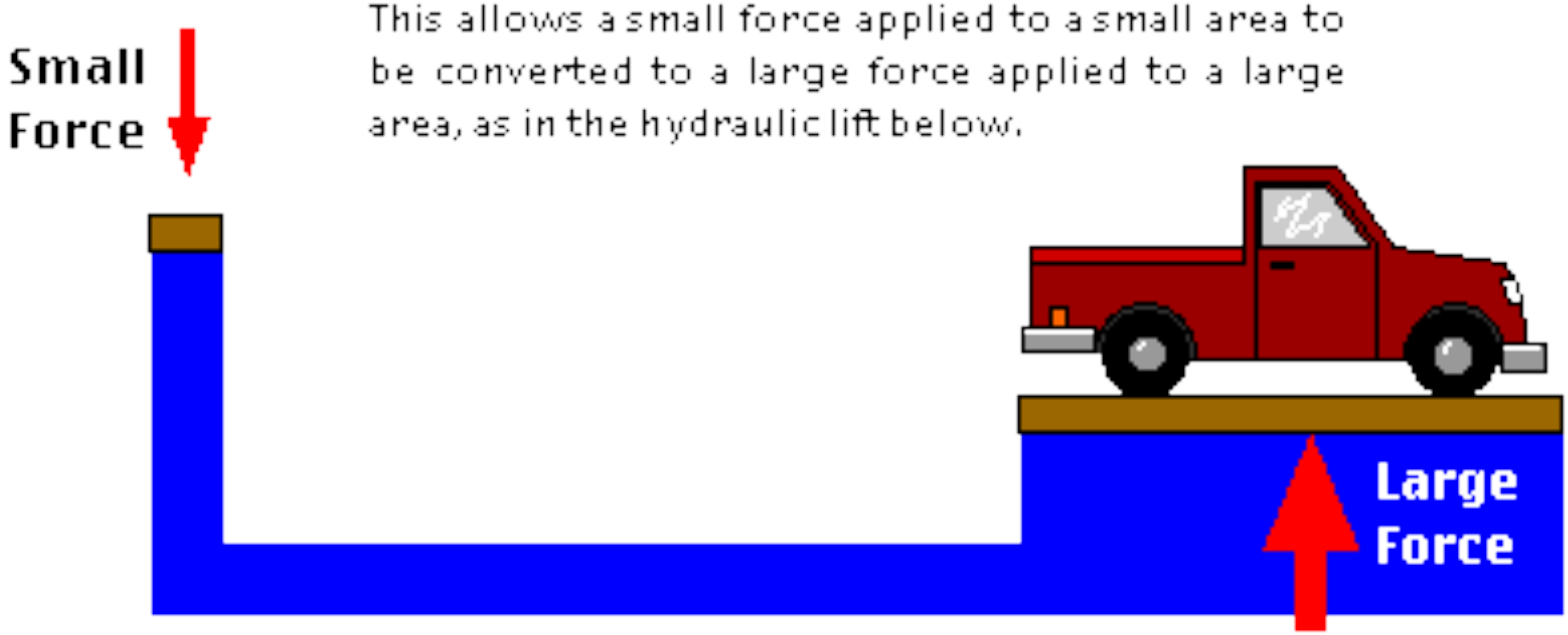


Pascal's Principle

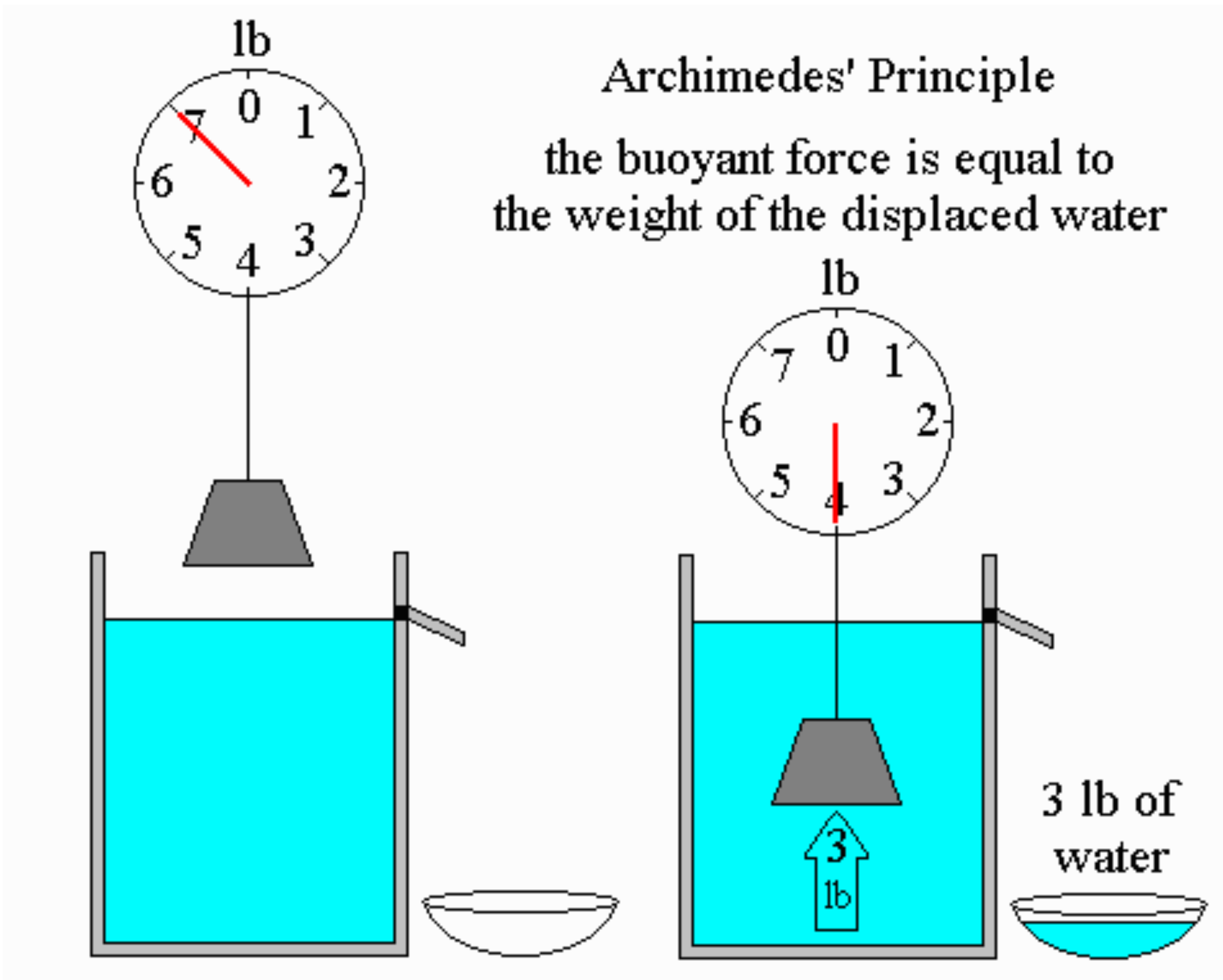
Pascal's Principle

"The pressure exerted at one surface of an incompressible fluid is equal to the pressure exerted on any other surface."

This allows a small force applied to a small area to be converted to a large force applied to a large area, as in the hydraulic lift below.



Buoyancy



lb
0
1
2
3
4
5
6
7
Archimedes' Principle
the buoyant force is equal to
the weight of the displaced water
lb
0
1
2
3
4
5
6
7
3
lb
3 lb of
water

Practice Question

The block A weighs 2 N (in air) and has a density twice that of water. When immersed in the beaker of water (but not dropped to the bottom), the reading on scale *E* will:

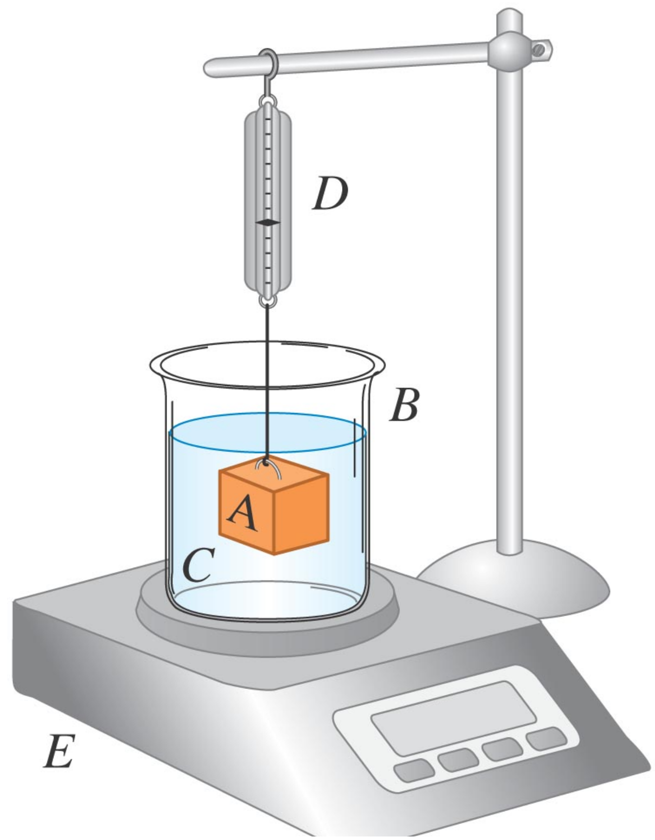
A) be unchanged

B) increase by 1 N

C) increase by 2 N

D) decrease by 1 N

E) cannot determine



Practice Question

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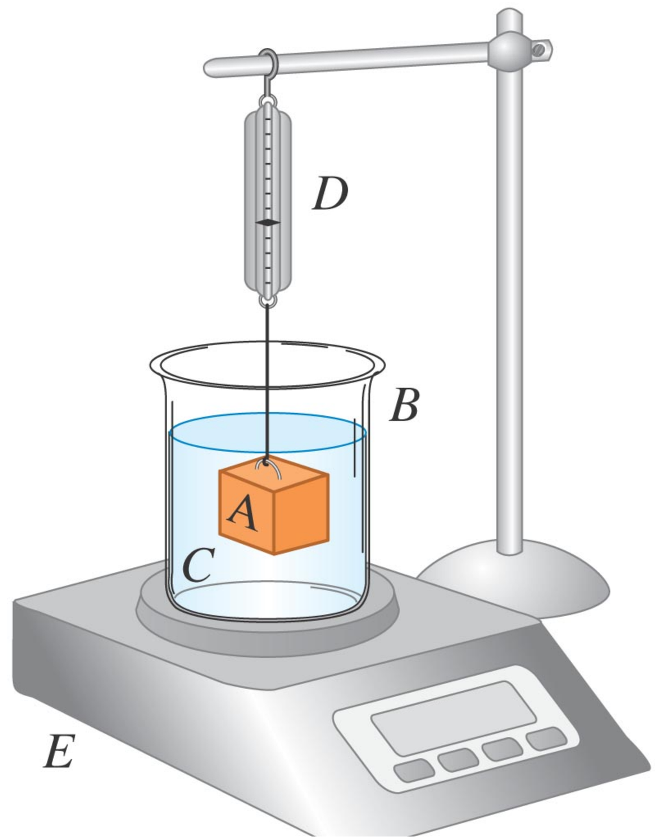

$\uparrow \; F_B = m_{fluid\_disp} \cdot g$

$\downarrow \; F_W = m_{block} \cdot g$

$$F_B = \rho_f \cdot V_b \cdot g$$

$$= \rho_b/2 \cdot V_b \cdot g$$

$$F_W = \rho_b \cdot V_b \cdot g$$

$$F_B = F_W/2 = 1\,N$$

- Force on water opposite F_B so weight on scale E is **+1 N**
- Weight on scale D decreases by 1 N.

Continuity