

$$1.a. E_y \cdot 0 + d/2 = \frac{q}{4\pi\epsilon_0 d^2}$$

$$\Rightarrow F = \frac{q^2}{4\pi\epsilon_0 d^2} \text{ outward}$$

$$1.b. u = -v$$

$$E_y' = \gamma E_y, \quad B_z' = \frac{\gamma v}{c^2} E_y$$

$$F_y' = qE_y' - qvB_z'$$

$$= \frac{q^2}{4\pi\epsilon_0 d^2} \gamma - \left(1 - \frac{v^2}{c^2}\right) = \frac{1}{\gamma} \frac{q^2}{4\pi\epsilon_0 d^2}$$

$$1.c. K^\mu = (0, 0, \frac{q^2}{4\pi\epsilon_0 d^2}, 0)$$

($\gamma = 1$ initially)

$K^{\mu'}$ = K^μ since y component of 4-vector doesn't change

$$\text{But } K^{\mu'} = \gamma \left(\frac{1}{c} \frac{dE'}{dt}, F_x', F_y', F_z' \right)$$

$$\Rightarrow F_y' = \frac{1}{\gamma} K^{2'} = \frac{1}{\gamma} \frac{q^2}{4\pi\epsilon_0 d^2}$$

2.a. Use Lorentz transformation
 $w/ u = -v$

$$E_z' = \gamma E_z = \gamma \sigma / 2\epsilon_0$$

$$B_y' = \gamma \frac{v}{c^2} E_z = -\gamma \frac{v\sigma}{c^2} E_z = \frac{-\gamma v\sigma}{2\epsilon_0 c^2}$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & \gamma \sigma / 2\epsilon_0 c \\ 0 & 0 & 0 & \gamma v\sigma / 2\epsilon_0 c^2 \\ 0 & 0 & 0 & 0 \\ -\frac{\gamma \sigma}{2\epsilon_0 c} & -\frac{\gamma v\sigma}{2\epsilon_0 c^2} & 0 & 0 \end{pmatrix}$$

2.b.

$$K^\mu = q \mathcal{Z}_\nu F^{\mu\nu}$$

$$\mathcal{Z}_\nu = (-c, 0, 0, v_p) / \sqrt{1 - v_p^2/c^2}$$

$$= \gamma_p (-c, 0, 0, v_p)$$

$$K^0 = q \gamma_p v_p \cdot \gamma \sigma / 2\epsilon_0 c = \gamma_p / c \frac{dE}{dt}$$

$$K^1 = q \gamma_p v_p \cdot \gamma v\sigma / 2\epsilon_0 c^2 = \gamma_p F_x$$

$$K^2 = 0$$

$$K^3 = -q \gamma_p c \cdot \frac{-\gamma \sigma}{2\epsilon_0 c} = \gamma_p F_z$$

F_x due to \vec{B} , F_z due to \vec{E}
 dE/dt due to motion through \vec{E}

$$3. A^\mu = \left(\frac{k}{rc}, 0, 0, 0 \right)$$

$$= \left(\frac{k}{c} \frac{1}{\sqrt{x^2 + y^2 + z^2}}, 0, 0, 0 \right)$$

$$F^{\mu\nu} = \partial A^\nu / \partial x^\mu - \partial A^\mu / \partial x^\nu$$

only non-zero derivatives are

$$\partial A^0 / \partial x = \frac{k}{c} \frac{-x}{(x^2 + y^2 + z^2)^{3/2}} = \frac{-kx}{cr^3}$$

$$\partial A^0 / \partial y = -ky / cr^3$$

$$\partial A^0 / \partial z = -kz / cr^3$$

$$F^{\mu\nu} = \frac{k}{cr^3} \begin{pmatrix} 0 & x & y & z \\ -x & 0 & 0 & 0 \\ -y & 0 & 0 & 0 \\ -z & 0 & 0 & 0 \end{pmatrix}$$