1. $\mathcal{E} = -d\Phi_B/dt$

$= -d/dt\,(B_z \cdot \pi a^2)$

$= -d/dt\,(\pi a^2 \mu n I_s)$

$= -\pi a^2 \mu n I_{s\_max}/\Delta t$

$I_{ind} = \mathcal{E}/R = -\pi a^2 \mu n I_{s\_max}/(R\Delta t)$

$\langle F \rangle = \langle \int \vec{I} \times \vec{B}\, dl \rangle = I_{ind} \cdot 2\pi a \cdot \langle B_r \rangle$

$\sim I_{ind} \cdot 2\pi a \cdot K \mu n I_{s\_max}/2$ ← time average

$\Delta p = F\Delta t = \pi^2 a^3 \mu^2 n^2 K I_{s\_max}^2/R$

$mg z_{max} = \Delta p^2/2m$

$\Rightarrow z_{max} = \Delta p^2/(2m^2 g)$

$$\boxed{z_{max} = \frac{\pi^4 a^6 \mu^4 n^4 K^2 I_{s\_max}^4}{2m^2 g R^2}}$$

2. $\vec{E} = \lambda/(2\pi\varepsilon s)\ \hat{s}$

$\vec{B} = \mu I/2\pi s\ \hat{\varphi}$

$\vec{S} = \frac{1}{\mu}(\vec{E} \times \vec{B})$

$= \dfrac{\lambda I}{4\pi^2 \varepsilon s^2}\ \hat{z}$

Be careful to use μ & ε, not μ_0 & ε_0

$P = \int \vec{S} \cdot d\vec{a} = \int \dfrac{\lambda I}{4\pi^2 \varepsilon s^2} \cdot 2\pi s\, ds$

$= \int \dfrac{\lambda I\, ds}{2\pi \varepsilon s} = \dfrac{\lambda I}{2\pi\varepsilon} \ln(b/a)$

But $\Delta V = -\int \vec{E} \cdot d\vec{l}$

$= -\int_a^b \dfrac{\lambda}{2\pi\varepsilon s}\, ds = \dfrac{-\lambda}{2\pi\varepsilon} \ln(b/a)$

$\Rightarrow \boxed{P = -\Delta V \cdot I}$ sign not critical

3. $\vec{B}_I(z,t) = \frac{E_0}{c}\cos(kz-\omega t)\,\hat{y}$

$\vec{B}_R(z,t) = E_0/c\,\cos(-kz-\omega t)\,\hat{y}$

$\vec{B}(z<0,t) = E_0/c\,[\cos(kz-\omega t)+\cos(-kz-\omega t)]\,\hat{y}$

$= 2E_0/c\,\cos(\omega t)\,\hat{y}$ @ $z=0$

$\Delta\vec{B}_{\parallel} = \mu_0\vec{K}\times\hat{n} = 2\, E_0/c\,\cos(\omega t)\,\hat{y}$

$\Rightarrow \vec{K} = \frac{2\,E_0}{\mu_0 c}\cos(\omega t)\,\hat{x}$

$\vec{f} = \vec{K}\times\vec{B}_{ave}$

Average of B at $z=0$

$= \frac{2\,E_0}{\mu_0 c}\cos(\omega t)\,\hat{x} \times \frac{E_0}{c}\cos(\omega t)\,\hat{y}$

$= \frac{2\,E_0^2}{\mu_0 c^2}\cos^2(\omega t)\,\hat{z}$

$$\boxed{P = \langle f\rangle = \varepsilon_0 E_0^2}$$

Twice the value for an absorber

4. $u = c\Delta\hat{r} - \vec{v}$

$\Delta\hat{r}\cdot\vec{u} = c - \Delta\hat{r}\cdot\vec{v} = c - v\cos\theta$

$= c(1-\beta\cos\theta)$

$\Delta\hat{r}\times(\vec{u}\times\vec{a}) = \vec{u}(\vec{a}\cdot\Delta\hat{r}) - \vec{a}(\Delta\hat{r}\cdot\vec{u})$

$|\Delta\hat{r}\times(\vec{u}\times\vec{a})|^2 = (\vec{a}\cdot\Delta\hat{r})^2u^2 + (\Delta\hat{r}\cdot\vec{u})^2a^2$

$- 2(\vec{a}\cdot\vec{u})(\vec{a}\cdot\Delta\hat{r})(\Delta\hat{r}\cdot\vec{u})$

$= a^2\sin^2\theta\cos^2\varphi\, u^2 + c^2(1-\beta\cos\theta)^2a^2$

$-2(\vec{a}\cdot\vec{u})\,a\sin\theta\cos\varphi\, c(1-\beta\cos\theta)$

$u^2 = c^2+v^2-2c\cos\theta = c^2(1+\beta^2-2\beta\cos\theta)$

$\vec{a}\cdot\vec{u} = ac\sin\theta\cos\varphi$

$\Rightarrow |\Delta\hat{r}\times(\vec{u}\times\vec{a})|^2 = a^2c^2\sin^2\theta\cos^2\varphi(1+\beta^2-2\beta\cos\theta)$

$+a^2c^2(1-\beta\cos\theta)^2$

$-2a^2c^2\sin^2\theta\cos^2\varphi(1-\beta\cos\theta)$

$= a^2c^2\left[(1-\beta\cos\theta)^2 - (1-\beta^2)\sin^2\theta\cos^2\varphi\right]$

4. continued

$$\frac{dP}{d\Omega} = \frac{q^2 a^2 c^2 \left[(1-\beta\cos\theta)^2 - (1-\beta^2)\sin^2\theta\cos^2\phi\right]}{16\pi^2\varepsilon_0 \cdot c^5 (1-\beta\cos\theta)^5}$$

$$\Rightarrow \boxed{\frac{dP}{d\Omega} = \frac{\mu_0 q^2 a^2}{16\pi^2 c} \frac{\left[(1-\beta\cos\theta)^2 - (1-\beta^2)\sin^2\theta\cos^2\phi\right]}{(1-\beta\cos\theta)^5}}$$

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5. In S $A^\mu = \left(\frac{q}{4\pi\varepsilon_0 rc}, 0, 0, 0\right)$

In S' $A^{0'} = \gamma A_0$, $A^{1'} = -\gamma\beta A_0$

so $V' = \gamma \frac{q}{4\pi\varepsilon_0 r}$, $\vec{A}' = \frac{-\gamma\vec{u}}{c} \frac{q}{4\pi\varepsilon_0 r}$

$$= \frac{\gamma\vec{v}}{c}\frac{q}{4\pi\varepsilon_0 r}$$

$$= \frac{\vec{v}}{c}V'$$

But $1/r = 1/\sqrt{x^2+y^2+z^2}$

$$= 1/\sqrt{\gamma^2(x'+ut')^2 + y'^2 + z'^2}$$

$$= 1/\sqrt{\gamma^2(x'-vt')^2 + y'^2 + z'^2}$$

$$\Rightarrow \gamma/r = 1/\sqrt{(x'-vt')^2 + (1-v^2/c^2)(y'^2+z'^2)}$$

$$\Rightarrow \boxed{\begin{aligned} V'(\vec{r}') &= \frac{q}{4\pi\varepsilon_0\sqrt{(x'-vt')^2 + (1-v^2/c^2)(y'^2+z'^2)}} \\ \vec{A}'(\vec{r}') &= \frac{\vec{v}}{c}V'(\vec{r}') \end{aligned}}$$

Can also write $V'(\vec{r}') = \frac{q}{4\pi\varepsilon_0 R}\frac{1}{\sqrt{1-\frac{v^2}{c^2}\sin^2\theta}}$