Electricity and Magnetism II [3812] Midterm 1
Wednesday February 20, 2018

Directions:

This exam is closed book. You are allowed a copy of the latest equation sheet posted on the course website. You may annotate your equation sheet.

Read all the questions carefully and answer every part of each question. Show your work on all problems – partial credit may be granted for correct logic or intermediate steps, even if your final answer is incorrect.

Unless otherwise instructed, express your answers in terms of fundamental constants like μ_0 and ε_0, rather than calculating numerical values.

If the question asks for an explanation, write at least one full sentence explaining your reasoning.

Please ask if you have any questions, including clarification about the instructions, during the exam.

<u>This test is designed to be gender and race neutral.</u>

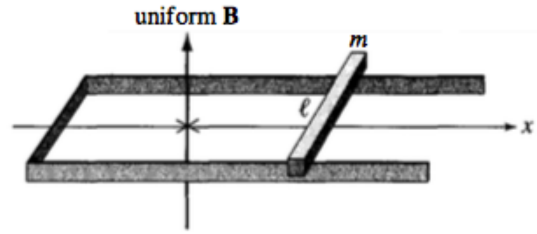
Good luck!

Honor Pledge: I understand that sharing information with anyone during this exam by talking, looking at someone else's test, or any other form of communication, will be interpreted as evidence of cheating. I also understand that if I am caught cheating, the result will be no credit (0 points) for this test, and disciplinary action may result.

Sign Your Name___________________________________

Print Your Name___________________________________

Question 1 (25 points): A bar with resistance R, length l, and mass m is placed on two conducting frictionless rails, which complete the circuit, as shown in the figure. A uniform magnetic field points perpendicular to the plane of the rails, and increases at a constant rate $dB/dt = K$.



1a (15 points): Using $F = ma$, derive (do not solve) the complete differential equation for the motion of the bar, in terms of the coordinate x and its derivatives dx/dt and d^2x/dt^2, the magnetic field B and its derivative $dB/dt = K$, and the properties of the bar.

1b (5 points): Which way does the bar move initially if it starts from rest? Why?

1c (5 points): What force must be exerted on the bar to keep it from moving (assuming it starts from rest at position x_0)?

Question 2 (35 points): A parallel-plate capacitor has two circular plates of radius *R*, separated by a distance *d*, with vacuum between them. The electric field between the plates is approximately uniform and equal to $\vec{E} = \frac{1}{\varepsilon_0}\frac{Q}{\pi R^2}\hat{z}$. Now, imagine that the charge on the plates *(+Q* on the bottom plate, *-Q* on the top plate) is increased at a constant rate $I = \frac{dQ}{dt}$.

2a (10 points): Find the total displacement current I_d between the plates (the displacement current density is $\overrightarrow{J_d} = \frac{\partial \vec{D}}{\partial t}$), and relate this to the conduction current *I*.

2b (10 points): Find the direction and magnitude of the induced magnetic field as a function of position in the volume between the capacitor plates.

2c (15 points): Compute the magnitude and direction of the Poynting vector $\vec{S}$ at the surface of the volume between the plates and from this find the total electromagnetic energy per unit time transported into this volume, as a function of *Q* and *I*.

Question 3 (40 points): A lamp produces an electromagnetic wave with angular frequency ω and amplitude E_0, with polarization in the y direction ($\hat{n} = \hat{y}$), propagating in the +x direction ($\vec{k} = k\hat{x}$).

3a (5 points): Write the time-dependent real vector electric and magnetic fields as a function of position and time.

3b (15 points): Compute the elements of the Maxwell stress tensor in Cartesian coordinates. *Hint: Since E and B are each confined to a single plane, only the diagonal elements T_{xx}, T_{yy}, T_{zz} can possibly have non-zero values.*

3c (20 points): Compute the momentum density $\vec{g}$, and show explicitly that the wave satisfies the momentum continuity equation $\frac{\partial \vec{g}}{\partial t} = \nabla \cdot \overleftrightarrow{T}$.