Electricity and Magnetism II [3812] Midterm 2
Wednesday April 10, 2018

Directions:

This exam is closed book. You are allowed a copy of the latest equation sheet posted on the course website. You may annotate your equation sheet.

Read all the questions carefully and answer every part of each question. Show your work on all problems – partial credit may be granted for correct logic or intermediate steps, even if your final answer is incorrect.

Unless otherwise instructed, express your answers in terms of fundamental constants like μ_0 and ε_0, rather than calculating numerical values.

If the question asks for an explanation, write at least one full sentence explaining your reasoning.

Please ask if you have any questions, including clarification about the instructions, during the exam.

This test is designed to be gender and race neutral.

Good luck!

Honor Pledge: I understand that sharing information with anyone during this exam by talking, looking at someone else's test, or any other form of communication, will be interpreted as evidence of cheating. I also understand that if I am caught cheating, the result will be no credit (0 points) for this test, and disciplinary action may result.

Sign Your Name____________________________________

Print Your Name____________________________________

Question 1 (30 points): Start with the complex forms for the electric and magnetic fields of an electromagnetic plane wave at normal incidence between two linear dielectric materials with permittivities ε_1 and ε_2, and permeabilities μ_1 and μ_2. Assume the incident wave is polarized in the x-direction and traveling in the z-direction, and that the boundary between the two materials is at z=0. Use the boundary conditions on the tangential electric and magnetic fields to find two relationships between the complex incident, reflected, and transmitted electric field amplitudes $\tilde{E}_{0I}$, $\tilde{E}_{0R}$, and $\tilde{E}_{0T}$.

Question 2 (35 points): This problem addresses the following scalar and vector potentials (*k*, *B,* and *C* are constants, *c* is the speed of light):

$$V(\vec{r},t) = kcB\cos\,(kx - kct)$$
$$\vec{A}(\vec{r},t) = kBcos(kx - kct)\hat{x} + C\,\sin(kx - kct)\,\hat{y}$$

2a (10 points). Find the electric and magnetic fields corresponding to these potentials.

2b. (10 points). Are these potentials in the Coulomb or Lorenz gauge, or both, or neither?

2c. (15 points). Find a gauge transformation that makes the scalar potential *V'* equal to zero. What is the gauge function λ, and what is the new vector potential *A'*?

Problem 3 (35 points): Imagine a point charge *q* accelerating uniformly along the z-axis, so that its position $z(t) = \frac{1}{2}a_0 t^2$, its velocity $\vec{v}(t) = a_0 t\,\hat{z}$, and its acceleration is $\vec{a}(t) = a_0\,\hat{z}$.

3a (10 points). Assuming $v << c$, so that $\vec{u} \sim c\widehat{\Delta r}$, find the radiation electric field $\vec{E}_{rad}$ for a retarded separation vector $\overrightarrow{\Delta r} = x\hat{x}$ perpendicular to the motion of the charge.

3b (5 points). Find the corresponding Poynting vector $\vec{S}_{rad}$.

3c (10 points). Where is the actual particle located when the separation vector is equal to the one given above, assuming the instantaneous retarded position is z_r=0?

3d (10 points). Compute the dipole moment of this configuration from $\vec{p} = \sum q_i \vec{r}_i$ and use the multipole approximation to find a first order approximation for the Poynting flux radiated from this configuration, as a function of position and time. Verify that it matches the answer from part **3b**.