Electricity and Magnetism II [3812] Final Exam
Monday May 11, 2020 – 3:00pm

Directions:

The exam will be posted on the course web page by 3:00pm. You must submit your answers to me by e-mail by 6:00pm. The exam is intended to take roughly two hours – the extra hour is grace period to check your work, scan it, and submit it.

This exam is open book and open notes. However, it is not open internet, open solutions manual, or open classmate. Please do not utilize solutions or consult with anyone to solve the problems. I trust you all not to abuse this unique situation.

Read all the questions carefully and answer every part of each question. Show your work on all problems. Partial credit may be granted for correct logic or intermediate steps, even if the final answer is incorrect. Make sure to clearly indicate (e.g. circle) your final answers.

The solution for each of the five problems is ~½ page. If it looks like your solution is going to require substantially more work, you may be doing it the hard way!

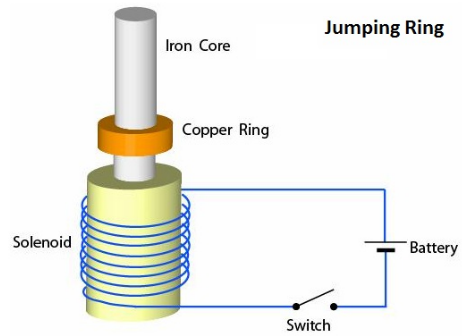
Unless otherwise instructed, express your answers in terms of fundamental constants like μ_0 and ε_0, rather than calculating numerical values.

Please ask if you have any questions, including clarification about the instructions, during the exam. A Zoom meeting (ID 992-770-55648) will be open during the exam.

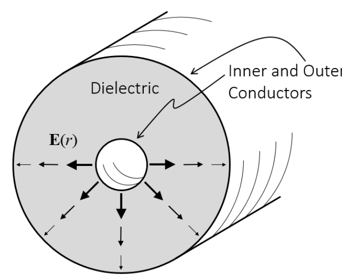
<u>This test is designed to be gender and race neutral.</u>

Good luck!

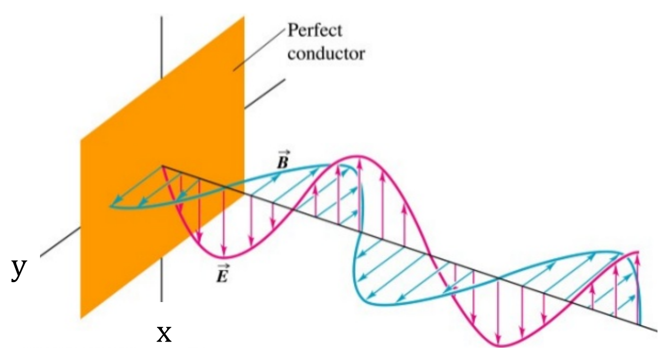
Problem 1 (20 points): A jumping ring demo uses a solenoid to generate an axial magnetic field $B_z = \mu n I_s$ through an iron core. If the solenoid current I_s is suddenly increased from zero to a maximum I_{s_max} over an interval Δt, an induced current I_{ind} is produced in the copper ring (mass *m*, radius *a,* resistance *R*). There is a radial component to the fringing magnetic field around the iron core, which can be approximated as $B_r = K\mu n I_s$ (K is a geometrical constant). Find I_{ind} in the ring, estimate the impulse $\Delta p = F\Delta t$ from the Lorentz force on the ring, and find the height z_{max} to which it jumps in terms of the parameters given. An answer correct to within a factor of two is acceptable.



Problem 2 (20 points): A coaxial cable consists of a center conductor (radius *a*), an outer conductor (radius *b*), and a dielectric material between them (*a* < *s* < *b*). The two conductors are held at a potential difference *ΔV*, and a current *I* flows axially along the inner conductor and back along the outer conductor. Assume the dielectric material is linear, with permittivity *ε* and permeability *μ*. Find the Poynting vector $\vec{S}(\vec{r})$ in the dielectric. From this, find the total energy transported through the coaxial cable per unit time (i.e. the power), expressed in terms of *ΔV* and *I*.



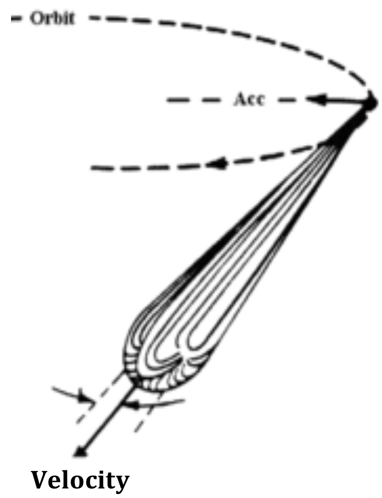
Problem 3 (20 points): To find the radiation pressure exerted on a reflecting surface, consider a plane wave reflecting from a perfect conductor at normal incidence in vacuum. The electric field (real form) for the incident and reflected waves (the total electric field is the sum of these) in the region *z* < *0* is:



$$\vec{E}_I(z,t) = E_0 \cos(kz - \omega t)\,\hat{x} \qquad \vec{E}_R(z,t) = -E_0 \cos(-kz - \omega t)\,\hat{x}$$

Find the corresponding incident, reflected, and total magnetic field in the region *z* < *0*. Then, using the fact that there must be zero field inside the perfect conductor, find the surface current $\vec{K}(t)$ that flows across the surface z = 0. Finally, find the magnetic Lorentz force per unit area exerted on this sheet of current (from $\vec{f} = \vec{K}\times\vec{B}_{ave}$), and show that its time-averaged value gives the correct value for the radiation pressure.

Problem 4 (20 points): The radiated power per solid angle for an accelerated point charge (Griffiths 11.72) is:



$$\frac{dP}{d\Omega} = \frac{q^2}{16\pi^2\varepsilon_0}\frac{\left|\widehat{\Delta \mathrm{r}}\times(\vec{u}\times\vec{a})\right|^2}{\left(\widehat{\Delta r}\cdot\vec{u}\right)^5}$$

For the case with perpendicular velocity and acceleration (*i.e. synchrotron radiation*), find the angular distribution of radiated power. Define your coordinates such that the velocity points along the z-axis ($\vec{v} = v\hat{z}$), the acceleration points along the x-axis ($\vec{a} = a\hat{x}$), and the separation unit vector $\widehat{\Delta r} = sin\theta\ cos\phi\ \hat{x} + sin\theta\ sin\phi\ \hat{y} + cos\theta\ \hat{z}$. Show that the angular distribution of synchrotron radiation is:

$$\frac{dP}{d\Omega} = \frac{\mu_0 q^2 a^2}{16\pi^2 c}\frac{[(1-\beta\ cos\theta)^2 - (1-\beta^2)\ sin^2\theta\ cos^2\phi]}{(1-\beta\ cos\theta)^5}$$

Problem 5 (20 points): The scalar and vector potentials for a charged particle in motion with a constant velocity can be derived by the use of the Lorentz transformation. Starting with the potentials for a charged particle at rest in frame *S*, apply the Lorentz transformation to the 4-vector $A^\mu = (\frac{V}{c}, A_x, A_y, A_z)$ to transform to a frame *S'* where the particle moves with velocity *v*. Express the resulting scalar and vector potentials in terms of the coordinates *x', y', z', t'* of the particle in *S'*.