

$$\begin{aligned}
 1. \quad \vec{E}_I &= \vec{E}_{0I} e^{i(\kappa_1 z - \omega t)} \hat{x} \\
 \vec{B}_I &= \vec{E}_{0I} \cdot \frac{1}{v_1} \cdot e^{i(\kappa_1 z - \omega t)} \hat{y} \\
 \vec{E}_R &= \vec{E}_{0R} e^{i(-\kappa_2 z - \omega t)} \hat{x} \\
 \vec{B}_R &= \vec{E}_{0R} \cdot \frac{1}{v_1} \cdot e^{i(-\kappa_2 z - \omega t)} \cdot (-\hat{y}) \\
 \vec{E}_T &= \vec{E}_{0T} e^{i(\kappa_2 z - \omega t)} \hat{x} \\
 \vec{B}_T &= \vec{E}_{0T} \cdot \frac{1}{v_2} e^{i(\kappa_2 z - \omega t)} \hat{y}
 \end{aligned}$$

$$\begin{aligned}
 \Delta \vec{E}_{||} &= 0 \quad @ \quad z=0 \\
 &\Rightarrow \vec{E}_{0I} + \vec{E}_{0R} = \vec{E}_{0T}
 \end{aligned}$$

$$\Delta \vec{H}_{||} = \Delta(\vec{B}_{||}/\mu) = 0 \quad @ \quad z=0$$

$$\Rightarrow \frac{\vec{E}_{0I}}{\mu_1 v_1} - \frac{\vec{E}_{0R}}{\mu_1 v_1} = \frac{\vec{E}_{0T}}{\mu_2 v_2}$$

$$\text{or } \vec{E}_{0I} - \vec{E}_{0R} = \frac{\mu_1 v_1}{\mu_2 v_2} \vec{E}_{0T} = \beta \vec{E}_{0T}$$

$$\begin{aligned}
 2.a. \quad \vec{E} &= -\nabla V - \frac{\partial \vec{A}}{\partial t} \\
 &= \kappa^2 c \beta \sin(\kappa x - \kappa c t) \hat{x} - \kappa^2 c \beta \sin(\kappa x - \kappa c t) \hat{x} \\
 &\quad + \kappa c \cos(\kappa x - \kappa c t) \hat{y} \\
 &= \kappa c \cos(\kappa x - \kappa c t) \hat{y}
 \end{aligned}$$

$$\begin{aligned}
 \vec{B} &= \nabla \times \vec{A} = \frac{\partial A_y}{\partial x} \hat{z} \\
 &= \kappa c \cos(\kappa x - \kappa c t) \hat{z}
 \end{aligned}$$

$$2.b. \quad \nabla \cdot \vec{A} = -\kappa^2 \beta \sin(\kappa x - \kappa c t) \neq 0$$

$$\text{p.e. } \frac{\partial V}{\partial t} = \frac{1}{c^2} \cdot \kappa^2 c^2 \beta \sin(\kappa x - \kappa c t)$$

$$\Rightarrow \text{Lorenz Gauge}$$

$$2.c. \quad V' = V - \kappa c \beta \cos(\kappa x - \kappa c t) \\ = V - \frac{\lambda}{g t} = 0$$

$$\text{if } \lambda = -\beta \sin(\kappa x - \kappa c t)$$

$$\vec{A}' = \vec{A} + \nabla \lambda \\ = \vec{A} - \beta \kappa \cos(\kappa x - \kappa c t) \hat{x} \\ = \left(\sin(\kappa x - \kappa c t) \hat{y} \right)$$

$$3.a. \quad \vec{E}_{\text{rad}} = \frac{q}{4\pi\epsilon_0} \frac{\Delta r}{(\Delta r - \vec{a})^3} [\vec{\Delta r} \times (\vec{a} \times \vec{a})] \\ = \frac{q}{4\pi\epsilon_0} \cdot \frac{x}{(c x)^3} [x \hat{x} \times (c \hat{x} \times a_0 \hat{z})] \\ = \frac{q}{4\pi\epsilon_0} \frac{a_0}{c^2 x} [\hat{x} \times (\hat{x} \times \hat{z})] \\ = \frac{-q a_0 \hat{z}}{4\pi\epsilon_0 c^2 x} = \frac{-\frac{q \mu_0 a_0}{4\pi x} \hat{z}}$$

$$3.b. \quad \vec{S}_{\text{rad}} = \frac{1}{\mu_0 c} E_{\text{rad}}^2 \hat{x} \\ = \frac{\mu_0 q^2 a_0^2}{16\pi^2 c x^2} \hat{x}$$

$$3.c. \quad t_r = t - \Delta r/c = t - \frac{x}{c} \\ \Rightarrow t = t_r + \frac{x}{c} = \frac{x}{c} \quad \text{since } t_r = 0 \text{ when } z_r = 0 \\ \Rightarrow z = \frac{1}{2} a_0 \left(\frac{x}{c}\right)^2$$

$$3.d. \quad \vec{p} = q z = \frac{1}{2} q a_0 t^2$$

$$\dot{\vec{p}} = q a_0 \\ \vec{S} \approx \frac{\mu_0 \dot{\vec{p}}^2}{16\pi^2 c} \frac{\sin^2 \theta}{r^2} \hat{r}$$

$$= \frac{\mu_0 q^2 a_0^2}{16\pi^2 c x^2} \hat{x} \quad \text{for } \vec{r} = x \hat{x} \\ (\theta = \pi/2)$$