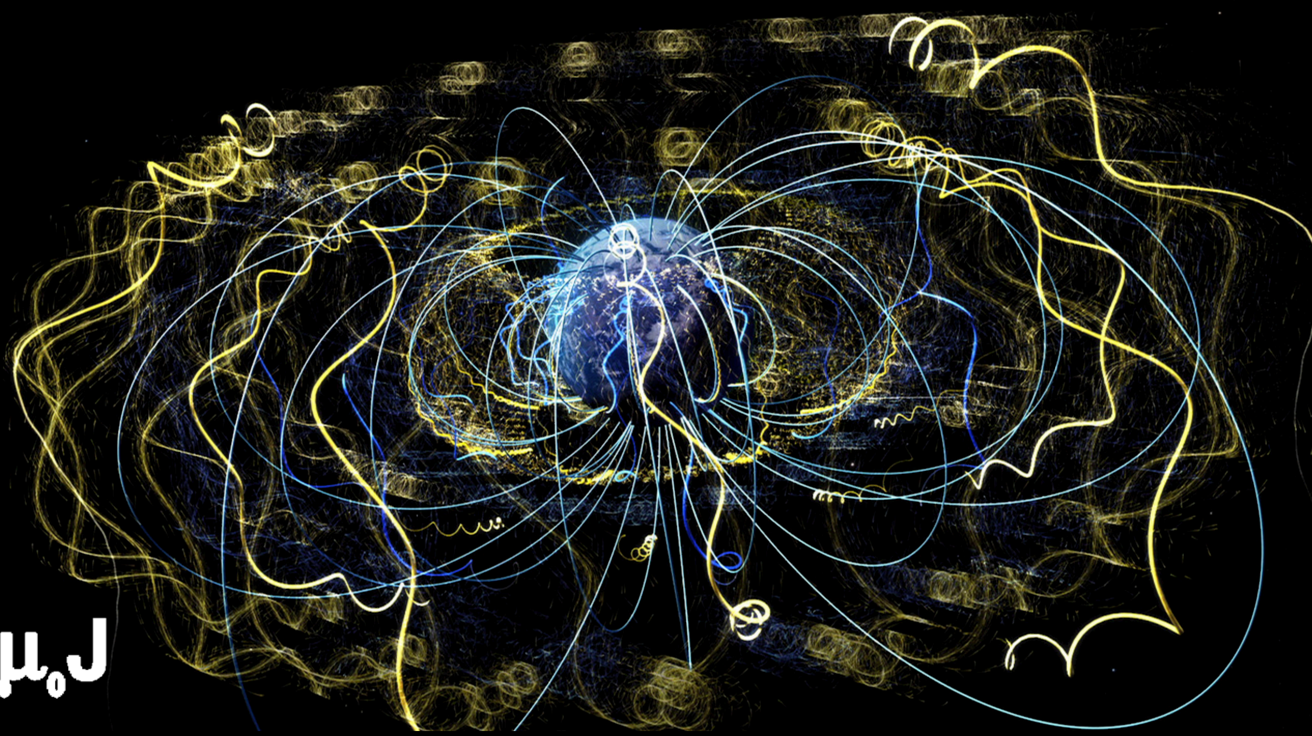
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism II: 3812

Professor Jasper Halekas
Van Allen 70
MWF 9:30-10:20 Lecture

Announcements I

- Midterm #1 this Wednesday Feb. 26
  - In this room, during normal class time
  - Covers all sections of Ch. 7-9.2 except:
    - No questions on 8.2.4 (angular momentum) and 8.3 (magnetic forces do no work)
- Equation Sheet posted
  - Review, print out, annotate as desired, and bring to exam
- Practice midterms and solutions posted

Maxwell's Equations

Maxwell's Equations:

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \oint\vec{E}\cdot\overrightarrow{dl} = -\frac{d}{dt}\int\vec{B}\cdot\overrightarrow{da}$$

$$\nabla\cdot\vec{E} = \rho/\varepsilon_0 \qquad \oint\vec{E}\cdot\overrightarrow{da} = Q_{enc}/\varepsilon_0$$

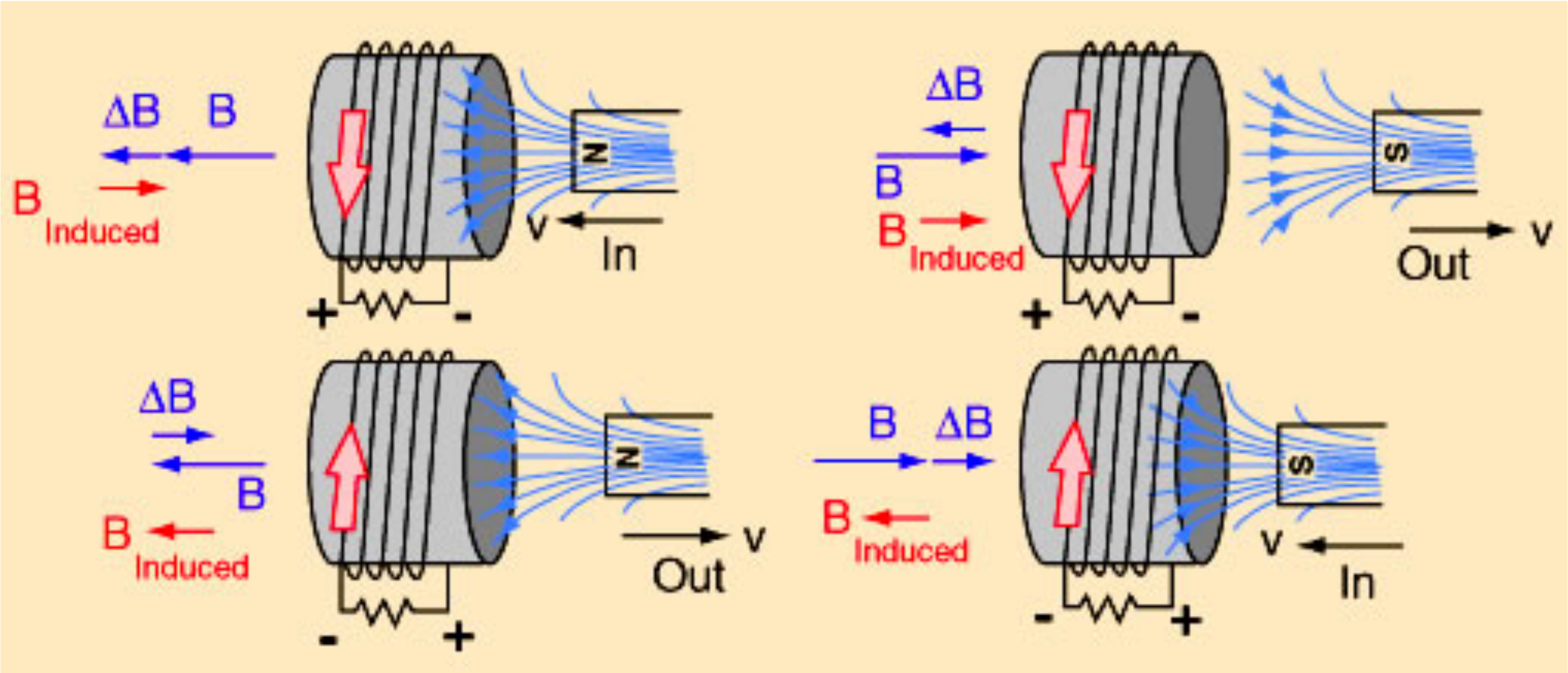
$$\nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} \qquad \oint\vec{B}\cdot\overrightarrow{dl} = \mu_0 I_{enc} + \mu_0\varepsilon_0\frac{d}{dt}\int\vec{E}\cdot\overrightarrow{da}$$

$$\nabla\cdot\vec{B} = 0 \qquad \oint\vec{B}\cdot\overrightarrow{da} = 0$$

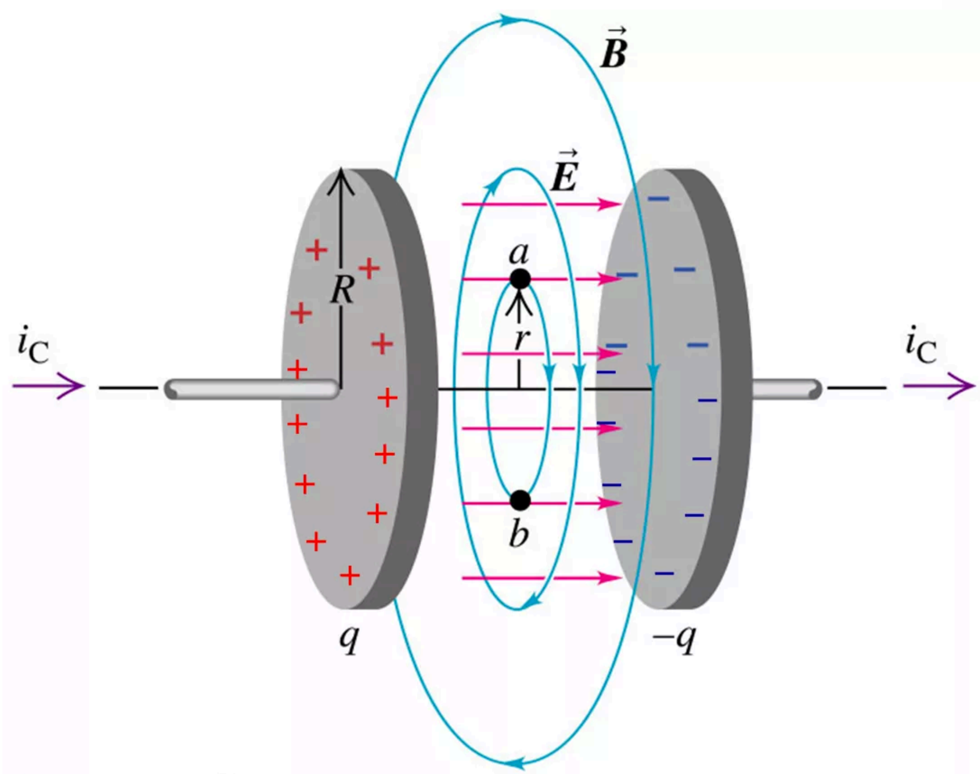
$$\nabla\cdot\vec{D} = \rho_f \qquad \oint\vec{D}\cdot\overrightarrow{da} = Q_{f\_enc}$$

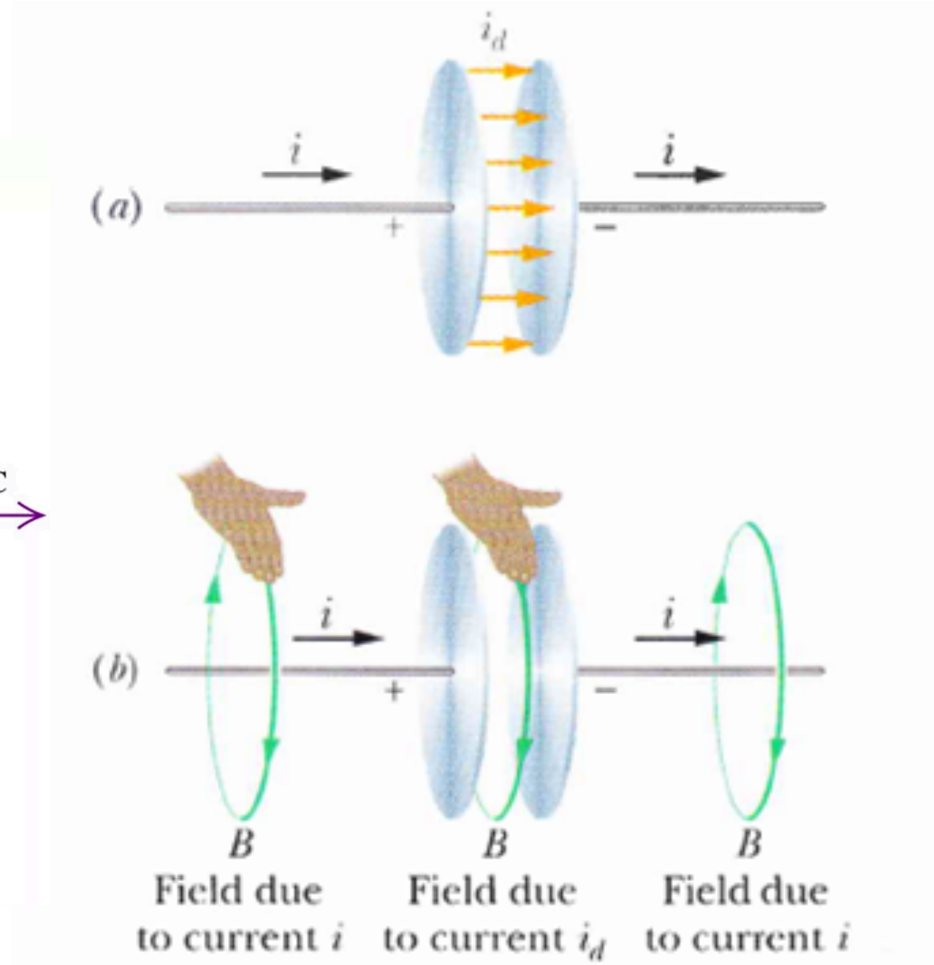
$$\nabla\times\vec{H} = \overrightarrow{J_f} + \frac{\partial\vec{D}}{\partial t} \qquad \oint\vec{H}\cdot\overrightarrow{dl} = I_{f_{enc}} + \frac{d}{dt}\int\vec{D}\cdot\overrightarrow{da}$$

Faraday's Law, Lenz's Law



Ampere's Law, Displacement Current



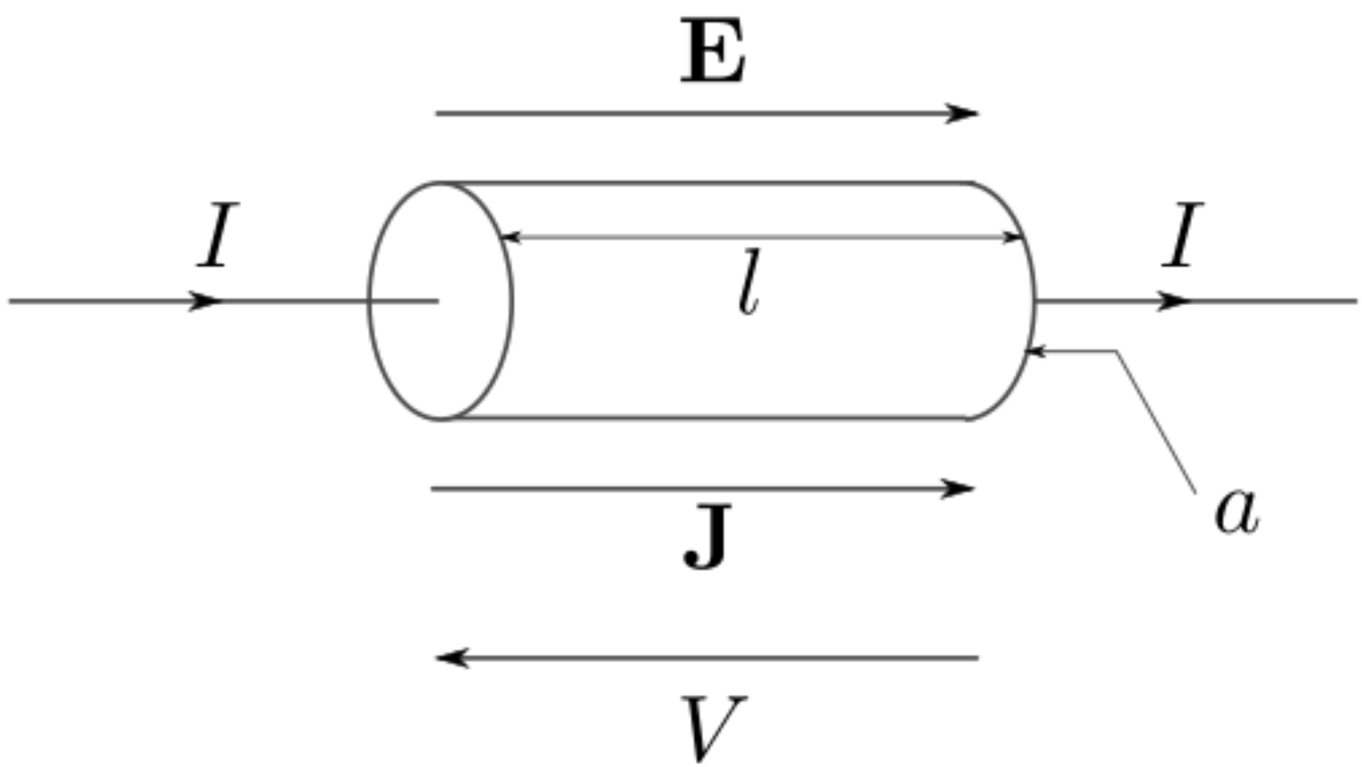


Ohm's Law, EMF, Inductance

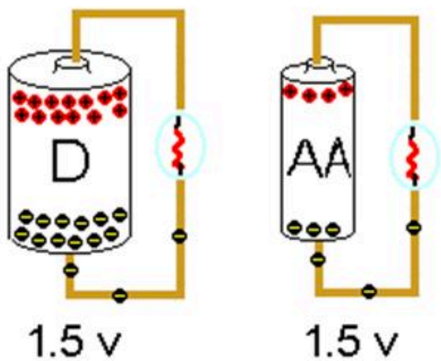
Ohm's Law and EMF: $\vec{J} = \sigma\vec{E}$ $\mathcal{E} = \oint(\vec{F}/q)\cdot\overrightarrow{dl}$ $\mathcal{E}_{motional} = -\frac{d\Phi_B}{dt}$

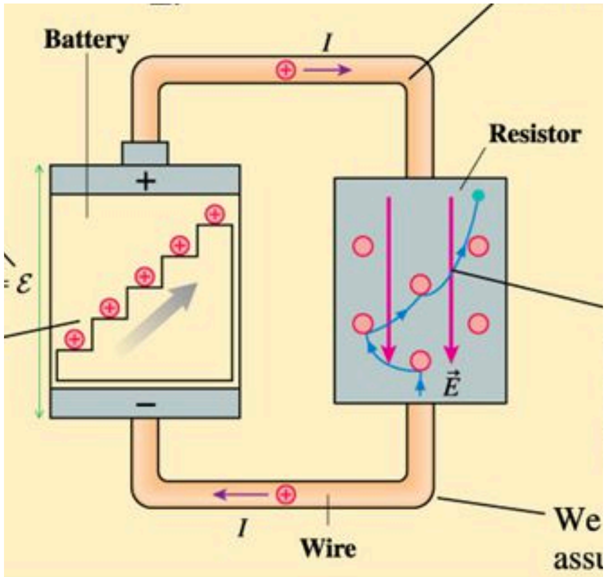
Inductance: $M = \frac{\Phi_{B2}}{I_1} = \frac{\Phi_{B1}}{I_2}$ $L = \frac{\Phi_{B1}}{I_1}$ $\mathcal{E}_{induced} = -L\frac{dI}{dt}$ $\frac{dW}{dt} = \mathcal{E}I$

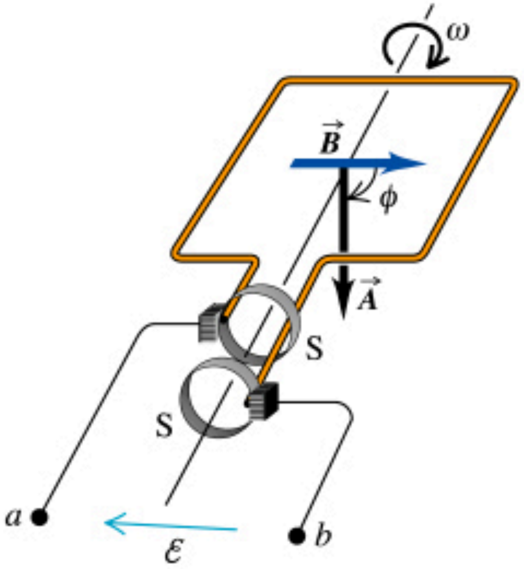
Ohm's Law

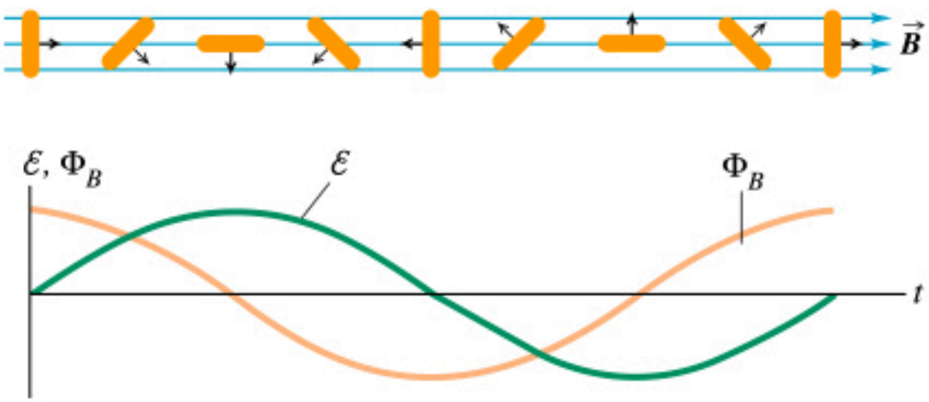


EMF

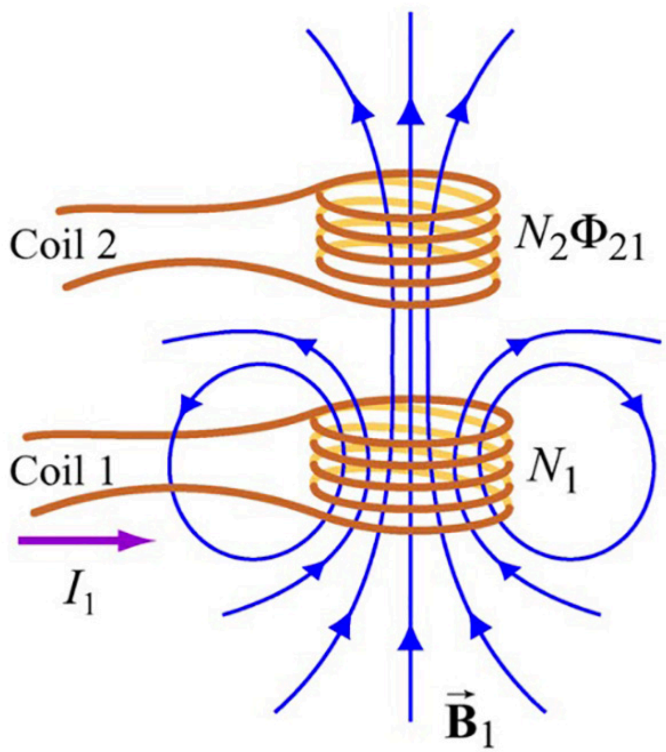


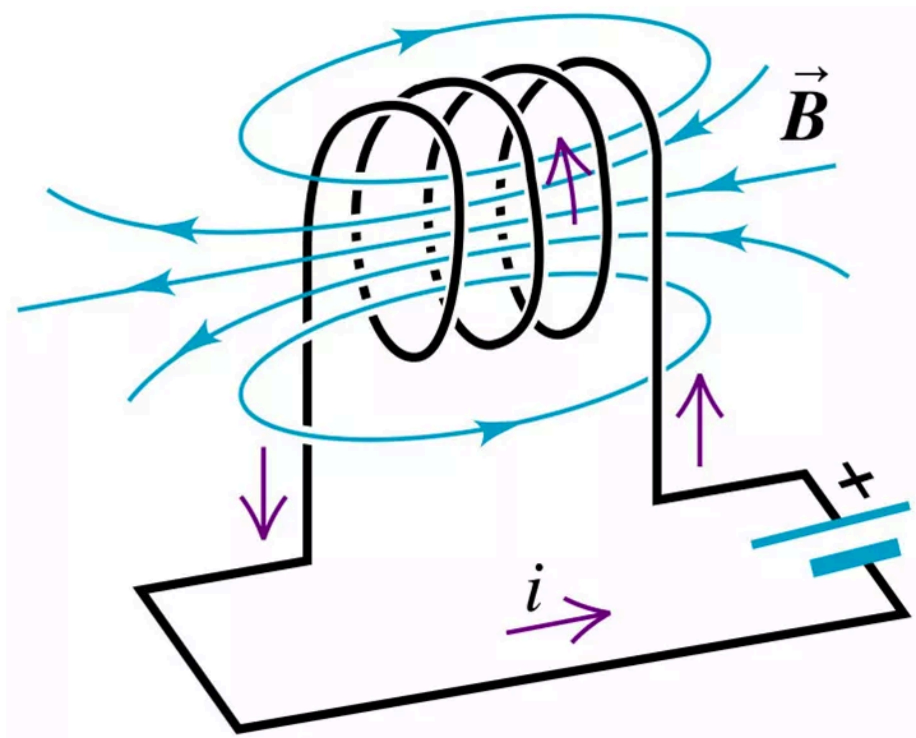






Inductance





Fields in Matter, Boundary Conditions

Fields in Matter: $\vec{P} = \vec{p}/volume$ $\quad \vec{D} = \varepsilon_0\vec{E} + \vec{P}$ $\quad \sigma_b = \vec{P}\cdot\hat{n}$ $\quad \rho_b = -\nabla\cdot\vec{P}$ $\quad J_p = \frac{\partial\vec{P}}{\partial t}$

$\vec{M} = \vec{m}/volume$ $\quad \vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$ $\quad \vec{K}_b = \vec{M}\times\hat{n}$ $\quad \vec{J}_b = \nabla\times\vec{M}$

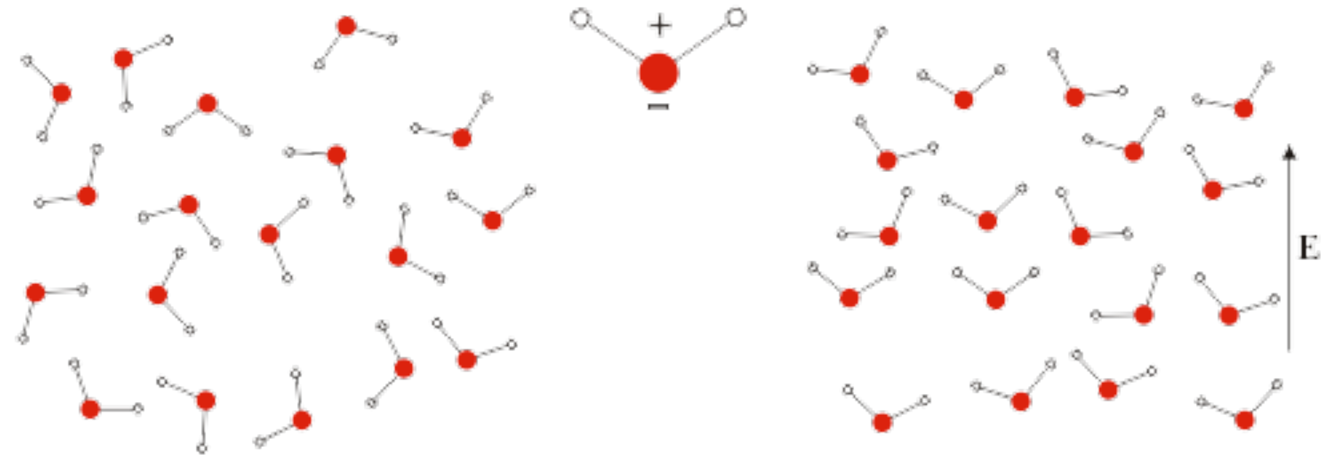
Linear Materials: $\vec{P} = \varepsilon_0\chi_e\vec{E}$ $\quad \vec{D} = \varepsilon\vec{E} = (1+\chi_e)\varepsilon_0\vec{E} = \varepsilon_r\varepsilon_0\vec{E}$

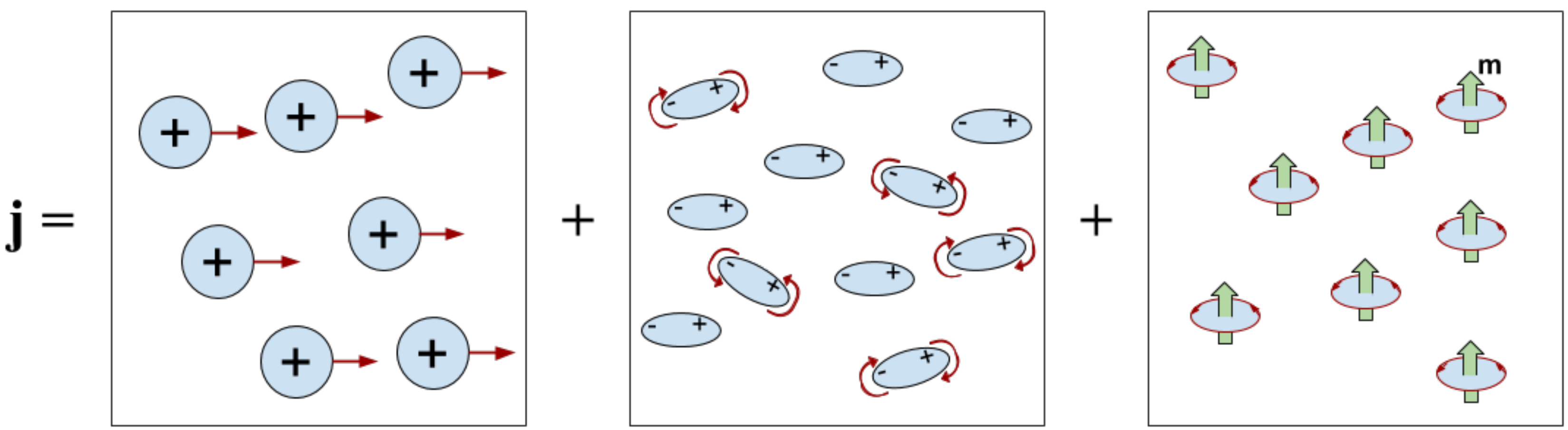
$\vec{M} = \chi_m\vec{H}$ $\quad \vec{B} = \mu\vec{H} = (1+\chi_m)\mu_0\vec{H}$

Boundary Conditions: $\Delta D_\perp = \sigma_f$ $\quad \Delta\vec{E}_{||} = 0$ $\quad \Delta\vec{D}_{||} = \Delta\vec{P}_{||}$

$\Delta\vec{H}_{||} = \vec{K}_f\times\hat{n}$ $\quad \Delta B_\perp = 0$ $\quad \Delta H_\perp = -\Delta M_\perp$

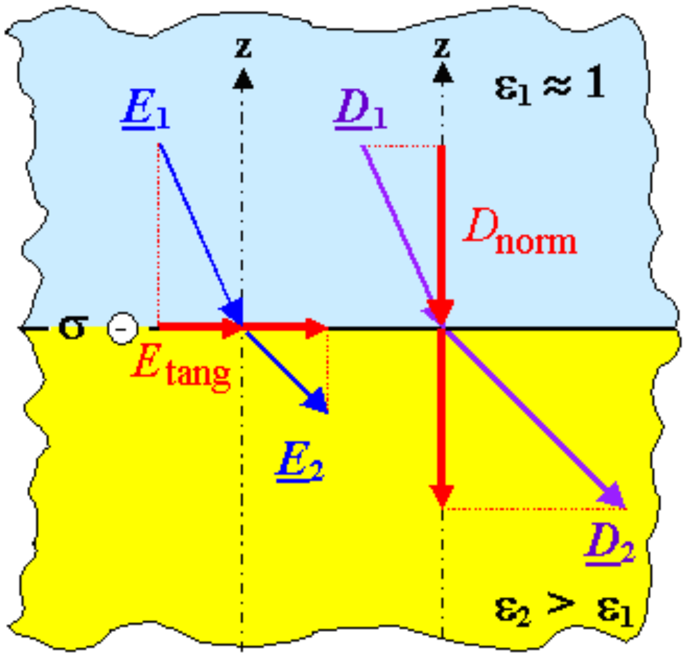
Fields in Matter

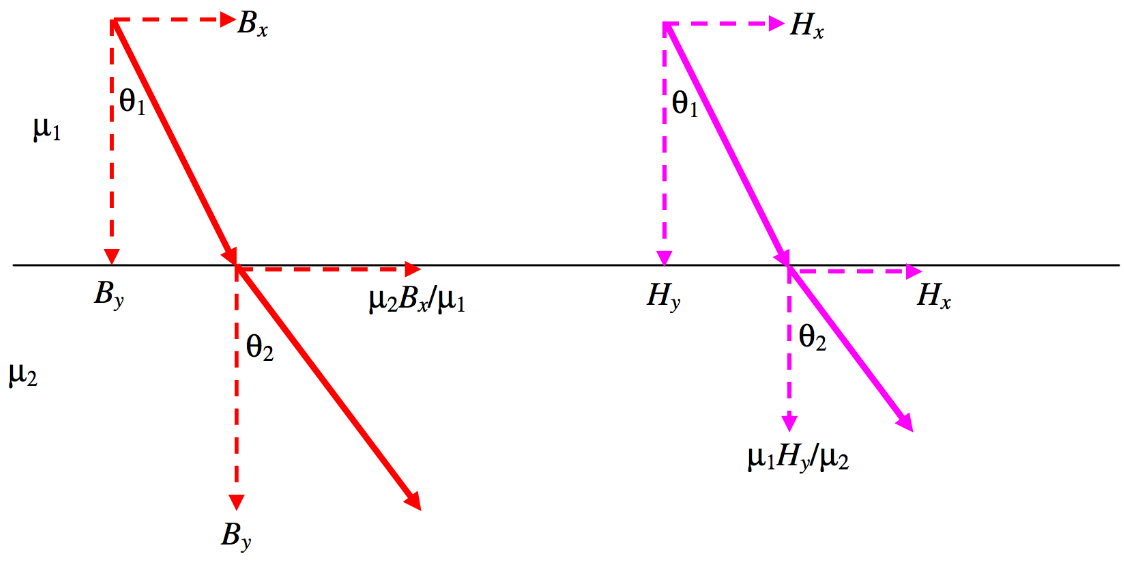




j_f: free current j_p: bound current j_m: magnetization current

Boundary Conditions





Images show boundary conditions for case w/ no free charge or current at the boundary

Continuity, Energy, Momentum

Continuity of Charge/Current: $\frac{\partial \rho}{\partial t} = -\nabla \cdot \vec{J}$ $\quad$ $\frac{\partial \rho_b}{\partial t} = -\nabla \cdot \overrightarrow{J_p}$

Energy & Momentum: $\vec{S} = \frac{1}{\mu_0}\left(\vec{E}\times\vec{B}\right)$ $\quad$ $u_{EM} = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right)$ $\quad$ $U_{EM} = \int u_{EM}\, d\tau$

$T_{ij} = \varepsilon_0\left(E_i E_j - \frac{1}{2}\delta_{ij}E^2\right) + \frac{1}{\mu_0}\left(B_i B_j - \frac{1}{2}\delta_{ij}B^2\right)$ $\quad$ $\vec{g} = \varepsilon_0\left(\vec{E}\times\vec{B}\right) = \mu_0\varepsilon_0\vec{S}$ $\quad$ $\vec{p}_{EM} = \int \vec{g}\, d\tau$

$\frac{dW}{dt} = -\oint \vec{S}\cdot\overrightarrow{da} - \frac{dU_{EM}}{dt}$ $\quad$ $\frac{\partial u_{EM}}{\partial t} = -\nabla\cdot\vec{S} \;\; if \;\frac{dW}{dt} = 0$

$\vec{F} = \frac{d\vec{p}_{mech}}{dt} = \oint \overleftrightarrow{T}\cdot\overrightarrow{da} - \frac{d\vec{p}_{EM}}{dt}$ $\quad$ $\vec{f} = \nabla\cdot\overleftrightarrow{T} - \frac{\partial\vec{g}}{\partial t}$ $\quad$ $\frac{\partial\vec{g}}{\partial t} = \nabla\cdot\overleftrightarrow{T} \;\; if\; \vec{f} = 0$

Charge Continuity Equation

Current I out of a volume is equal to rate of decrease of charge Q contained in that volume:

$$I = -\frac{dQ}{dt} = -\frac{d}{dt}\int_{\mathcal{V}} \rho_v \, d\mathcal{V}$$

$$\oint_S \mathbf{J}\cdot d\mathbf{s} = -\frac{d}{dt}\int_{\mathcal{V}} \rho_v \, d\mathcal{V}$$

$$\oint_S \mathbf{J}\cdot d\mathbf{s} = \int_{\mathcal{V}} \nabla\cdot\mathbf{J}\, d\mathcal{V} = -\frac{d}{dt}\int_{\mathcal{V}} \rho_v \, d\mathcal{V}$$

Used Divergence Theorem

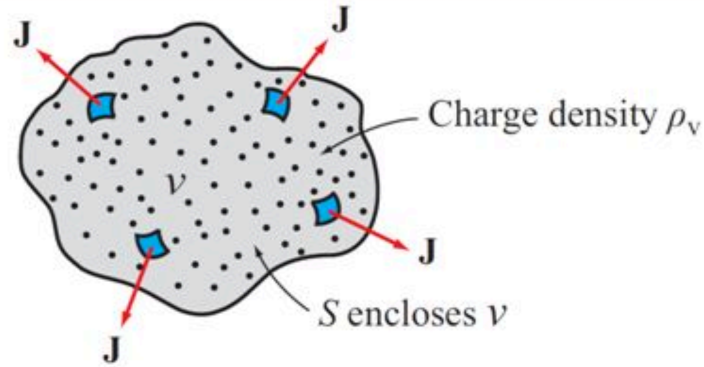
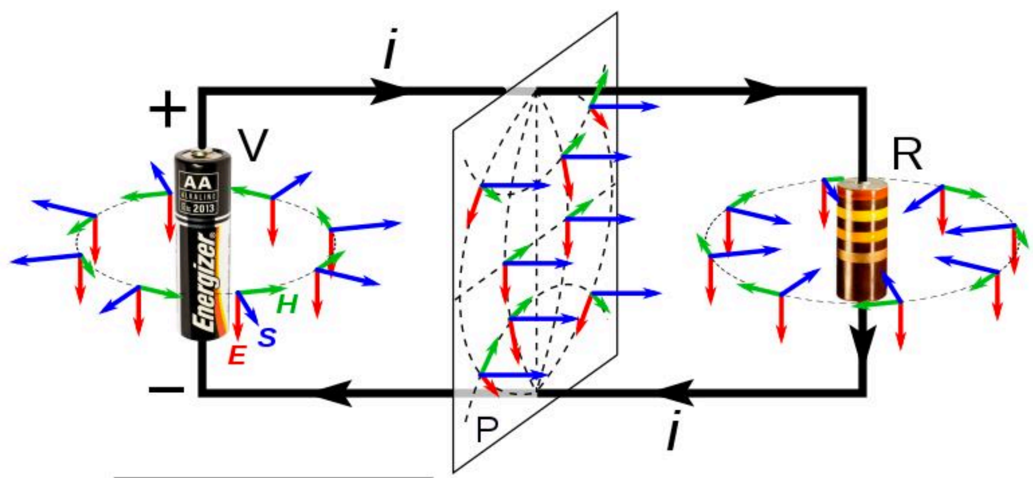


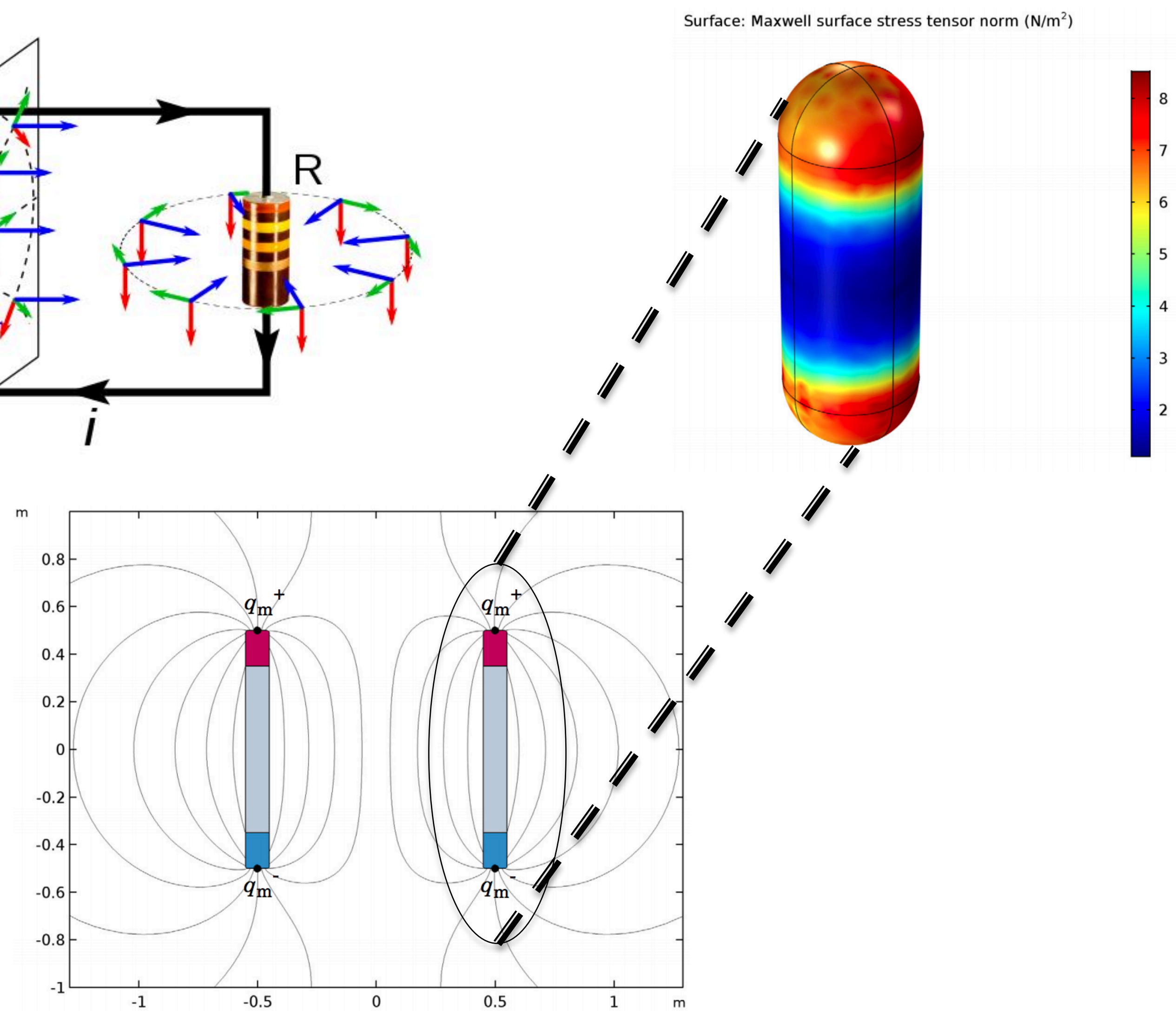
Figure 6-14: The total current flowing out of a volume $\mathcal{V}$ is equal to the flux of the current density **J** through the surface S, which in turn is equal to the rate of decrease of the charge enclosed in $\mathcal{V}$.

$$\nabla\cdot\mathbf{J} = -\frac{\partial\rho_v}{\partial t}, \qquad (6.54)$$

which is known as the ***charge-current continuity relation***, or simply the ***charge continuity equation***.

EM Energy and Momentum





EM Energy and Momentum Summary

$$\frac{dW}{dt} = -\frac{d}{dt}\int_{\mathcal{V}} u_{\text{em}}\, d\tau - \oint_{\mathcal{S}} \mathbf{S}\cdot d\mathbf{a} \quad \longleftrightarrow \quad \frac{d\mathbf{p}_{\text{mech}}}{dt} = -\epsilon_0\mu_0 \frac{d}{dt}\int_{\mathcal{V}} \mathbf{S}\, d\tau + \oint_{\mathcal{S}} \overleftrightarrow{\mathbf{T}}\cdot d\mathbf{a}$$

$$\frac{\partial}{\partial t}\left(u_{\text{mech}} + u_{\text{em}}\right) = -\nabla\cdot\mathbf{S} \quad \longleftrightarrow \quad \frac{\partial}{\partial t}\left(\mathbf{P}_{\text{mech}} + \mathbf{P}_{\text{em}}\right) = -\nabla\cdot\left(-\overleftrightarrow{\mathbf{T}}\right)$$

$$\mathbf{P}_{\text{em}} = \int_{\mathcal{V}} \left(\varepsilon_0\mu_0\mathbf{S}\right) d\tau = \int_{\mathcal{V}} \varepsilon_0\left(\mathbf{E}\times\mathbf{B}\right) d\tau$$

$$u_{\text{em}} = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) \quad \longleftrightarrow \quad \mathbf{g} = \varepsilon_0\mu_0\mathbf{S} = \varepsilon_0\left(\mathbf{E}\times\mathbf{B}\right)$$

Poynting Vector S

S : *Energy per unit area (Energy flux density), per unit time transport by EM fields*

$\varepsilon_0\mu_0 S$: *Momentum per unit volume (Momentum density) stored in EM fields*

Stress Tensor $\overleftrightarrow{\mathbf{T}}$

$\overleftrightarrow{\mathbf{T}}$: *EM field stress (Force per unit area) acting on a surface*

$-\overleftrightarrow{\mathbf{T}}$: *Flow of momentum (momentum per unit area, unit time) carried by EM fields*

Continuity Equations of EM fields in empty space

$$\frac{\partial\rho}{\partial t} = -\left(\nabla\cdot\mathbf{J}\right)$$

$$\frac{\partial u_{em}}{\partial t} = -\left(\nabla\cdot\mathbf{S}\right) \qquad (\mathbf{S}) \text{ playing the part of } \mathbf{J} \rightarrow \textbf{Local conservation of field energy}$$

$$\frac{\partial\mathbf{g}}{\partial t} = -\nabla\cdot\left(-\overleftrightarrow{\mathbf{T}}\right) \qquad \left(-\overleftrightarrow{\mathbf{T}}\right) \text{ playing the part of } \mathbf{J} \rightarrow \textbf{Local conservation of field momentum}$$

EM Waves

EM Waves:

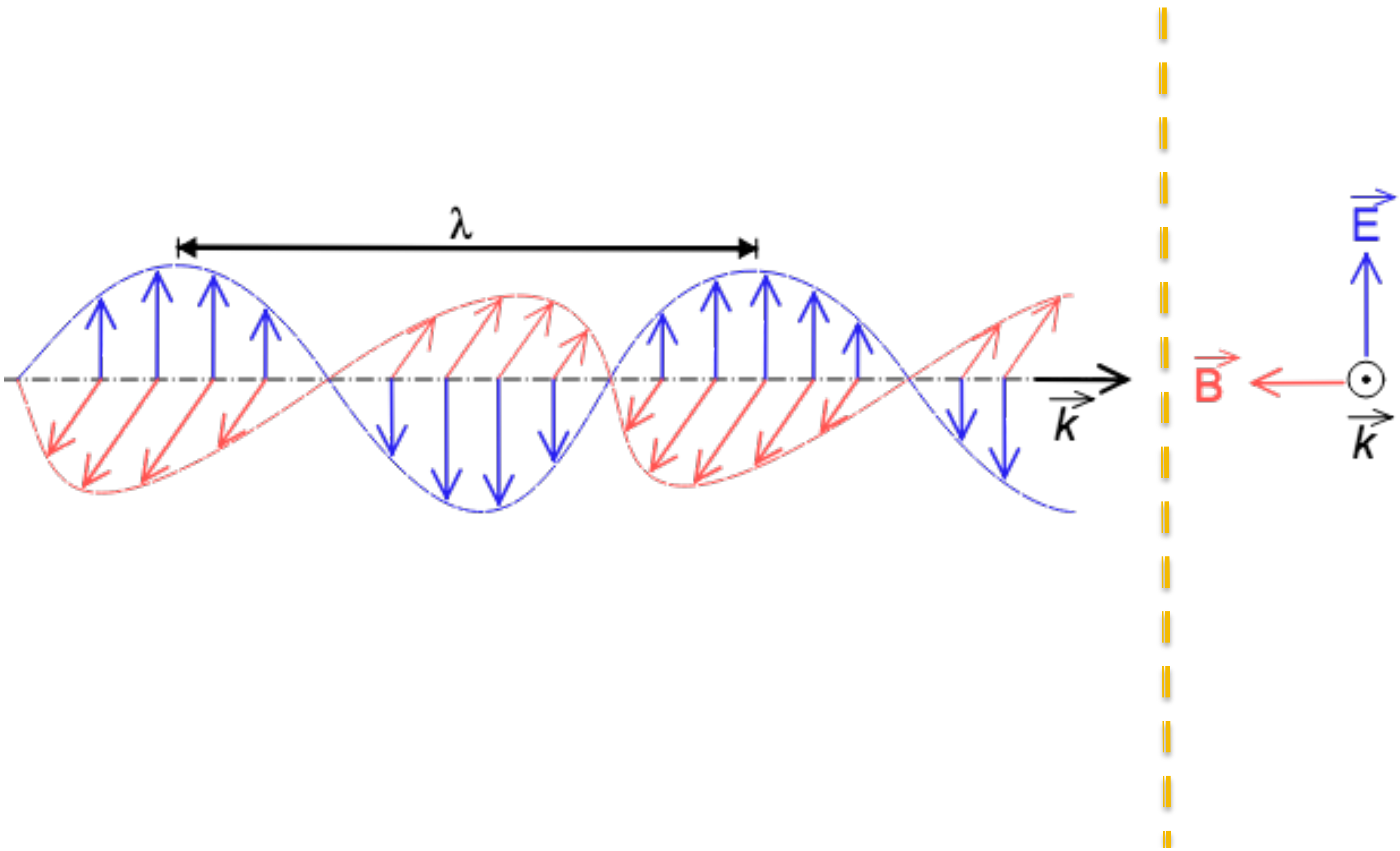
Complex: $\vec{E}(\vec{r},t) = \widetilde{E_0}\exp\left(i(\vec{k}\cdot\vec{r}-\omega t)\right)\hat{n}$ $\quad$ $\vec{B}(\vec{r},t) = \frac{k}{\omega}\widetilde{E_0}\exp\left(i(\vec{k}\cdot\vec{r}-\omega t)\right)\left(\hat{k}\times\hat{n}\right)$

Real: $\vec{E}(\vec{r},t) = E_0\cos\left(\vec{k}\cdot\vec{r}-\omega t+\delta\right)\hat{n}$ $\quad$ $\vec{B}(\vec{r},t) = \frac{k}{\omega}E_0\cos\left(\vec{k}\cdot\vec{r}-\omega t+\delta\right)\left(\hat{k}\times\hat{n}\right)$

$\frac{\omega}{k} = c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ $\quad$ $\langle u\rangle = \frac{1}{2}\varepsilon_0 {E_0}^2$ $\quad$ $\langle\vec{S}\rangle = c\langle u\rangle\hat{k} = I\hat{k} = \frac{1}{2}c\varepsilon_0{E_0}^2\hat{k}$ $\quad$ $\langle\vec{g}\rangle = \frac{\langle\vec{S}\rangle}{c^2} = \frac{1}{2c}\varepsilon_0{E_0}^2\hat{k}$

$P = \frac{I}{c} = \frac{\langle S\rangle}{c}$

EM Waves



Wave Energy, Momentum, Etc.

EM waves transport energy: $$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$$

They also transport momentum: $$p = U / c$$

They exert a pressure: $$P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt} = \frac{1}{cA}\frac{dU}{dt} = \frac{S}{c}$$

(Double this for a reflecting surface)