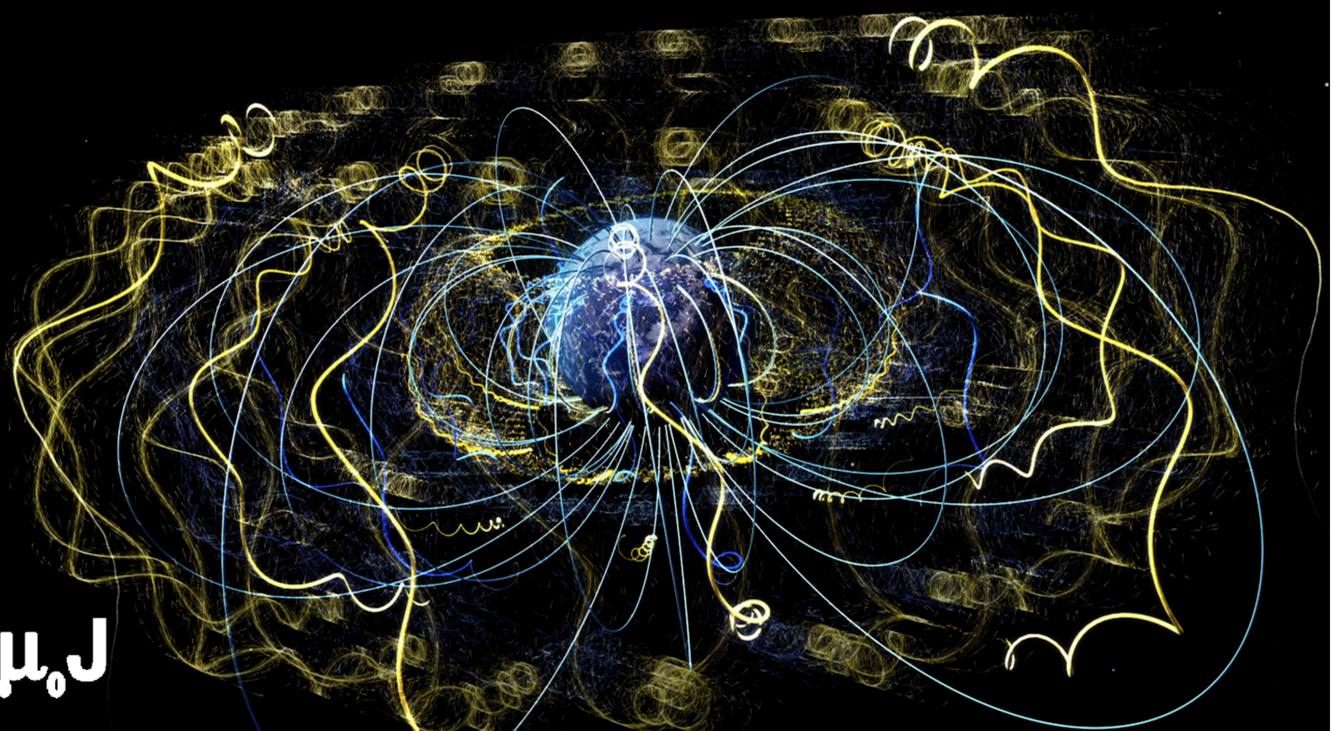
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

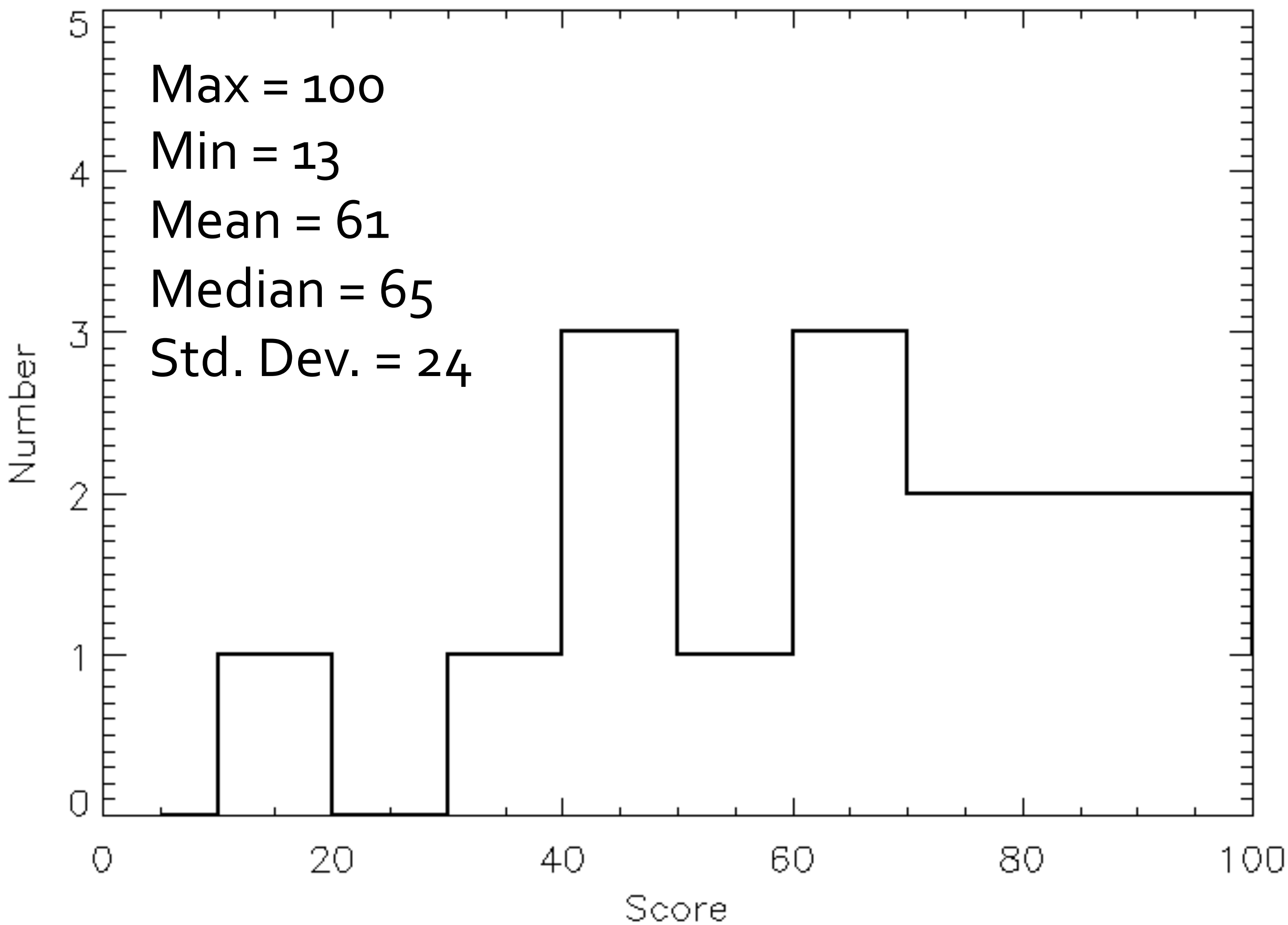
$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism II: 3812

Professor Jasper Halekas
Van Allen 70
MWF 9:30-10:20 Lecture

Midterm 1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Min = 13
Mean = 61
Median = 65
Std. Dev. = 24
Number
Score
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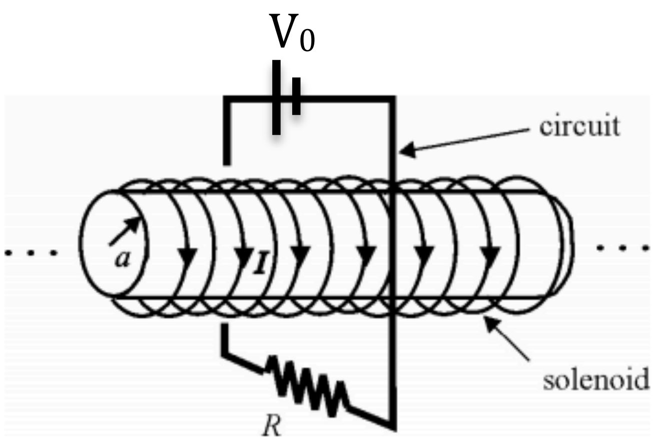
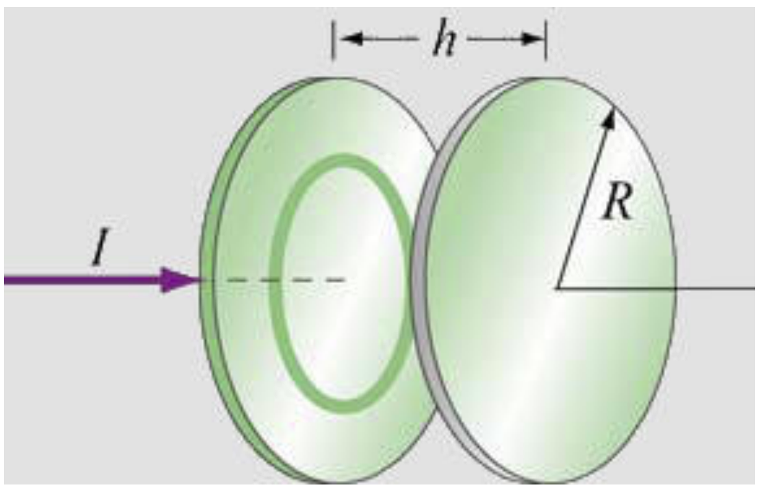
1
2
3
4
5
0
20
40
60
80
100

Question 1



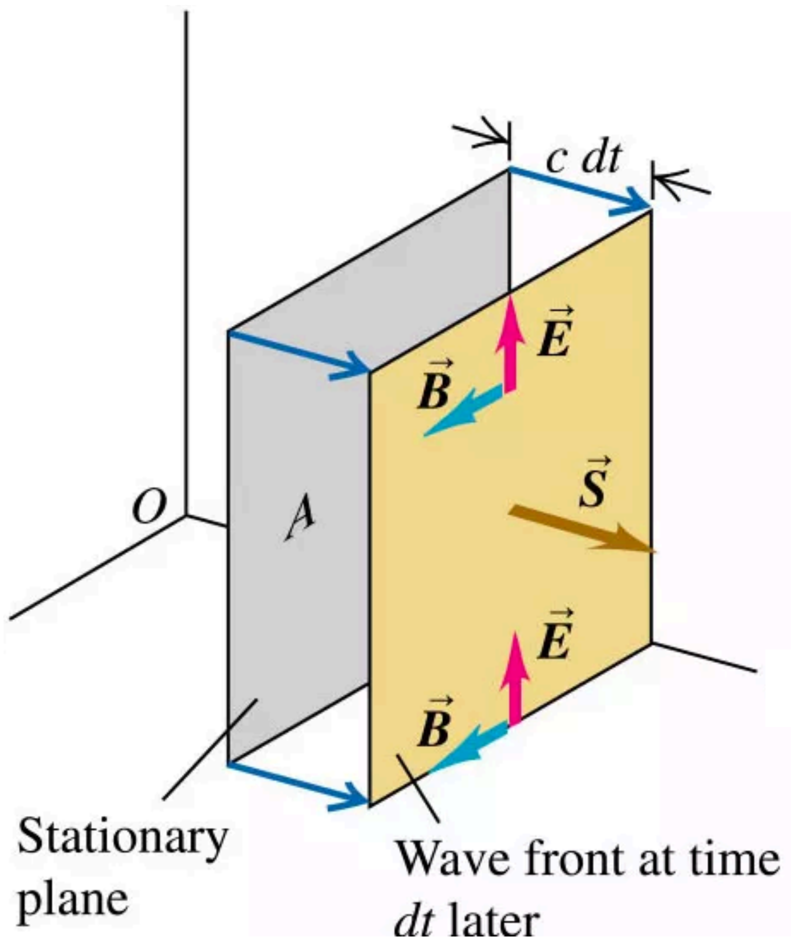
- Changing B produces E
- The resulting EMF opposes the change in B
- Total EMF is sum of all individual EMFs, including induced EMF
- I can’t draw electric circuits right

Question 2



- Changing E produces B
- Increasing electromagnetic energy in a volume filled w/ vacuum requires Poynting flux into volume

Question 3



- E, B, and k are mutually perpendicular for EM waves
- EM wave carries both energy and momentum, in propagation direction
- EM wave absorbed by surface exerts pressure on surface

9.3 EM Waves in Matter

No sources: $\rho_f , \vec{J}_f = 0$

Assume linear & homogeneous
so $\vec{D} = \varepsilon \vec{E}$, $\vec{B} = \mu \vec{H}$

$$\nabla \cdot \vec{D} = 0 \Rightarrow \nabla \cdot \vec{E} = 0$$
$$\nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{E} = -\partial \vec{B}/\partial t$$
$$\nabla \times \vec{H} = \partial \vec{D}/\partial t \Rightarrow \nabla \times \vec{B} = \mu \varepsilon \, \partial \vec{E}/\partial t$$

Same as vacuum case,
but $\mu_0 \to \mu$, $\varepsilon_0 \to \varepsilon$

Solution: EM waves w/
$v = 1/\sqrt{\mu\varepsilon}$

Index of Refraction

$$n = c/v = \sqrt{\mu\varepsilon} / \sqrt{\mu_0 \varepsilon_0}$$

$n \geq 1$ always

Many materials $\mu \sim \mu_0$
$\Rightarrow n \sim \sqrt{\varepsilon/\varepsilon_0} = \sqrt{1 + \chi_e} = \sqrt{\varepsilon_r}$

ε_r (sometimes κ) is "dielectric constant"

Energy & Momentum

You showed:

$$\vec{S} = \vec{E} \times \vec{H}$$

In linear media $\vec{S} = (\vec{E} \times \vec{B})/\mu$

$$u_{EM} = \frac{1}{2}(\vec{E} \cdot \vec{D} + \vec{B} \cdot \vec{H})$$

$$= \frac{1}{2}\varepsilon E^2 + \frac{1}{2}\frac{B^2}{\mu} \quad \text{in linear media}$$

Phase Velocity $v = \frac{1}{\sqrt{\mu\varepsilon}} = \omega/k$

$$|\vec{B}| = |\vec{E}|/v = \sqrt{\mu\varepsilon}\,|\vec{E}|$$

$|\vec{B}|/|\vec{E}|$ bigger in matter

But $u_B = \frac{1}{2}B^2/\mu = \frac{1}{2}\mu\varepsilon E^2/\mu$

$$= \frac{1}{2}\varepsilon E^2$$

so $u_B = u_E$ still

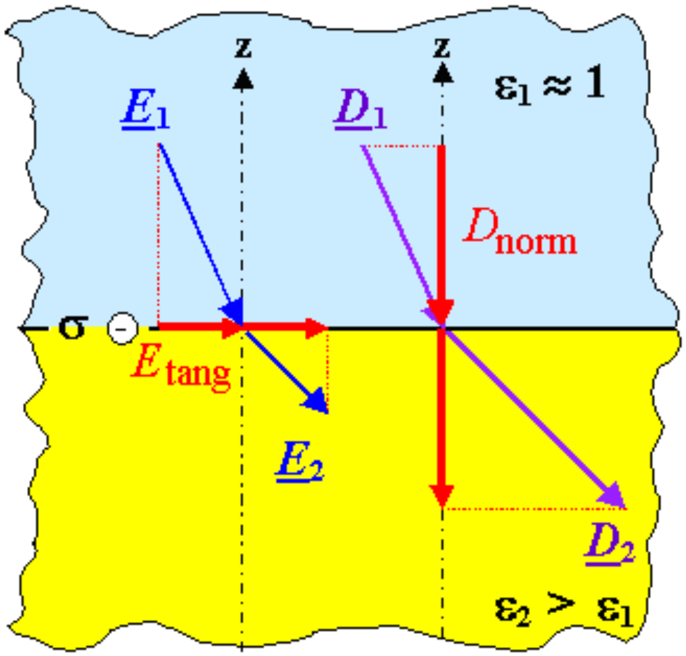
$$I = \langle S \rangle = \tfrac{1}{2} E_0 \cdot \sqrt{\mu\varepsilon}\, E_0/\mu$$

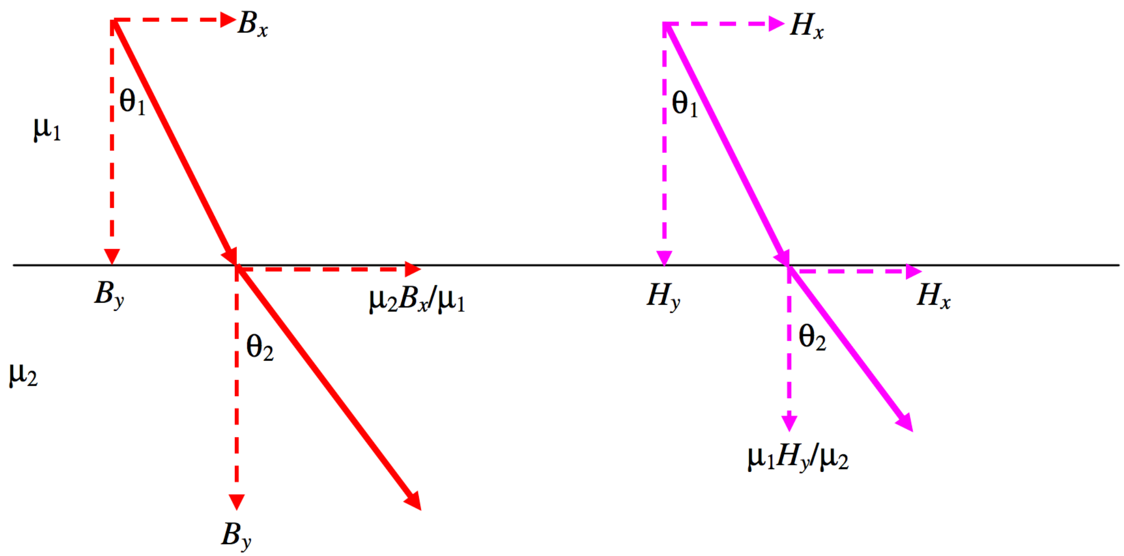
$$= \tfrac{1}{2}\varepsilon E_0^2/\sqrt{\mu\varepsilon}$$

$$= \tfrac{1}{2}\varepsilon E_0^2 \cdot v$$

(compare to $\frac{1}{2}\varepsilon_0 E_0^2 c$ in vacuum)

Boundary Conditions





Images show boundary conditions for case w/ no free charge or current at the boundary

Boundary Conditions

Interface w/ no free charge or current

$$\Delta D_\perp = \Delta(\varepsilon E_\perp) = 0$$
$$\Delta \vec{E}_\parallel = 0$$
$$\Delta B_\perp = 0$$
$$\Delta \vec{H}_\parallel = \Delta(\vec{B}_\parallel / \mu) = 0$$

Normal Incidence

- Boundary at x-y plane ($z=0$)
- Pick $\vec{k} = k\hat{z}$, $\hat{n} = \hat{x}$
- $\vec{E}, \vec{B} \perp \vec{k}$ so $E_\perp = B_\perp = 0$

Incident: $$\tilde{\vec{E}}_I(z,t) = \tilde{E}_{0I}\, e^{i(k_1 z - \omega t)}\,\hat{x}$$
$$\tilde{\vec{B}}_I(z,t) = \frac{\tilde{E}_{0I}}{v_1}\, e^{i(k_1 z - \omega t)}\,\hat{y}$$

Reflected: $$\tilde{\vec{E}}_R(z,t) = \tilde{E}_{0R}\, e^{i(-k_1 z - \omega t)}\,\hat{x}$$
$$\tilde{\vec{B}}_R(z,t) = \frac{\tilde{E}_{0R}}{v_1}\, e^{i(-k_1 z - \omega t)}\,(-\hat{y})$$

Transmitted: $$\tilde{\vec{E}}_T(z,t) = \tilde{E}_{0T}\, e^{i(k_2 z - \omega t)}\,\hat{x}$$
$$\tilde{\vec{B}}_T(z,t) = \frac{\tilde{E}_{0T}}{v_2}\, e^{i(k_2 z - \omega t)}\,\hat{y}$$