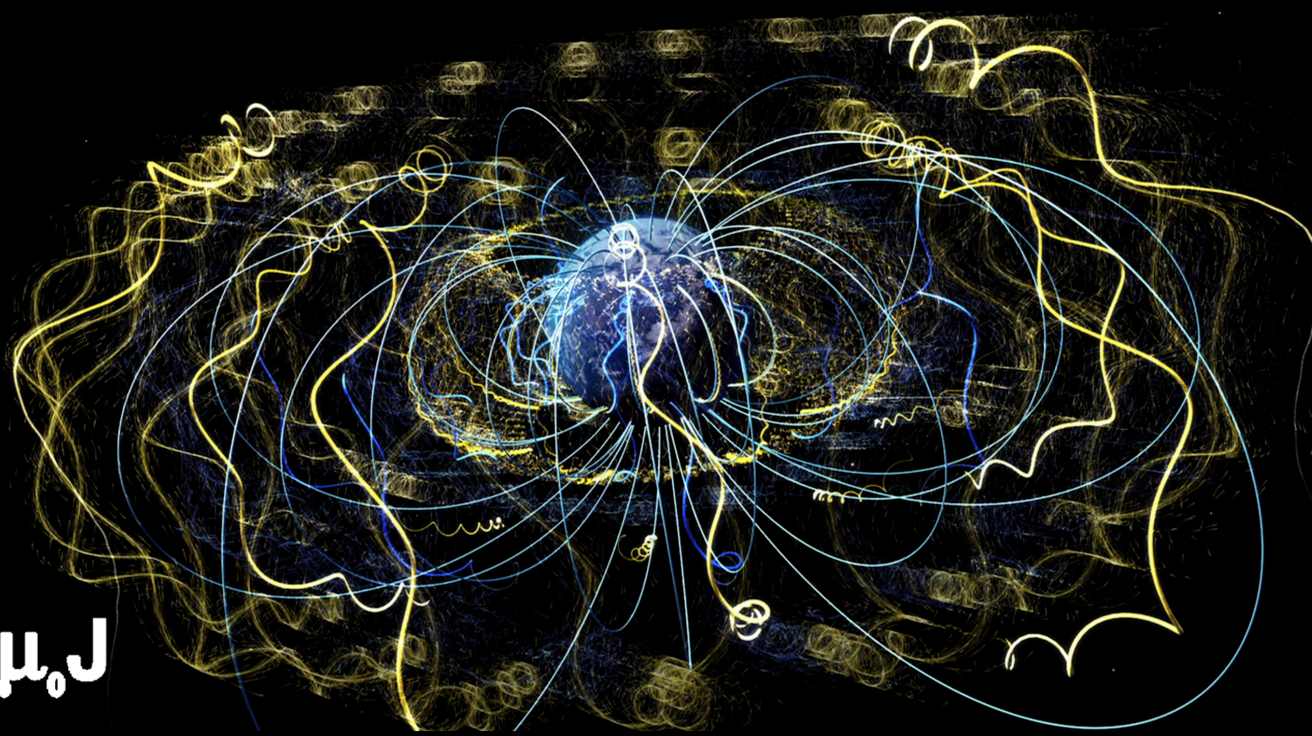
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism II: 3812

Professor Jasper Halekas
Van Allen 70

MWF 9:30-10:20 Lecture

Boundary Conditions

Interface w/ no free charge or current

$$\Delta D_\perp = \Delta(\varepsilon E_\perp) = 0$$
$$\Delta \vec{E}_\parallel = 0$$
$$\Delta B_\perp = 0$$
$$\Delta \vec{H}_\parallel = \Delta(\vec{B}_\parallel/\mu) = 0$$

Normal Incidence

- Boundary at x-y plane ($z=0$)
- Pick $\vec{k} = k\hat{z}$, $\hat{n} = \hat{x}$
- $\vec{E}, \vec{B} \perp \vec{k}$ so $E_\perp = B_\perp = 0$

Incident: $\tilde{\vec{E}}_I(z,t) = \tilde{E}_{0I}\, e^{i(k_1 z - \omega t)}\,\hat{x}$

$\tilde{\vec{B}}_I(z,t) = \frac{\tilde{E}_{0I}}{v_1}\, e^{i(k_1 z - \omega t)}\,\hat{y}$

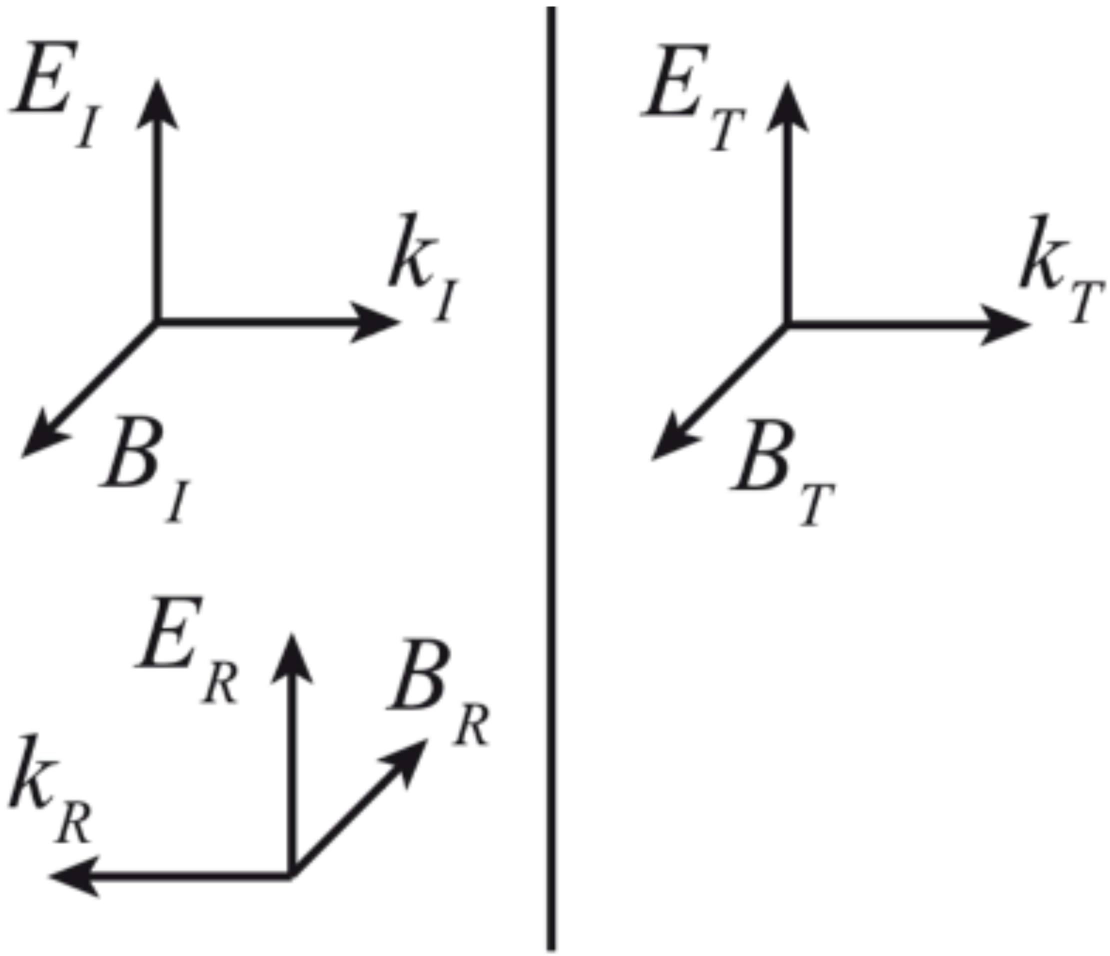
Reflected: $\tilde{\vec{E}}_R(z,t) = \tilde{E}_{0R}\, e^{i(-k_1 z - \omega t)}\,\hat{x}$

$\tilde{\vec{B}}_R(z,t) = \frac{\tilde{E}_{0R}}{v_1}\, e^{i(-k_1 z - \omega t)}\,(-\hat{y})$

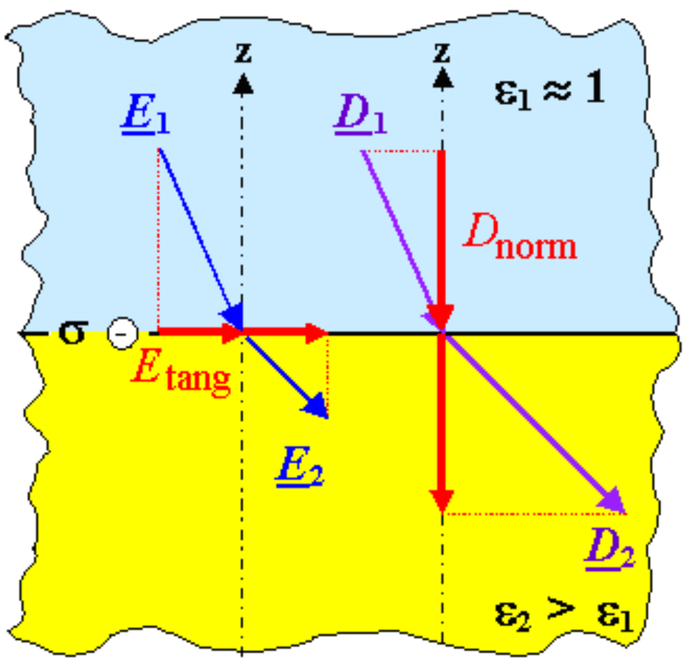
Transmitted: $\tilde{\vec{E}}_T(z,t) = \tilde{E}_{0T}\, e^{i(k_2 z - \omega t)}\,\hat{x}$

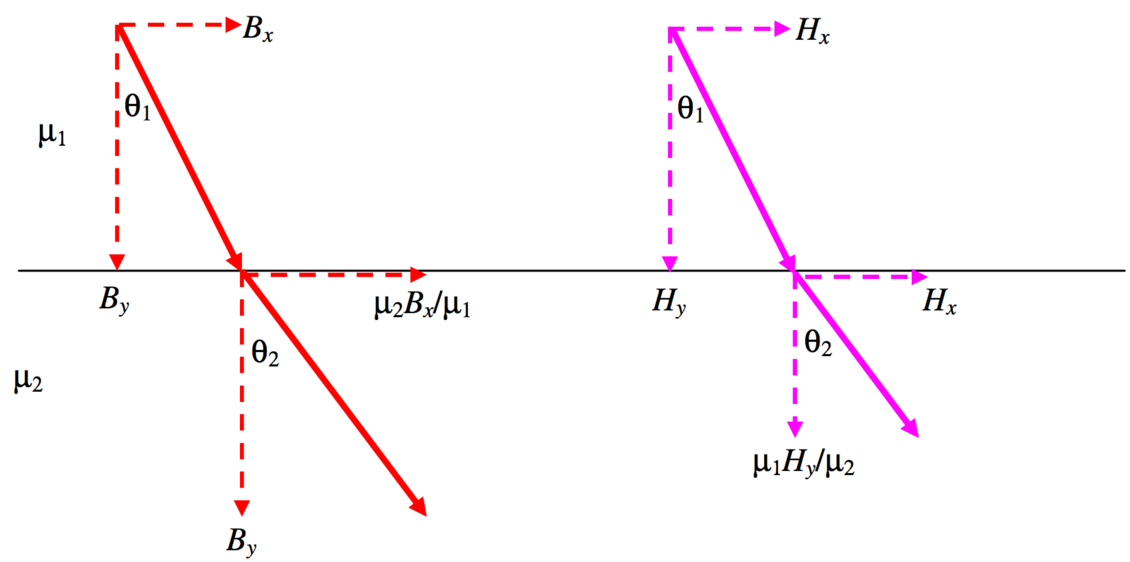
$\tilde{\vec{B}}_T(z,t) = \frac{\tilde{E}_{0T}}{v_2}\, e^{i(k_2 z - \omega t)}\,\hat{y}$

Reflection & Transmission at Normal Incidence



Boundary Conditions





Images show boundary conditions for case w/ no free charge or current at the boundary

$$\Delta \vec{E}_{\parallel} = 0$$
$$\Rightarrow \quad \tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T}$$

$$\Delta \vec{H}_{\parallel} = 0$$
$$\Rightarrow \quad \frac{1}{\mu_1}\left(\frac{\tilde{E}_{0I}}{v_1} - \frac{\tilde{E}_{0R}}{v_1}\right) = \frac{1}{\mu_2}\frac{\tilde{E}_{0T}}{v_2}$$

Write $\beta = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}$

Solution:
$$\boxed{\begin{aligned}\tilde{E}_{0R} &= \frac{1-\beta}{1+\beta}\tilde{E}_{0I} \\ \tilde{E}_{0T} &= \frac{2}{1+\beta}\tilde{E}_{0I}\end{aligned}}$$

If $\mu_1 \sim \mu_2 \Rightarrow \beta = \frac{v_1}{v_2} = \frac{n_2}{n_1}$

$$\tilde{E}_{0R} = \frac{v_2 - v_1}{v_2 + v_1}\tilde{E}_{0I}, \quad \tilde{E}_{0T} = \frac{2v_2}{v_1 + v_2}\tilde{E}_{0I}$$

$v_2 > v_1$: $E_{0R} = \frac{v_2 - v_1}{v_2 + v_1}E_{0I}$, $E_{0T} = \frac{2v_2}{v_1+v_2}E_{0I}$

In phase reflection

$v_2 < v_1$: $E_{0R} = \frac{v_1 - v_2}{v_1 + v_2}E_{0I}$, $E_{0T} = \frac{2v_2}{v_1+v_2}E_{0I}$

Inverted reflection

Same as wave on a string!

In terms of n

$$E_{0R} = \left|\frac{n_1 - n_2}{n_1 + n_2}\right| E_{0I}$$

$$E_{0T} = \frac{2n_1}{n_1 + n_2} E_{0I}$$

Reflection & Transmission

$$I = \frac{1}{2} \varepsilon E_0^2 v$$

$$R = I_R / I_I = \frac{E_{0R}^2}{E_{0I}^2} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

If $\mu_1 = \mu_2$

$$T = I_T / I_I = \frac{\varepsilon_2 v_2 E_{0T}^2}{\varepsilon_1 v_1 E_{0I}^2}$$

$$= \frac{n_2^2}{n_1^2} \frac{n_1}{n_2} \left(\frac{2n_1}{n_1 + n_2}\right)^2$$

$$= \frac{4 n_1 n_2}{(n_1 + n_2)^2} \quad \text{If } \mu_1 = \mu_2$$

Note $$R + T = \frac{n_1^2 - 2n_1 n_2 + n_2^2 + 4n_1 n_2}{(n_1 + n_2)^2}$$

$$= 1 \quad \text{(Always True)}$$

Oblique Incidence

$$\tilde{\vec{E}}_I(\vec{r},t) = \tilde{E}_{0I}\, e^{i(\vec{k}_I\cdot\vec{r} - \omega t)}\, \hat{n}_I$$

$$\tilde{\vec{B}}_I(\vec{r},t) = \frac{\tilde{E}_{0I}}{v_1}\, e^{i(\vec{k}_I\cdot\vec{r} - \omega t)}\, \hat{k}_I \times \hat{n}_I$$

$$\tilde{\vec{E}}_R(\vec{r},t) = \tilde{E}_{0R}\, e^{i(\vec{k}_R\cdot\vec{r} - \omega t)}\, \hat{n}_R$$

$$\tilde{\vec{B}}_R(\vec{r},t) = \frac{\tilde{E}_{0R}}{v_1}\, e^{i(\vec{k}_R\cdot\vec{r} - \omega t)}\, \hat{k}_R \times \hat{n}_R$$

$$\tilde{\vec{E}}_T(\vec{r},t) = \tilde{E}_{0T}\, e^{i(\vec{k}_T\cdot\vec{r} - \omega t)}\, \hat{n}_T$$

$$\tilde{\vec{B}}_T(\vec{r},t) = \frac{\tilde{E}_{0T}}{v_2}\, e^{i(\vec{k}_T\cdot\vec{r} - \omega t)}\, \hat{k}_T \times \hat{n}_T$$

$$k_I v_1 = k_R v_1 = k_T v_2 = \omega$$

$$\Rightarrow \quad k_I = k_R = \frac{v_2}{v_1} k_T = \frac{n_1}{n_2} k_T$$

$$\tilde{\vec{E}}_{I\parallel}(z=0) + \tilde{\vec{E}}_{R\parallel}(z=0) = \tilde{\vec{E}}_{T\parallel}(z=0)$$

Must hold for all x, y, t

$\Rightarrow$ exponential factors equal @ $z = 0$

$$\Rightarrow \quad \vec{k}_I\cdot\vec{r} = \vec{k}_R\cdot\vec{r} = \vec{k}_T\cdot\vec{r} \quad @\ z=0$$

$$\Rightarrow \quad k_{Ix}x + k_{Iy}y = k_{Rx}x + k_{Ry}y$$

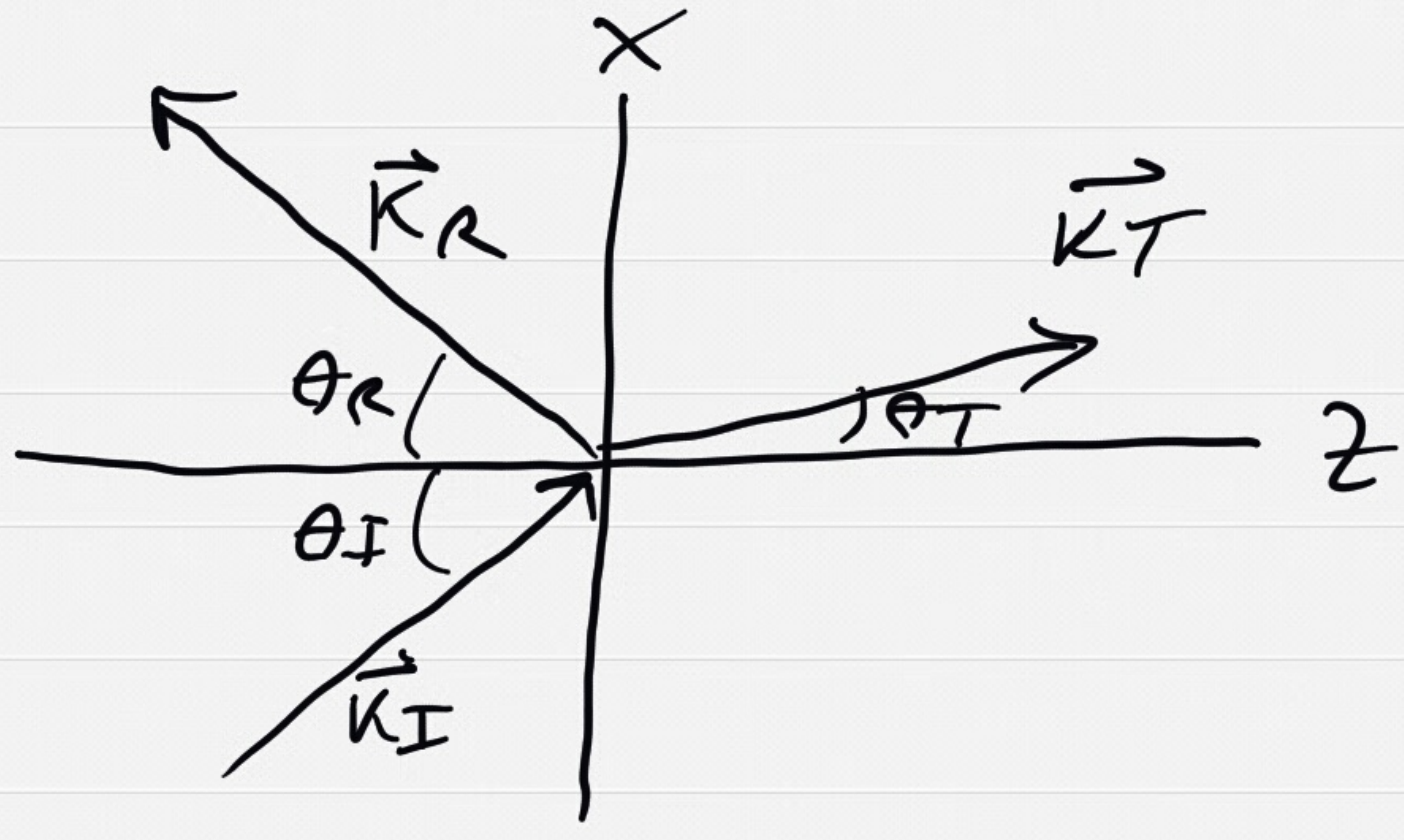
$$= k_{Tx}x + k_{Ty}y$$

Must hold for all x, y

$$\Rightarrow \quad k_{Ix} = k_{Rx} = k_{Tx}$$
$$k_{Iy} = k_{Ry} = k_{Ty}$$

This constraint is equivalent to saying that wave fronts have to line up with each other.

Since only k_z changes, all k vectors are in a plane. For convenience, we say this is x-z plane



$$k_{Rx} = k_{Ix} \Rightarrow k_I \sin\theta_I = k_R \sin\theta_R$$

but $k_I = k_R \Rightarrow$ $\boxed{\sin\theta_I = \sin\theta_R}$

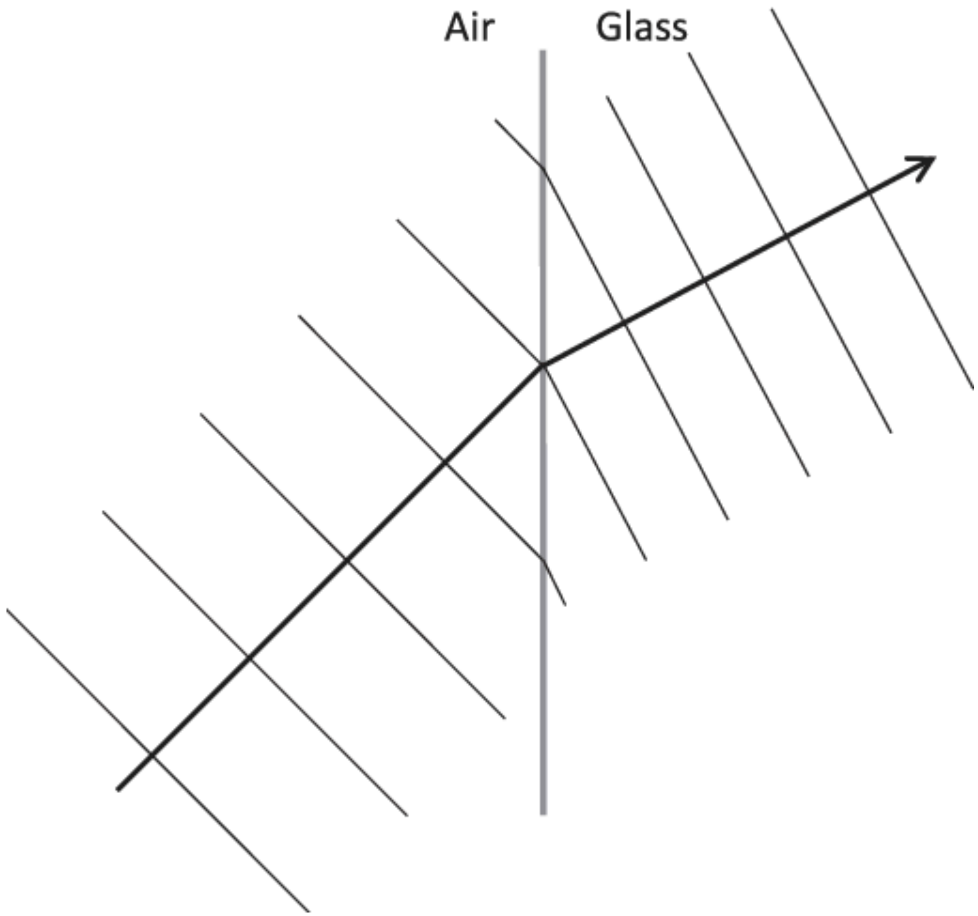
Equal Incidence/Reflection

$$k_{Ix} = k_{Tx} \Rightarrow k_I \sin\theta_I = k_T \sin\theta_T$$

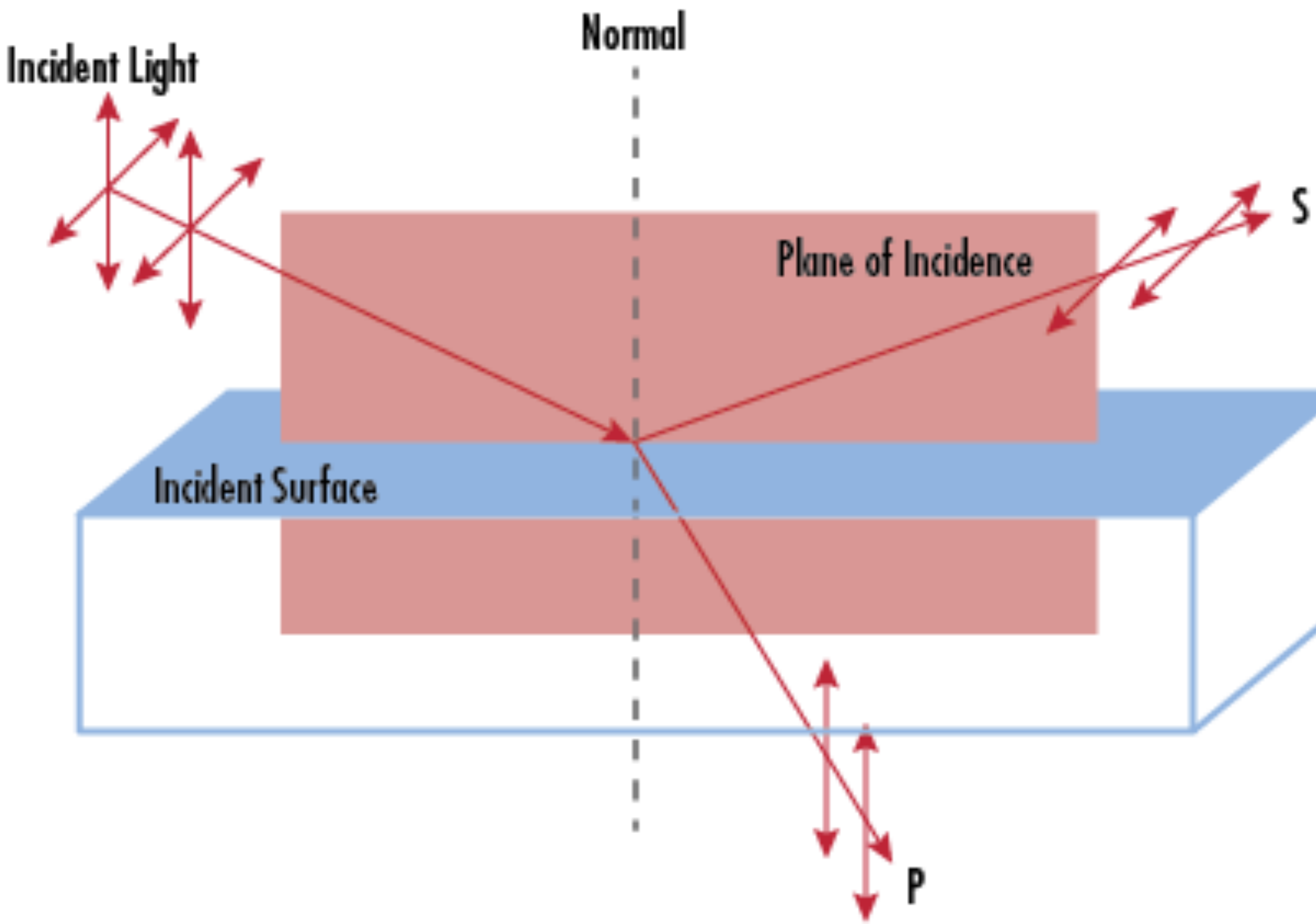
$k_I = \frac{n_1}{n_2} k_T \Rightarrow$ $\boxed{\dfrac{\sin\theta_T}{\sin\theta_I} = \dfrac{n_1}{n_2}}$

Snell's Law

Snell's Law: Geometric Proof



S & P Polarization



P-polarized (from the German parallel) light has an electric field polarized parallel to the plane of incidence, while s-polarized (from the German senkrecht) light is perpendicular to this plane.

Reflection & Transmission at Oblique Incidence

Out-of-Plane (S) Polarization | In-Plane (P) Polarization