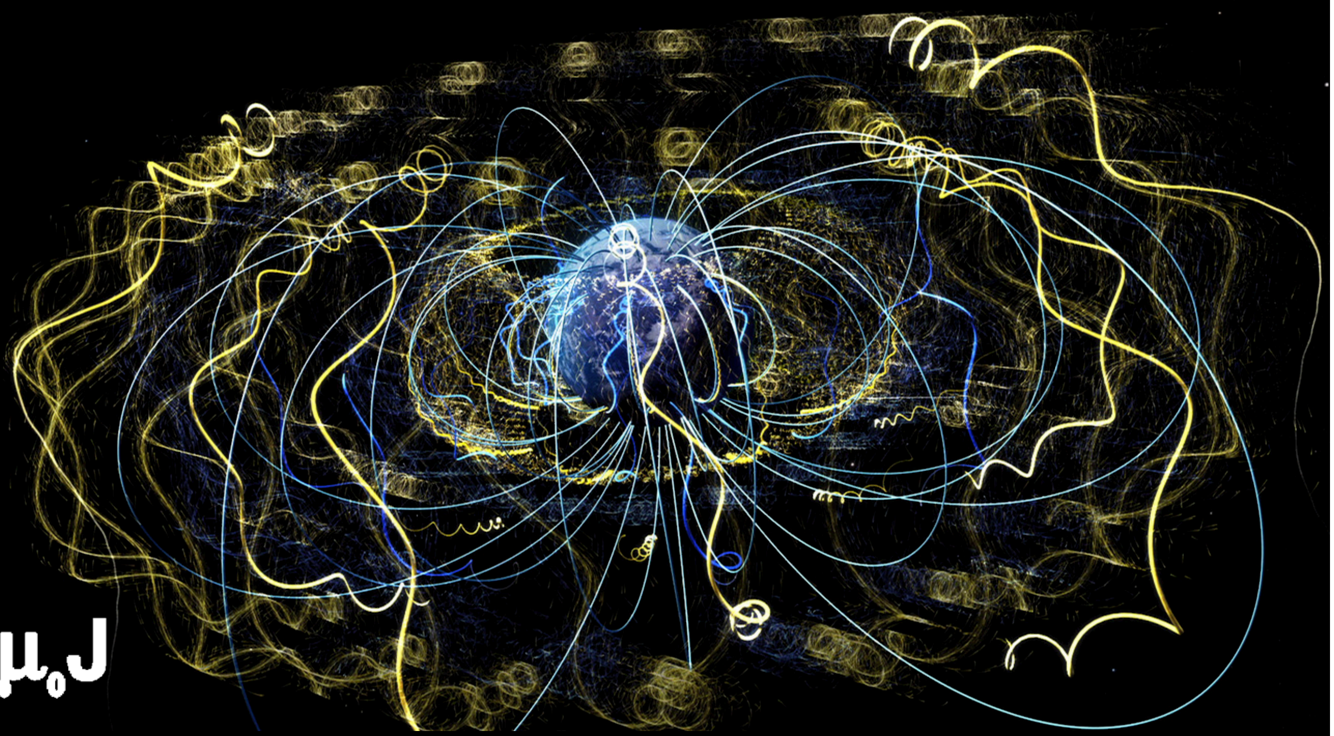
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism II: 3812

Professor Jasper Halekas
Van Allen 70
MWF 9:30-10:20 Lecture

$$\nabla \cdot \vec{E} = 0 \qquad \nabla \times \vec{E} = -\partial \vec{B}/\partial t$$

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = = \mu\varepsilon\, \partial \vec{E}/\partial t + \mu\sigma \vec{E}$$

$$\nabla \times (\nabla \times \vec{B}) = \nabla(\nabla \cdot \vec{B}) - \nabla^2 \vec{B}$$

$$= -\nabla^2 \vec{B}$$

$$= \mu\varepsilon\, \nabla \times (\partial \vec{E}/\partial t) + \mu\sigma\, \nabla \times \vec{E}$$

$$= \mu\varepsilon\, \partial/\partial t\, (\nabla \times \vec{E}) + \mu\sigma\, \nabla \times \vec{E}$$

$$= \mu\varepsilon \cdot -\partial^2 \vec{B}/\partial t^2 + \mu\sigma \cdot -\partial \vec{B}/\partial t$$

$$\Rightarrow \nabla^2 \vec{B} = \mu\varepsilon\, \partial^2 \vec{B}/\partial t^2 + \mu\sigma\, \partial \vec{B}/\partial t$$

similarly $\nabla^2 \vec{E} = \mu\varepsilon\, \partial^2 \vec{E}/\partial t^2 + \mu\sigma\, \partial \vec{E}/\partial t$

Damped Wave Equations!

Assume solutions: $\vec{E}(\vec{r},t) = \tilde{\vec{E}}_0\, e^{i(\tilde{\vec{k}} \cdot \vec{r} - \omega t)}$

$\vec{B}(\vec{r},t) = \tilde{\vec{B}}_0\, e^{i(\tilde{\vec{k}} \cdot \vec{r} - \omega t)}$

$\tilde{k} = (k_r + i k_i)\hat{k}$ (book uses kappa for k_i)

$$\nabla^2 \vec{E} = i\tilde{\vec{k}} \cdot i\tilde{\vec{k}} \cdot \tilde{\vec{E}}$$

$$\mu\varepsilon\, \partial^2 \vec{E}/\partial t^2 = (-i\omega)^2 \cdot \mu\varepsilon\, \tilde{\vec{E}}$$

$$\mu\sigma\, \partial \vec{E}/\partial t = (-i\omega) \cdot \mu\sigma\, \tilde{\vec{E}}$$

$$\Rightarrow -\tilde{k}^2 = -\mu\varepsilon\omega^2 - i\mu\sigma\omega$$

$$\Rightarrow \boxed{\begin{aligned} k_r &= \omega\sqrt{\frac{\varepsilon\mu}{2}}\sqrt{1 + \sqrt{1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2}} \\ k_i &= \omega\sqrt{\frac{\varepsilon\mu}{2}}\sqrt{\sqrt{1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2} - 1} \end{aligned}}$$

EM wave in Conductor

say $\vec{\tilde{k}} = \tilde{k}\,\hat{z} = (k_r + i k_i)\,\hat{z}$

$$\vec{\tilde{E}}(z,t) = \vec{\tilde{E}}_0\, e^{i((k_r + i k_i)z - \omega t)}$$
$$= \vec{\tilde{E}}_0\, e^{-k_i z}\, e^{i(k_r z - \omega t)}$$

$$\vec{\tilde{B}}(z,t) = \vec{\tilde{B}}_0\, e^{-k_i z}\, e^{i(k_r z - \omega t)}$$

- Plane wave multiplied by Damping term

- skin depth = e-folding distance

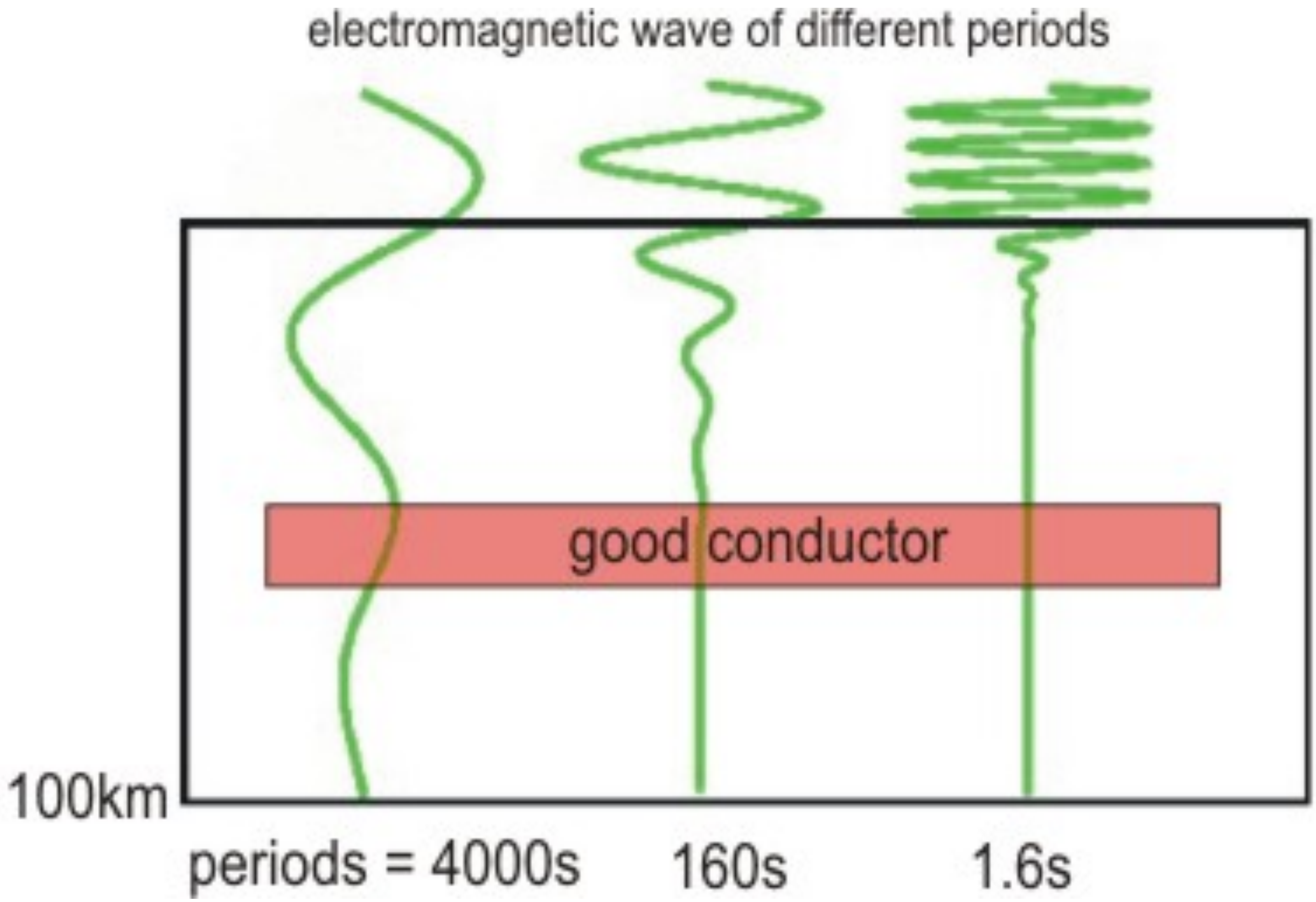
$$d = 1/k_i$$

$d \sim 1/\omega \Rightarrow$ high frequency waves have shorter skin depth

~ wave properties all from k_r

$$\lambda = 2\pi/k_r\,,\quad v = \omega/k_r\,,\quad n = c k_r/\omega$$

Skin Depth


electromagnetic wave of different periods
good conductor
100km
periods = 4000s
160s
1.6s

Phase Relationship in Conductor

$$\nabla \times \tilde{\vec{E}} = i\tilde{\vec{k}} \times \tilde{\vec{E}}$$
$$i\tilde{k}\,\hat{k} \times \hat{n}\,\tilde{E}$$

$$-\partial \vec{B}/\partial t = i\omega \tilde{\vec{B}}$$

$$\Rightarrow \tilde{k}\,(\hat{k} \times \hat{n})\,\tilde{E}_0 = \omega \tilde{\vec{B}}_0$$

So if
$$\tilde{\vec{E}}(\vec{r},t) = \tilde{E}_0\, e^{i(\tilde{\vec{k}}\cdot\vec{r} - \omega t)}\,\hat{n}$$
$$\tilde{\vec{B}}(\vec{r},t) = \frac{\tilde{k}}{\omega}\tilde{E}_0\, e^{i(\tilde{\vec{k}}\cdot\vec{r} - \omega t)}\,\hat{k}\times\hat{n}$$

$\vec{E}$, $\vec{B}$, $\hat{k}$ still all perpendicular

But $\tilde{B}_0 = \frac{\tilde{k}}{\omega}\tilde{E}_0$

So $B_0 e^{i\delta_B} = \frac{|\tilde{k}|}{\omega} e^{i\varphi} E_0 e^{i\delta_E}$

$$\text{w/ } |\tilde{k}| = \sqrt{|\tilde{k}^* \tilde{k}|} = \sqrt{k_r^2 + k_i^2}$$
$$= \omega\sqrt{\varepsilon\mu\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}}$$
$$\varphi = \tan^{-1}(k_i/k_r)$$

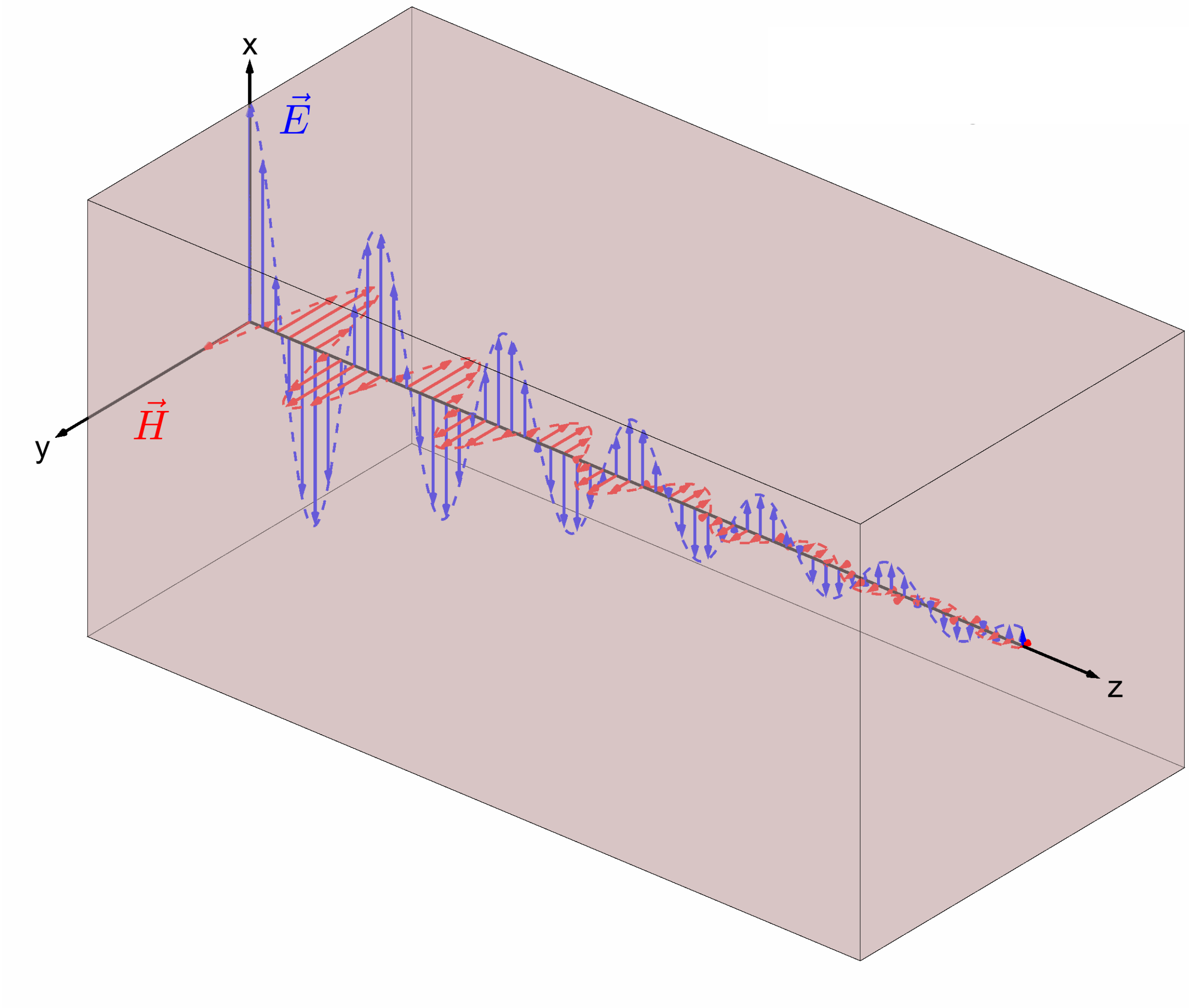
So $$B_0 = \frac{|\tilde{k}|}{\omega} E_0 = \sqrt{\varepsilon\mu\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}}\; E_0$$

And $\delta_B = \delta_E + \varphi \Rightarrow \vec{B}$ lags behind $\vec{E}$!

Real fields for $\tilde{\vec{k}} = \tilde{k}\,\hat{z}$

$$\vec{E}(z,t) = E_0 e^{-k_i z}\cos(kz - \omega t + \delta_E)\,\hat{n}$$
$$\vec{B}(z,t) = \frac{|\tilde{k}|}{\omega} E_0 e^{-k_i z}\cos(kz - \omega t + \delta_E + \varphi)\,\hat{k}\times\hat{n}$$

Waves in Conductors



Reflection @ Conducting surface

Normal Incidence

$E_\perp, B_\perp = 0$

$E_\perp = 0 \Rightarrow \sigma_f = 0$

Ohm's Law $\vec{J}_f = \sigma \vec{E}$

$\Rightarrow \vec{K}_f = 0$ or else would need a δ-function electric field on surface

$$\tilde{\vec{E}}_I(z,t) = \tilde{E}_{0I}\, e^{i(k_1 z - \omega t)}\,\hat{x}$$

$$\tilde{\vec{B}}_I(z,t) = \frac{\tilde{E}_{0I}}{v_1}\, e^{i(k_1 z - \omega t)}\,\hat{y}$$

$$\tilde{\vec{E}}_R(z,t) = \tilde{E}_{0R}\, e^{i(-k_1 z - \omega t)}\,\hat{x}$$

$$\tilde{\vec{B}}_R(z,t) = \frac{\tilde{E}_{0R}}{v_2}\, e^{i(-k_1 z - \omega t)}\,(-\hat{y})$$

$$\tilde{\vec{E}}_T(z,t) = \tilde{E}_{0T}\, e^{i(\tilde{k}_2 z - \omega t)}\,\hat{x}$$

$$\tilde{\vec{B}}_T(z,t) = \frac{\tilde{k}_2}{\omega}\,\tilde{E}_{0T}\, e^{i(\tilde{k}_2 z - \omega t)}\,\hat{y}$$

$$\Delta \vec{E}_{\parallel} = 0 \Rightarrow \tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T}$$

$$\Delta\left(\frac{\vec{B}_{\parallel}}{\mu}\right) = 0 \Rightarrow \frac{1}{\mu_1 v_1}\left(\tilde{E}_{0I} - \tilde{E}_{0R}\right) = \frac{\tilde{k}_2}{\omega} \cdot \frac{\tilde{E}_{0T}}{\mu_2}$$

Define $\tilde{\beta} = \frac{\mu_1 v_1}{\mu_2} \cdot \frac{\tilde{k}_2}{\omega}$

$$\tilde{E}_{0R} = \left(\frac{1-\tilde{\beta}}{1+\tilde{\beta}}\right)\tilde{E}_{0I}$$

$$\tilde{E}_{0T} = \left(\frac{2}{1+\tilde{\beta}}\right)\tilde{E}_{0I}$$

same as usual but w/ complex $\tilde{\beta}$

For highly conductive material

$|\tilde{k}| \to \infty$

$\Rightarrow |\tilde{\beta}| \to \infty$

$$\Rightarrow \quad \tilde{E}_{0R} = -\tilde{E}_{0I}$$
$$\tilde{E}_{0T} = 0$$

Perfect reflection w/ 180° phase shift

We will not cover 9.4.3 (Frequency dependence of permittivity)