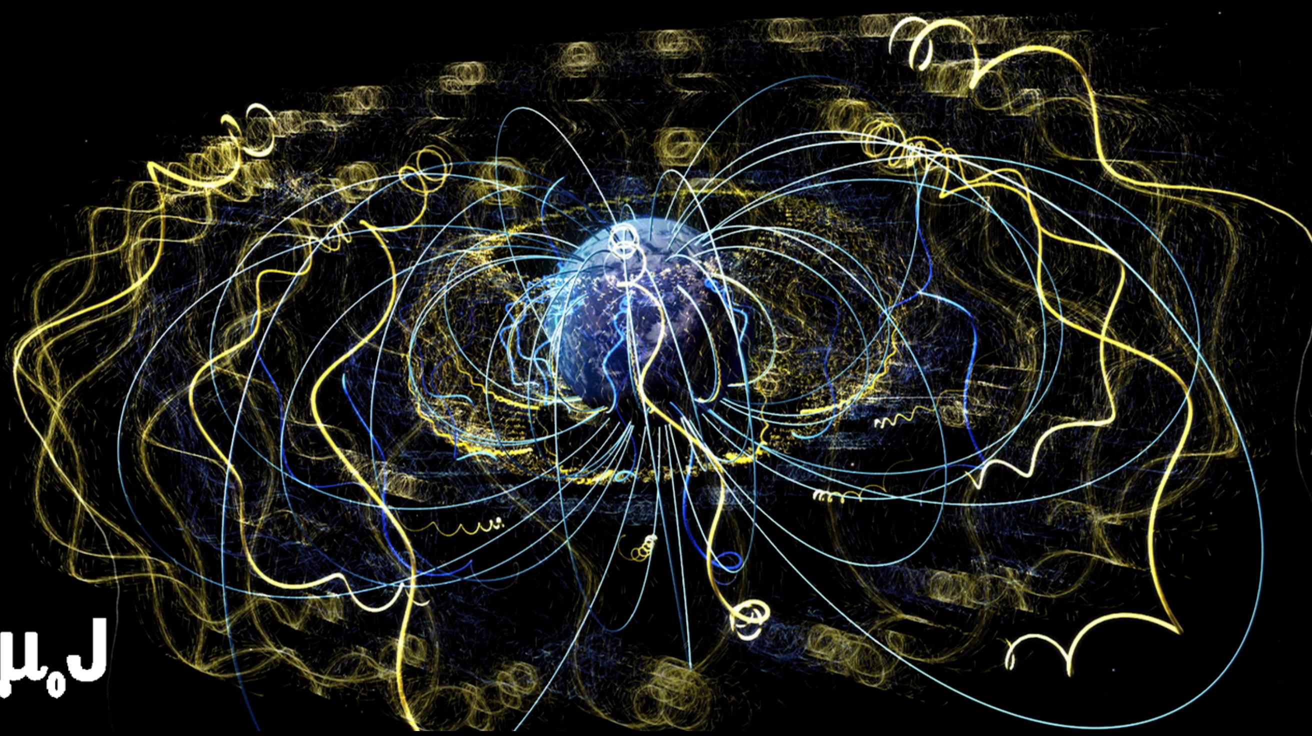
$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} + \mu_0 J$$



Electricity and Magnetism II: 3812

Professor Jasper Halekas
Virtual by Zoom!
MWF 9:30-10:20 Lecture

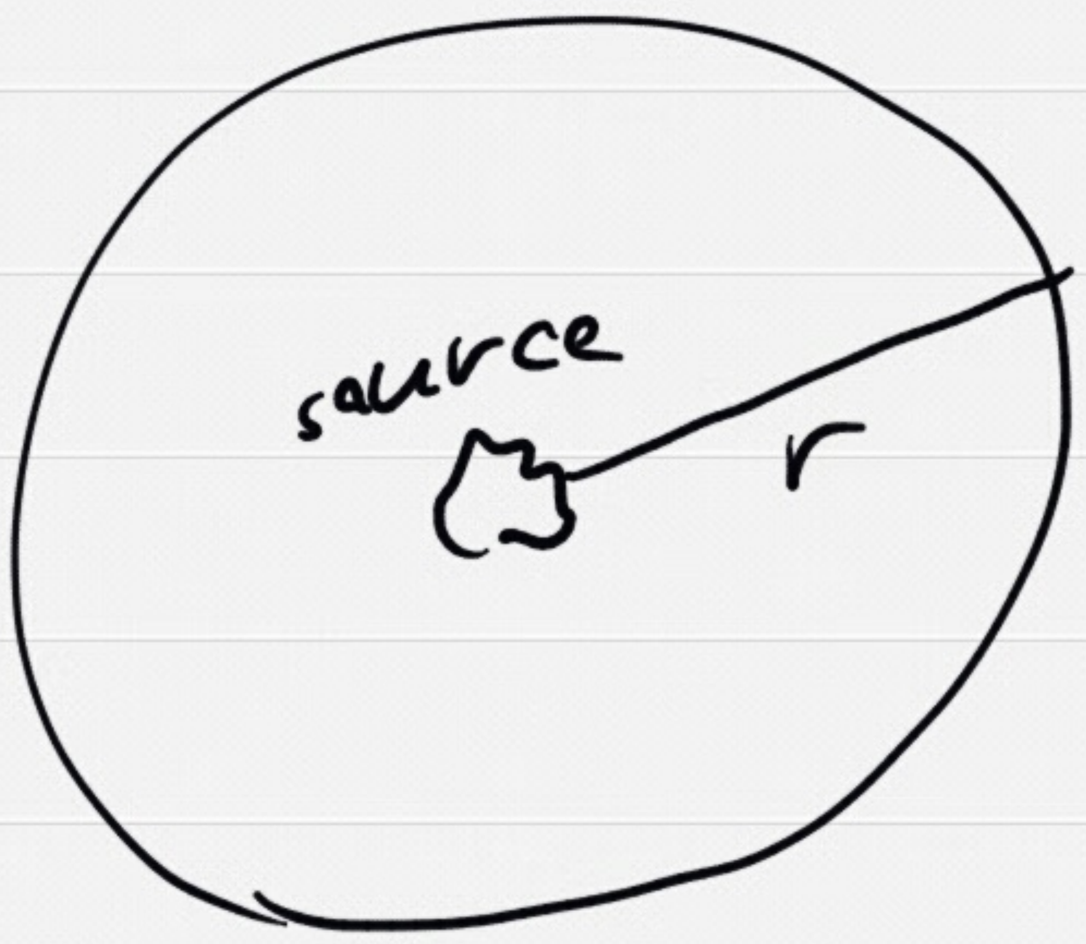
Ch. 11	Radiation
11.1	Dipole Radiation

- We have studied propagating electromagnetic waves
- We've looked at the electromagnetic fields of moving point charges
- Now it's time to show how moving point charges generate waves
  - A.K.a. "Radiation"

- What about static charges?
  - No good!

Consider EM energy transferred through spherical surface

$$P(r,t) = \oint \vec{S}\cdot d\vec{a} = -dU/dt$$
$$= \frac{1}{\mu_0}\oint (\vec{E}\times\vec{B})\cdot d\vec{a}$$



$P_{rad}(t_0) = \lim_{r\to\infty} P(r, t_0 + r/c)$

is energy lost to radiation (not returning to source)

$A_{sphere} = 4\pi r^2$

Finite P_{rad} requires $|\vec{S}| \sim 1/r^2$ or less steep

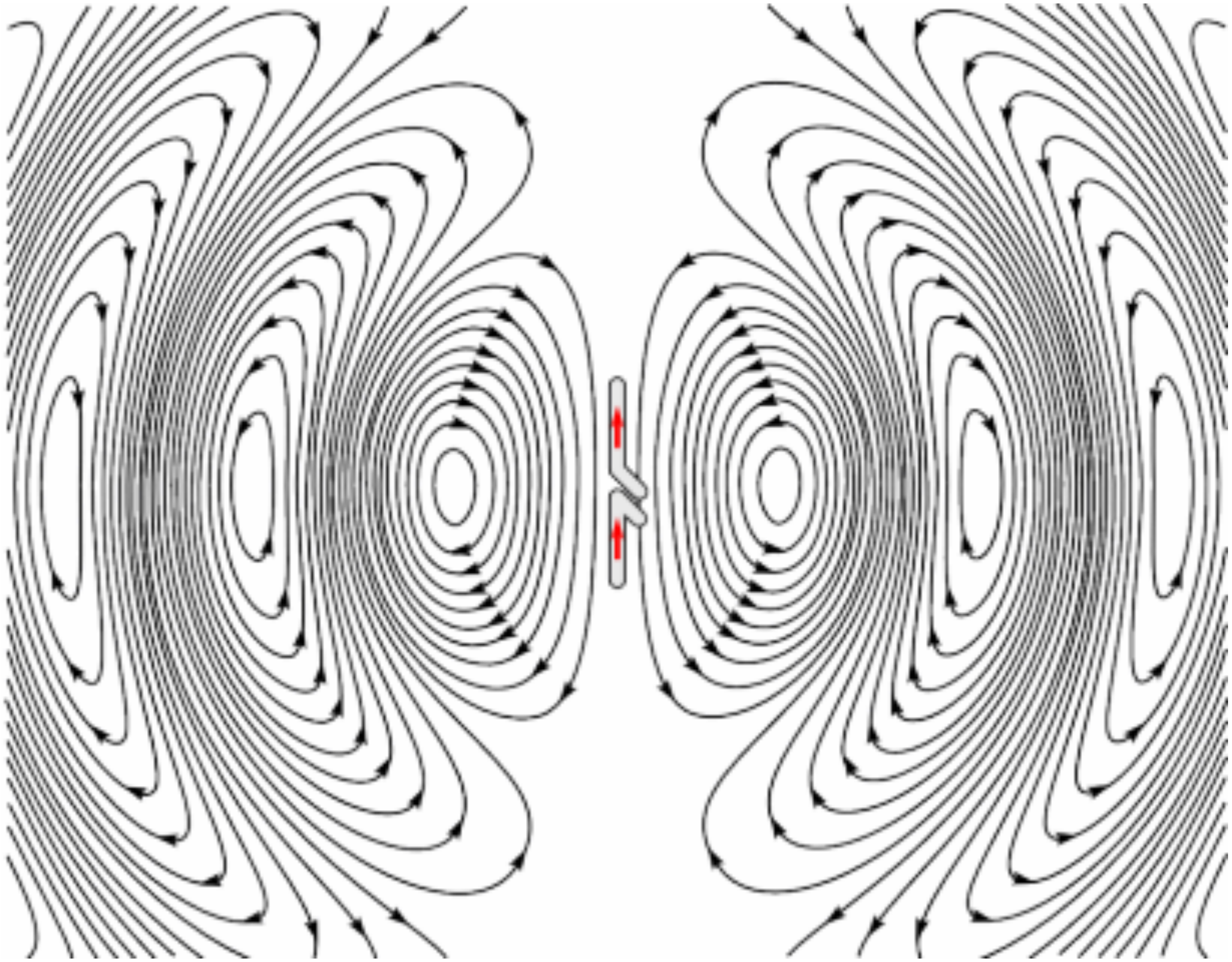
But static sources have

$$E \sim \frac{Q}{4\pi\varepsilon_0 r^2} \qquad B \sim \frac{\mu_0 I}{4\pi r^2}$$

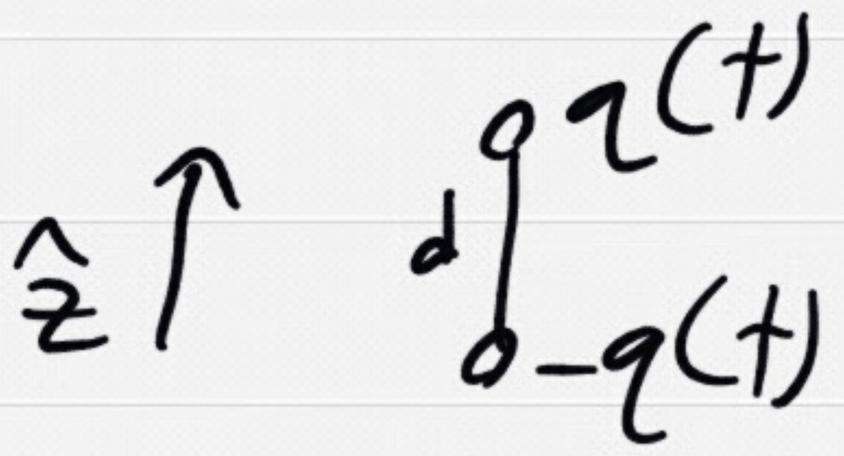
$\Rightarrow S \sim 1/r^4$ so $P_{rad} = 0$

Need $E, B \sim 1/r$ or slower

Electric Dipole Radiation



Dipole Radiation

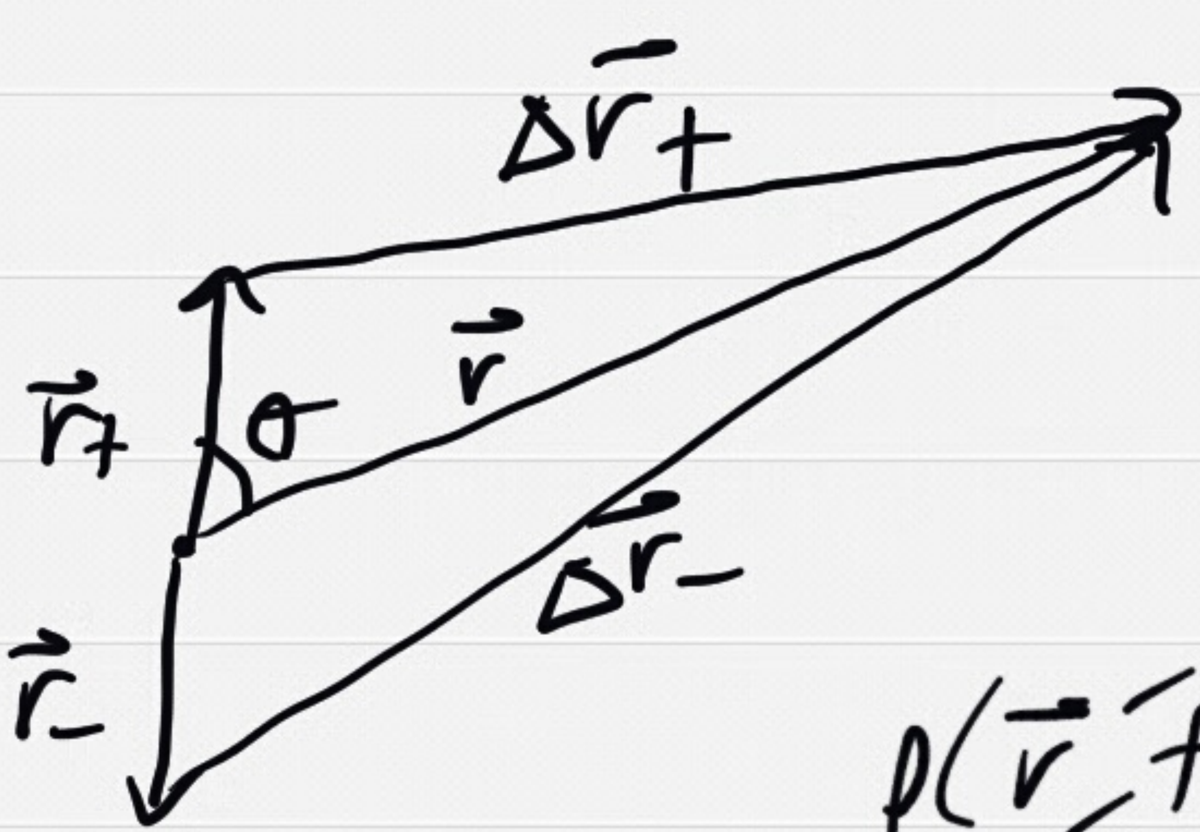


$$q(t) = q_0 \cos(\omega t)$$

$$\Rightarrow \vec{p}(t) = q_0 d \cos(\omega t)\,\hat{z}$$

$$= p_0 \cos(\omega t)\,\hat{z}$$

$$= \vec{p}_0 \cos(\omega t)$$



$$V(\vec{r},t) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r}\,',t_r)}{\Delta r}\, d\tau'$$

$$\rho(\vec{r}\,',t_r) = q_0 \cos\left(\omega\left(t - \frac{\Delta r}{c}\right)\right) \delta(\vec{r}\,' - \vec{r}_+)$$

$$- q_0 \cos\left(\omega\left(t - \frac{\Delta r}{c}\right)\right) \delta(\vec{r}\,' - \vec{r}_-)$$

$$\Rightarrow V(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_0 \cos\left(\omega\left(t - \frac{\Delta r_+}{c}\right)\right)}{\Delta r_+} - \frac{q_0 \cos\left(\omega\left(t - \frac{\Delta r_-}{c}\right)\right)}{\Delta r_-}\right]$$

$$\Delta r_\pm = \sqrt{r^2 + \left(\frac{d}{2}\right)^2 \mp r d \cos\theta}$$

$$= r\sqrt{1 + \left(\frac{d}{2r}\right)^2 \mp \frac{d}{r}\cos\theta}$$

For $d \ll r \Rightarrow \Delta r_\pm \sim r\left(1 \mp \frac{d}{2r}\cos\theta\right)$

$$\Rightarrow \frac{1}{\Delta r_\pm} \sim \frac{1}{r}\left(1 \pm \frac{d}{2r}\cos\theta\right)$$

Next $\cos(\omega(t - \Delta r_\pm / c))$

$$\sim \cos\left(\omega(t - r/c) \pm \frac{\omega d}{2c} \cos\theta\right)$$

$$= \cos(\omega(t - r/c)) \cos\left(\frac{\omega d}{2c} \cos\theta\right) \mp \sin(\omega(t - r/c)) \sin\left(\frac{\omega d}{2c} \cos\theta\right)$$

Now, assume $d << \lambda = c/\omega$
(small antenna)

$$\Rightarrow \cos(\omega(t - \Delta r_\pm / c)) \sim \cos(\omega(t - r/c)) \mp \sin(\omega(t - r/c)) \cdot \frac{\omega d}{2c} \cos\theta$$

$$\Rightarrow V(\vec{r}, t) \sim \frac{q_0}{4\pi\varepsilon_0}\Big[\Big\{ \cos(\omega(t - r/c)) - \frac{\omega d}{2c} \cos\theta \sin(\omega(t - r/c)) \Big\} \cdot \frac{1}{r} \cdot \Big(1 + \frac{d}{2r} \cos\theta\Big)$$

$$- \Big\{ \cos(\omega(t - r/c)) + \frac{\omega d}{2c} \cos\theta \sin(\omega(t - r/c)) \Big\} \cdot \frac{1}{r} \cdot \Big(1 - \frac{d}{2r} \cos\theta\Big) \Big]$$

$$\Rightarrow V(\bar{r}, t) \sim \frac{q_0}{4\pi\varepsilon_0}\Big[2 \cos(\omega(t - r/c)) \cdot \frac{d}{2r^2} \cos\theta - 2 \frac{\omega d}{2c} \cos\theta \sin(\omega(t - r/c)) \cdot \frac{1}{r} \Big]$$

$$\boxed{V(\vec{r}, t) \sim \frac{q_0 d \cos\theta}{4\pi\varepsilon_0 r}\Big[\frac{\cos(\omega(t - r/c))}{r} - \frac{\omega}{c} \sin(\omega(t - r/c)) \Big]}$$

Finally, assume $r >> \lambda = c/\omega$
(so $r >> \lambda >> d$)

$$\Rightarrow \boxed{V(\bar{r}, t) \sim \frac{-q_0 d \omega}{4\pi\varepsilon_0 c} \frac{\cos\theta}{r} \sin(\omega(t - r/c))}$$

Note: $\omega \to 0$

$$\Rightarrow V(\vec{r},t) \to \frac{q_0 d \cos\theta}{4\pi\varepsilon_0 r^2}$$

static Dipole potential

What about $\vec{A}(\vec{r},t)$?

$$\vec{I}(t) = dq/dt\,\hat{z} = -q_0\omega\sin(\omega t)\,\hat{z}$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{I}(\vec{r}',t_r)}{\Delta r}\,dl'$$

$$= \frac{\mu_0}{4\pi}\int_{-d/2}^{d/2} \frac{-q_0\omega\sin(\omega(t-\frac{\Delta r}{c}))\,\hat{z}\,dz}{\Delta r}$$

$$\sim \frac{\mu_0}{4\pi} d\,\frac{-q_0\omega\sin(\omega(t-\frac{r}{c}))\,\hat{z}}{r}$$

$$= \boxed{-\frac{\mu_0 q_0 d\,\omega}{4\pi r}\sin(\omega(t-\frac{r}{c}))\,\hat{z}}$$

Summary:

$$\boxed{\begin{aligned} V(\vec{r},t) &\sim \frac{-p_0\omega}{4\pi\varepsilon_0 c}\frac{\cos\theta}{r}\sin(\omega(t-r/c)) \\ \vec{A}(\vec{r},t) &\sim -\frac{\mu_0 p_0\omega}{4\pi}\frac{1}{r}\sin(\omega(t-r/c))\,\hat{z} \end{aligned}}$$

Dipole Radiation Fields

$$\vec{E} = -\nabla V - \partial \vec{A}/\partial t$$

$$\nabla V = \partial V/\partial r \,\hat{r} + \frac{1}{r}\,\partial V/\partial\theta\,\hat{\theta}$$

$$= \frac{-p_0 w}{4\pi\varepsilon_0 c}\Big[\cos\theta\Big\{-\frac{1}{r^2}\sin(w(t-r/c)) - \frac{w}{rc}\cos(w(t-r/c))\Big\}\hat{r} - \frac{\sin\theta}{r^2}\sin(w(t-r/c))\,\hat{\theta}\Big]$$

$$\sim \frac{p_0 w^2}{4\pi\varepsilon_0 c^2}\,\frac{\cos\theta}{r}\cos(w(t-r/c))\,\hat{r}$$

$$\partial\vec{A}/\partial t = -\frac{\mu_0 p_0 w^2}{4\pi r}\cos(w(t-r/c))\,\hat{z}$$

$$= -\frac{\mu_0 p_0 w^2}{4\pi r}\cos(w(t-r/c))(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta})$$

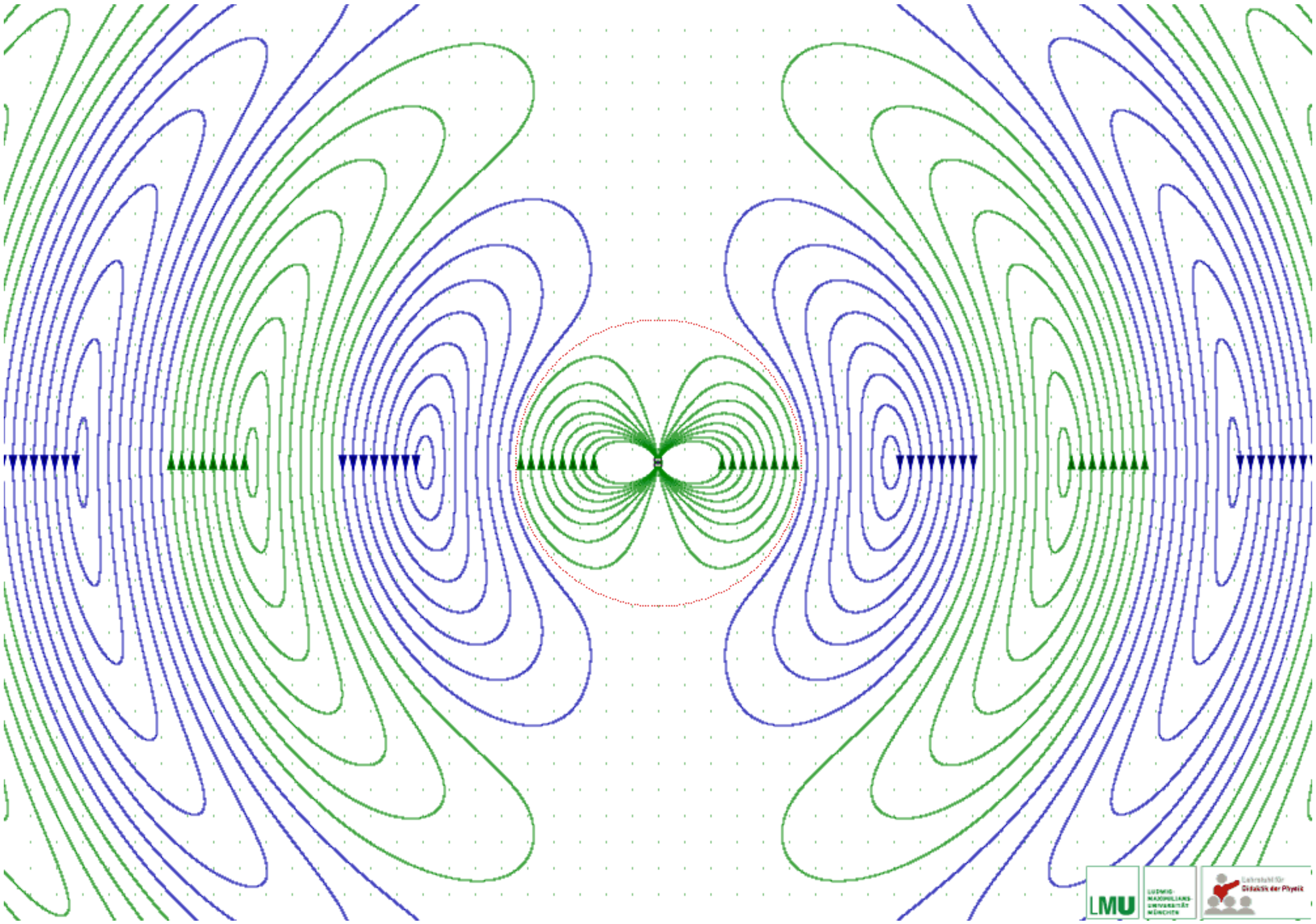
$$\Rightarrow \boxed{\vec{E} \cong -\frac{\mu_0 p_0 w^2}{4\pi}\,\frac{\sin\theta}{r}\cos(w(t-r/c))\,\hat{\theta}}$$

$$\vec{B} = \nabla\times\vec{A} = \frac{1}{r}\Big[\partial/\partial r(rA_\theta) - \partial A_r/\partial\theta\Big]\hat{\varphi}$$

$$= -\frac{\mu_0 p_0 w^2}{4\pi r}\Big\{\frac{w}{c}\sin\theta\cos(w(t-r/c)) + \frac{\sin\theta}{r}\sin(w(t-r/c))\Big\}\hat{\varphi}$$

$$\Rightarrow \boxed{\vec{B} \cong -\frac{\mu_0 p_0 w^2}{4\pi c}\,\frac{\sin\theta}{r}\sin(w(t-r/c))\,\hat{\varphi}}$$

Electric Dipole Radiation



Electric Dipole Radiation